Interference in the Photodetachment of a Negative Ion near Two Perpendicular Elastic Surfaces

Dehua Wang

School of Physics and Optoelectronic Engineering, Ludong University, Yantai 264025, China
(Received May 7, 2014; Revised July 31, 2014)

The interference in the photodetachment of a negative hydrogen ion near two perpendicular elastic surfaces has been studied using the theoretical imaging method. Firstly, we analyze the images of the detached electron caused by the surfaces. Then we put forward an analytic formula for calculating the detached electron probability density and electron flux distribution at a given observation plane. Due to the influence of the two perpendicular elastic surfaces, four detached electron waves which propagate from the ion source to the same point on the detector plane will interfere and lead to an interference pattern in the electron’s probability density and flux distribution. Our calculation results suggest that the electron flux distributions on the detector plane are not only related to the electron energy, but also related to the position of the ion relative to the surfaces. Compared to the photodetachment of a H\(^-\) ion near one elastic surface, the oscillatory structure in the flux distribution of our system becomes much more complicated. This study provides a new understanding of the photodetachment process of negative ions in the vicinity of surfaces and may guide future research in photodetachment microscopy experiments.

DOI: 10.6122/CJP.20140901C   PACS numbers: 32.80.Gc, 31.15.xg, 34.35.+a

I. INTRODUCTION

Over the past several decades much research has studied the photodetachment of negative ions in different external environments. As a simple model system, the photodetachment of a H\(^-\) ion has been studied both theoretically and experimentally. From 1987 onwards, people began to study the photodetachment of H\(^-\) ions in different electric and magnetic fields and found oscillatory structures appearing in the photodetachment cross section [1–10]. Du and Delos have put forward a semiclassical closed orbit theory to analyze these oscillatory structures [11]. According to quantum mechanics, the photodetachment cross section is related to the outgoing electron flux across a large enclosure where the bound H\(^-\) ion sits, therefore the oscillation in the cross section is a consequence of the oscillation in the detached electron flux distribution [12, 13]. This predication has already been observed in photodetachment microscopy experiments [14, 15]. Recently, with the development of surface physics, many researchers found that the surface can affect the photodetachment of negative ions significantly. Firstly, Yang et al. applied closed orbit theory to study the pho-

*Electronic address: lduwdh@163.com
toddetachment of H\(^{-}\) near an elastic surface [16–18]. Subsequently, Afaq and Du developed a theoretical imaging method and applied it to study the same system [19]. The correspondence of the above two results suggests the correctness of the theoretical imaging method. Later, Wang and Yang et al. studied the photodetachment of a H\(^{-}\) ion near two parallel elastic surfaces [20, 21]. Then, what will happen if we change the geometric configuration of the two surfaces? In Ref. [22], Zhao and Du studied the escape of quantum particles inside an open cavity in the shape of a wedge. They calculated the total escape rate of the detached electron in the wedge cavity. As for the photodetached electron flux distribution near two surfaces, no one has given a report. In this paper, we study the photodetachment of a H\(^{-}\) ion near two perpendicular elastic surfaces for the first time. In our system, the interaction potential between the detached electron and the surfaces has been neglected and the collision of the electron with the surface is elastic. According to the theoretical imaging method [19], when the detached electron is put near two perpendicular surfaces, three images of the detached electron will appear. Therefore, the theoretical treatment of our system is more difficult compared to the case of the photodetachment of H\(^{-}\) near one elastic surface. Our calculation results suggest that oscillatory structures appear in the detached electron flux distribution, which is a consequence of the interference effect between four electron waves propagating from the ion source and arriving at the same point on the detector plane. Although no experiments on the photodetachment of a negative ion near two surfaces have been carried out so far, our predictions show that there are some interesting phenomena here, and we hope that our calculations may guide future experimental research.

This paper is organized as follows: In Section II, we describe the images of the detached electron and analyze the propagation of the electron waves near two perpendicular surfaces. Then we derive the formula for calculating the detached electron’s probability density and flux distribution. Some numerical results and discussion are presented in Section III. Finally, Section IV presents some of the conclusions of this paper. Atomic units are used throughout this work unless indicated otherwise.

