Ground State Phase Diagram of an Extended Hubbard Chain with Spin-Dependent Interaction

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A half-filled one-dimensional (1D) extended Hubbard model ($U - V_{\perp} - V_{\parallel}$) with on-site and nearest-neighbor spin-dependent interactions is studied analytically. The ground state phase diagram is obtained in the weak-coupling regime by performing the bosonization and renormalization group methods. The result shows that the nearest-neighbor spin-dependent attraction influences the low-energy physics of the 1D correlated electron system, and changes the topological structure of the ground state phase diagram of the $U-V$ model.

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I. INTRODUCTION

The one-dimensional conventional Hubbard model ($U-V$) with on-site interaction $U$ and nearest-neighbor Coulomb interaction $V$ has been intensively investigated for years [1–24]. As an important model, it has accounted for some unusual properties of low-dimensional correlated electronic systems related to organic Bechgaard salts [1], conducting polymers [2], and the high-$T_c$ cuprate superconductors [3], especially in the case of a commensurate filling band. Generally, the exact solution of the $U-V$ model is not readily derived, except for the Bethe-ansatz solution of the special case $V = 0$ [4]. Therefore, much attention has been attracted to explore the low-energy physics of these systems, and much effort has been devoted to investigating the corresponding ground state phase diagram both analytically [5, 6] and numerically [7, 8]. Moreover, most studies of the $U-V$ model focused on the repulsive interactions. On the other hand, the $U-V$ model with attraction represents another realistic situation, reflecting the unconventional screening effect on negative charge [see Fig. 1] [22]. Supposed a negative charge locates at the coordinate origin, it may attract so many positive charges that the magnitude of the positive trapped charges is much higher than that of the negative charge, and therefore other electrons away from the coordinate origin are attracted by the negative charge. Such an unconventional screening effect can be at least realized in some crystal structures when the oscillating frequency of the negative charge is lower than the resonance frequency of the lattice. Such an attractive interaction between electrons is indeed possible only for those renormalized “effective” electrons [22, 23]. Indeed, the extended Hubbard chain with $V < 0$ has been investigated

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FIG. 1: Response to a negative charge located at the coordinate origin with (a) the conventional screening effect, and (b) the unconventional screening effect. The direction of the electronic field is denoted by arrows [22].

to interpret the competition between the spin-density-wave (SDW) and superconducting correlations in the TMTSF system [6, 11, 12, 24], where metallic Luttinger-liquid (LL) and singlet-superconductivity (SS) correlations may be realized. The dynamical generation of an energy-gap in the spin excitation spectrum leads to the suppression of the SDW and LL correlations in the case of $U < 2V$ [6]. Whereas the regime with both massless spin and charge excitations corresponds to the realization of dominated LL ordering for $|U| < -2V$, where the SS correlation with a weak logarithmic correction to the power law decay at large distances is suppressed [6, 11, 12].

The earlier investigations of the $U$-$V$ model with nearest-neighbor interactions were based mainly on the assumption with spin-SU(2) symmetry preserved in the models. A recent weak-coupling study of the $U$-$V_\perp$-$V_\parallel$ model with inter-site spin-dependent repulsion shows that due to spin-SU(2) symmetry breaking, the effect of spin-dependent repulsion leads to the removal of the degeneracy of SDW orderings and to the occurrence of semi-gapped transverse bond-spin-density-wave (bd-SDW$^\pm$) orderings, but to the absence of a long-range ordered bond-charge-density-wave (bd-CDW) correlation in the ground state [21]. Typically, the sign of the Coulomb-driven interactions is repulsive $U, V(V_\perp, V_\parallel) > 0$. However, from the viewpoint of a theoretical study, below we will deal with these interaction parameters as phenomenological ones, and extend these interactions to a general case including $U, V_\perp, V_\parallel < 0$. The model considered here is defined on a single-band tight-binding chain, and the corresponding Hamiltonian has the same form as that in Ref. [21]
in terms of lattice fermion operators,
\[ H = -t \sum_{i,\alpha} (c_{i+1,\alpha}^\dagger c_{i,\alpha} + H.c.) + \frac{U}{2} \sum_{i,\alpha} c_{i,\alpha}^\dagger c_{i,\alpha} c_{i,-\alpha}^\dagger c_{i,-\alpha} \]
\[ + V_\parallel \sum_{i,\alpha} n_{i,\alpha} n_{i+1,\alpha} + V_\perp \sum_{i,\alpha} n_{i,\alpha} n_{i+1,-\alpha}, \]
where all the operators and indices have their usual meanings \[17, 21, 24–30\]. \( t \) is the nearest-neighbor hopping amplitude, and within the one-band assumption two- and many-body hopping terms are neglected, since they do not change the properties of the system qualitatively \[30\]. \( U \) denotes the on-site interaction strength. \( V_\parallel = \langle i, i+1 | \Lambda_{\alpha,\alpha} | i, i+1 \rangle \), and \( V_\perp = \langle i, i+1 | \Lambda_{\alpha,-\alpha} | i, i+1 \rangle \) correspond to nearest-neighbor interaction matrix elements with parallel spin and opposite spin, respectively. The operator \( \Lambda_{\alpha,\beta} \) describes the interaction between electrons \[21\]. In a realistic system, it is proposed that the \( V_\parallel \) attraction be closely related to conducting effect and that the \( V_\perp \) attraction be related to intermediate effect \[24, 31\].

In this paper, we discuss the question whether the usual CDW (charge-density-wave), SDW, and bd-SDW orderings are stable against such a spin-dependent attraction, and whether the superconducting ordering TS (triplet-superconductivity) and/or SS absent from the repulsive interactions can be realized, exploring further effects of the spin-dependent attraction on the topological structure of the ground-state phase diagram of the conventional \( U-V \) model. We restrict ourselves to the weak-coupling regime \(|U|, |V_\parallel|, |V_\perp| \ll t\) so that the bosonization technique together with the renormalization group (RG) analysis may be safely applied. As will be displayed below, the triplet superconducting phases TS\(^0\) (in the \( S_z=0 \) channel) and TS\(^\pm\) (in the \( S_z=\pm1 \) channels) are realized in the case of \( V + U/2 < 0 \) with \( U > 0 \), while an insulating SDW\(^\pm\) (transverse SDW) phase is absent in favor of an SS ordering for \( U < 0 \). The interplay between on-site interaction and nearest-neighbor spin-dependent attraction will provide a new competing mechanism of superconducting and insulating orderings, since it is different from the cases of ferromagnetic spin anisotropy \[27, 32\], correlated hopping \[25, 26, 33\], and nearest-neighbor spin-dependent repulsion \[21\]. The result shows that the nearest-neighbor spin-dependent attraction influences the low-energy physics of the 1D correlated electron system, and changes the topological structure of the ground state phase diagram of the \( U-V \) model.

II. CONTINUUM-LIMIT THEORY AND BOSONIZATION

We first establish the bosonized version of the model Hamiltonian (1) at half filling. Although this procedure has been reviewed in many places \[6, 20, 27, 34, 35\], for clarity we briefly sketch the most important points.

The bosonization approach is a most convenient tool for analyzing a low-dimensional electron system. In one dimension, the low-energy physics is governed mainly by the states
near the Fermi points, $-k_F$ (left) and $+k_F$ (right). Around these points we can introduce two species of chiral fermions, left-moving $\psi_{\alpha,-}$ and right-moving $\psi_{\alpha,+}$, which correspond to the states near $-k_F$ and $+k_F$, respectively. Thus we can obtain two branches of linear spectra $E(k) = v_F(\pm k - k_F)$, where $v_F$ is the Fermi velocity. At half-filling $k_F = \pi/2a$ and $v_F = 2ta$, with $a$ being the lattice constant. These low-energy excitations are of particle-hole pair ones, falling into either left-moving or right-moving sectors, respectively. In the vicinity of the Fermi level, the initial fermion is explicitly decomposed into

$$c_{j,\alpha} \to \psi_{j,\alpha,-} + \psi_{j,\alpha,+}. \quad (2)$$

When reformulated in terms of the chiral fermions $\psi_{j,\alpha,\pm}$, the interaction between electrons is classified as forward-scattering, backward-scattering, and umklapp-scattering (only for the commensurability filling case) processes.