II. THEORETICAL METHOD

The schematic diagram of the system is shown in Fig. 1. The H\(^{-}\) ion sits at the origin. Two perpendicular elastic surfaces are placed in the y-z plane, one is parallel to the y-axis and the other is parallel to the z-axis. The position of the H\(^{-}\) ion relative to the two surfaces is determined by the angle \(\alpha\) and the distance \(d\). The detector plane is placed perpendicular to the z-axis and the laser light is polarized along the +z direction. When a laser light is applied to the H\(^{-}\) ion, it may absorb a photon. When the photon energy is larger than the binding energy of the H\(^{-}\) ion, the active electron is detached. According to the theoretical imaging method, for each charged particle near an elastic surface, there is a mirror image [19]. In our system, there are two perpendicular elastic surfaces and the number of the images is three. The three images are denoted as 2, 3, 4. For a common format, we denote the detached electron as 1. The detached electron and their images make
up a rectangle.

FIG. 1: Schematic plot for the detached electron waves propagation near two perpendicular elastic surfaces. The solid circle at the origin denotes the detached electron and the three dotted circles denote the images of the detached electron. Two perpendicular elastic surfaces are parallel to the \( y \)-axis and \( z \)-axis. The position of the detached electron relative to the surfaces is denoted by the distance \( d \) and angle \( \alpha \). The observation plane is placed perpendicular to \( z \)-axis and the laser light is polarized along the \( +z \) axis. Four outgoing waves \( \psi_1^+, \psi_2^+, \psi_3^+, \) and \( \psi_4^+ \) propagate away from the detached electron and its images.

The detached electron propagates in the form of an outgoing wave. Sufficiently far from the \( \text{H}^- \) ion, the wave propagates according to semiclassical mechanics, and it is correlated with classical trajectories. Due to the influence of the two perpendicular surfaces, four detached electron waves can reach the same point on the detector plane. The first electron wave propagates directly toward the detector plane, which is denoted as \( \psi_1^+ \). The second electron wave propagates toward the horizontal surface first, after being reflected by this surface, it then travels toward the detector plane. This electron wave can be considered as propagating from the “image 2” of the detached electron, which is denoted as \( \psi_2^+ \). The third electron wave propagates toward the vertical surface first and then reflects and finally reaches the detector plane. This electron wave can be considered as propagating from the “image 3” of the detached electron, which is denoted as \( \psi_3^+ \). The fourth electron wave propagates toward the horizontal surface first, after being reflected by this surface, it then travels toward the vertical surface and then reflects and finally reaches the detector plane. This electron wave can be considered as propagating from the “image 4” of the detached electron, which is denoted as \( \psi_4^+ \).

Using \((r, \theta, \phi), (r_2, \theta_2, \phi_2), (r_3, \theta_3, \phi_3), \) and \((r_4, \theta_4, \phi_4)\) as the spherical coordinates of
the detached electron and its images, we have [19]:

\[ \psi_1^+(r, \theta, \phi) = U(k, \theta, \phi) \frac{e^{ikr_1}}{r}, \]

(1)

\[ \psi_2^+(r_2, \theta_2, \phi_2) = U(k, \theta_2, \phi_2) \frac{e^{ikr_2}}{r_2}, \]

(2)

\[ \psi_3^+(r_3, \theta_3, \phi_3) = U(k, \theta_3, \phi_3) \frac{e^{ikr_3}}{r_3}, \]

(3)

\[ \psi_4^+(r_4, \theta_4, \phi_4) = U(k, \theta_4, \phi_4) \frac{e^{ikr_4}}{r_4}. \]

(4)

The factors \( U(k, \theta, \phi) \) for the laser polarization along the +z axis can be written as:

\[ U(k, \theta, \phi) = C \cos \theta, \]

(5)

where \( C = \frac{4kB_i}{(k^2 + k^2)^2} \), \( k = \sqrt{2E} \) is the momentum of the detached electron and \( E \) is the energy. \( B = 0.31552, k_b = \sqrt{2E_b}, \) with \( E_b = 0.754 \) eV being the binding energy of the detached electron.

\( \mu_j \) is the Maslov index. After each reflection by the elastic surface, there is a phase loss \( \pi \) of the wave function. Thus \( \mu_1 = 0, \mu_2 = \mu_3 = 1, \mu_4 = 2. \)

The total wave function of the detached electron at a given point on the detector plane can be written as a linear combination of the above four detached electron’s wave function:

\[ \psi_f = \psi_1^+ + \psi_2^+ + \psi_3^+ + \psi_4^+. \]

(6)

Substituting Eqs. (1–4) into the above formula, we get

\[ \psi_f = C \left\{ \cos \theta \frac{e^{ikr}}{r} + \cos \theta_2 \frac{e^{ikr_2}}{r_2} + \cos \theta_3 \frac{e^{ikr_3}}{r_3} + \cos \theta_4 \frac{e^{ikr_4}}{r_4} \right\}. \]

(7)

Since the distance between the observation plane and the H\(^-\) ion is much larger than the distance between the H\(^-\) ion and the two surfaces (\( L \gg d \)), the above formula can be simplified further. We use \( r_2 \approx r + 2d \cos \alpha \cos \theta,\ r_3 \approx r + 2d \sin \alpha \sin \theta, \) and \( r_4 \approx r + 2d \cos(\alpha - \theta) \) for the phase terms, and in the denominator use \( r_2 \approx r_3 \approx r_4 \approx r. \) The outgoing angles can be approximated as \( \theta_2 \approx \pi - \theta, \theta_3 \approx \theta, \theta_4 \approx \pi - \theta. \)

With these approximations, Equation (7) becomes

\[ \psi_f = C \frac{e^{ikr}}{r} \cos \theta \left\{ 1 + e^{i2kd \cos \alpha \cos \theta} - e^{i2kd \sin \alpha \sin \theta} - e^{i2kd \cos(\alpha - \theta)} \right\} 
   = -4iC \frac{e^{ikr}}{r} \cos \theta e^{ikd \cos(\alpha - \theta)} \cos(kd \cos \alpha \cos \theta) \sin(kd \sin \alpha \sin \theta). \]

(8)
According to quantum mechanics, the electron probability density distribution on the detector plane is

\[ P = |\psi_f|^2 = 16 |C|^2 \frac{\cos^2 \theta}{r^2} \cos^2 (kd \cos \alpha \cos \theta) \sin^2 (kd \sin \alpha \sin \theta). \]  

(9)

For a given detector plane perpendicular to the z-axis, the electron flux on that plane is defined as \( \vec{j} = \frac{i}{2} (\psi_f \nabla \psi_f^* - \psi_f^* \nabla \psi_f) \). The detached electron flux in the radial direction at large \( r \) is

\[ j_r = \frac{i}{2} \left( \psi_f \frac{\partial \psi_f^*}{\partial r} - \psi_f^* \frac{\partial \psi_f}{\partial r} \right). \]  

(10)

Substituting Eq. (8) into the above equation, we get

\[ j_r = 16 |C|^2 \frac{k \cos^2 \theta}{r^2} \cos^2 (kd \cos \alpha \cos \theta) \sin^2 (kd \sin \alpha \sin \theta). \]  

(11)

In order to obtain the flux on the observation plane, we project the radial flux in Eq. (11) in the normal direction of the plane: \( j_z = j_r \cos \theta \). We get

\[ j_z = 16 |C|^2 \frac{k \cos^3 \theta}{r^2} \cos^2 (kd \cos \alpha \cos \theta) \sin^2 (kd \sin \alpha \sin \theta). \]  

(12)

From the above equations, we find that the detached electron’s probability density and the detached electron flux distribution on the observation plane are not only related to the electron’s energy, but are also related to the ion-surface distance.

## III. RESULTS AND DISCUSSION

In the following calculations, we keep the distance from the ion to the observation plane, \( L = 10000 \) a.u., unchanged. Using Eq. (9) and Eq. (12), we calculate the detached electron’s probability density and the electron flux distribution on the observation plane. We choose the detached electron’s energy to be \( E = 2.0 \) eV, the distance \( d = 200 \) a.u., and \( \alpha = \pi/4 \). The results are given in Fig. 2. Fig. 2(a) is the electron probability density distribution curve, while Fig. 2(b) is the electron flux distribution curve. We found that oscillatory structures appear in the above two figures, and these structures can be well understood by four path interference. These four distinct paths of detached electron waves interfere with each other and consequently interference patterns appear. The peaks are caused by the constructive interference of the detached electron waves, and the valleys are caused by the destructive interference of the detached electron waves. By comparing the above two figures, we find that the electron probability density and the electron flux distributions have nearly the same oscillatory structures. Therefore, we only calculate the electron flux distributions in the following study.

Firstly, we keep the distance \( d = 200 \) a.u. and \( \alpha = \pi/4 \) unchanged, and show how the detached electron flux distribution varies with the change of the detached electron’s
FIG. 2: The detached electron’s probability density and the electron flux distribution on the observation plane. The detached electron’s energy $E = 2.0$ eV, the distance $d = 200$ a.u., and $\alpha = \pi/4$.

(a) is the electron flux distribution curve; (b) is the electron probability density distribution curve.

... energy. The results are shown in Fig. 3. From this figure, we find that the interference patterns in the electron flux distributions vary with the electron energy. With an increase of the detached electron’s energy, the number of peaks in the electron flux distribution is also increased, and the oscillatory structures become much more complex. For comparison, we plot the detached electron flux distribution for the photodetachment of a H$^-$ ion near one elastic surface [19]. We find that the oscillating amplitude of the detached electron flux distribution near two perpendicular elastic surfaces is always larger than the case of the detached electron flux distribution near one elastic surface. In addition, the oscillatory structures become much more complex. The reason for this can be interpreted as follows: For the photodetachment of a H$^-$ ion near one elastic surface, there are only two detached electron waves arriving at the same point on the detector plane, and the oscillatory structure in the electron flux distribution is caused by the interference of the two detached waves; however, for the photodetachment of a H$^-$ ion near two perpendicular surfaces, the oscillatory structure in the electron flux distribution is caused by the interference of four detached electron waves. When the number of detached electron waves increases, the oscillatory amplitude in the electron flux distribution is enlarged.

Next, we fix the detached electron’s energy to be $E = 1.0$ eV and the distance from the ion to the intersection of the two surfaces as $d = 200$ a.u.. Then we change the angle $\alpha$ and show the variation of the detached electron’s flux distribution on the detector plane. The results are given in Fig. 4. We find as $0 < \alpha < \pi/4$ the amplitude of the oscillating peaks increases with an increase of the angle $\alpha$, as $\alpha = \pi/4$, the oscillating amplitude is
FIG. 3: The detached electron flux distribution on the observation plane for different electron energies: (a) $E = 0.1$ eV, (b) $E = 0.5$ eV, (c) $E = 0.8$ eV, (d) $E = 1.0$ eV. The distance $d = 200$ a.u. and $\alpha = \pi/4$. The black line is the electron flux distribution near two perpendicular elastic surfaces, while the dashed line is the electron flux distribution near one elastic surface.

The largest; as $\pi/4 < \alpha < \pi/2$, the amplitude of the oscillating peaks decreases again.

Thirdly, we keep the detached electron’s energy $E = 1.0$ eV and $\alpha = \pi/4$ unchanged. Then we show how the detached electron flux distribution varies with the distance $d$ from the ion to the two surfaces. The results are shown in Fig. 5. It is also clear from this figure that the number of the oscillating peaks in the electron flux distribution increases with an increase of the distance from the ion to the two surfaces.

Finally, in order to show the dependence of the electron flux distribution on the position of the negative ion clearly, we plot the three dimensional surface plot of the electron flux on the detector plane. The detached electron’s energy is $E = 2.0$ eV. The result are shown in Fig. 6. From this figure, we find that at a given point on the detector plane, the detached electron flux is varied with the position of the ion sensitively. For example, at a given point ($y = 600$, $z = 10000$) on the detector plane, when the distance $d = 290$ a.u., the detached electron flux is the largest; however, at $d = 830$ a.u., the detached electron flux is the smallest.

IV. CONCLUSIONS

In conclusion, we have studied the interference of the detached electron wave propagation near two perpendicular elastic surfaces on the basis of the theoretical imaging
FIG. 4: Variation of the detached electron flux distribution on the observation plane with the angle $\alpha$. The detached electron’s energy $E = 1.0$ eV, $d = 200$ a.u. (a) $\alpha = \pi/18$, (b) $\alpha = \pi/9$, (c) $\alpha = \pi/4$, (d) $\alpha = \pi/3$, (e) $\alpha = 5\pi/12$, (f) $\alpha = 17\pi/36$.

FIG. 5: Variation of the detached electron flux distribution on the observation plane with the distance $d$. The detached electron’s energy $E=1.0$ eV, $\alpha = \pi/4$. (a) $d = 100$ a.u., (b) $d = 200$ a.u., (c) $d = 300$ a.u., (d) $d = 500$ a.u.
method. Analytic formulas for calculating the detached electron probability density and electron flux distribution at a given observation plane have been derived and calculated. Our results suggest that due to the interference of the four detached electron’s waves which propagate from the source to the same point on the detector plane, oscillatory structures appear in the electron’s probability density and flux distribution. In addition, the detached electron flux distribution on the detector plane depends both on the electron energy and the distance from the ion to the two surfaces. Therefore, we can change the detached electron flux propagation by changing the electron’s energy and the ion-surface distance. Our study provides a new understanding on the electron wave propagation in the vicinity of surfaces and may help to guide future research in photodetachment microscopy experiments.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant Nos. 11374133 and 61307067), and a Project of Shandong Province Higher Educational Science and Technology Program of China (Grant No. J13LJ04).

References