On the other hand, the bosonization procedure depends on the continuum-limit theory and is appropriate to deal with a 1D electron system where the interactions are small compared to the Fermi energy. In the continuum limit $a \to 0$, the lattice number $N \to \infty$ (with $Na$ finite), the continuous chiral fermion fields $\psi_{\alpha,\pm}(x)$ are introduced by making the replacement

$$\psi_{j,\alpha,\pm}/\sqrt{a} \to \psi_{\alpha,\pm}(x) \quad (x = ja). \quad (3)$$

Behind the bosonization treatment, the right-moving and left-moving electron fields $\psi_{\alpha,\pm}(x)$ can be defined by chiral bosonic fields $\varphi_{\alpha,\pm}(x)$ in a standard way [28]:

$$\psi_{-\alpha}(x) = \frac{1}{\sqrt{2\pi a}} e^{-ik_F x - i\varphi_{-\alpha}(x)}, \quad \psi_{+\alpha}(x) = \frac{1}{\sqrt{2\pi a}} e^{ik_F x + i\varphi_{\alpha}(x)}. \quad (4)$$

In terms of $\psi_{\alpha,\pm}(x)$, the charge ($c$) and spin ($c$) bosonic fields are introduced as

$$\phi_{c,r} = \frac{\varphi_{r\uparrow} + \varphi_{r\downarrow}}{2}, \quad \phi_{s,r} = \frac{\varphi_{r\uparrow} - \varphi_{r\downarrow}}{2}. \quad (5)$$

Finally we define a pair of conjugate scalar fields

$$\phi_{\alpha}(x) = \varphi_{+\alpha}(x) + \varphi_{-\alpha}(x), \quad \theta_{\alpha}(x) = \varphi_{+\alpha}(x) - \varphi_{-\alpha}(x), \quad (6)$$

which satisfy the commutation relation

$$[\phi_{\alpha}(x), \theta_{\alpha'}(x')] = i\delta_{\alpha,\alpha'} \text{sgn}(x - x'). \quad (7)$$

We rescale the bosonic phase fields after substituting the fermionic model (1). Thus a bosonized Hamiltonian for low-energy state with the charge, spin, and charge-spin coupling parts is given by

$$H = H_c + H_s + H_{cs}, \quad (8)$$
with
\[ H_c = \frac{v_c}{2\pi} \int dx [(\partial_x \phi_{c,-})^2 + (\partial_x \phi_{c,+})^2] - \frac{g_c}{2\pi^2} \int dx (\partial_x \phi_{c,+}) (\partial_x \phi_{c,-}) \]
\[ + \frac{g_c}{2a^2\pi^2} \int dx \cos 2\phi_c, \]
\[ H_s = \frac{v_s}{2\pi} \int dx [(\partial_x \phi_{s,-})^2 + (\partial_x \phi_{s,+})^2] - \frac{g_s}{2\pi^2} \int dx (\partial_x \phi_{s,+}) (\partial_x \phi_{s,-}) \]
\[ + \frac{g_s}{2a^2\pi^2} \int dx \cos 2\phi_s, \]
\[ H_{cs} = -\frac{g_{cs}}{2a^2\pi^2} \int dx \cos 2\phi_c \cos 2\phi_s, \]
in which the total bosonic phase fields \( \phi_{s/c} = \phi_{s/c,+} + \phi_{s/c,-} \). The weak-coupling “gology” notation adopted here is analogous to other conventional Hubbard models [6, 12, 20–27, 32]. Up to the lowest orders in \( U, V_\perp, \) and \( V_\parallel, \) the Luttinger parameters are \( g_\sigma = (U + 2V_\perp - 4V_\parallel) a, g_\rho = -(U + 2V_\perp + 4V_\parallel) a, \) describing low-energy physics of the collective spin- and charge-mode, accordingly. The renormalized charge and spin velocities are \( v_s = a[2t - (U + 2V_\perp - 4V_\parallel)/2\pi], v_c = a[2t + (U + 2V_\perp + 4V_\parallel)/2\pi], \) respectively. The coefficients \( g_c \) and \( g_s \) are endowed with \( g_c = -(U - 2V_\perp) a, g_s = (U - 2V_\perp) a, \) corresponding to the amplitude of the Umklapp and backward scattering of electrons with opposite spins, respectively. \( g_{cs} \) represents the charge-spin coupling term, which will be ignored as a strongly irrelevant operator at weak coupling, as in the case of the works [17, 21, 26–30].

### III. THE RG ANALYSIS

With the charge-spin separation hypothesis, the \( t-U-V_\perp-V_\parallel \) model is completely controlled by four independent coupling constants: \( g_c, g_\rho, g_\sigma, \) and \( g_s \). The RG analysis is appropriate for studying the relative importance of these couplings. The low-energy physics of model (1) is determined by the scaling behavior of these couplings during the scale transformations of the cutoff \( a \rightarrow ae^{dl} \). We focus on the following coupled RG equations [20, 28, 29, 35, 36],
\[ \frac{dg_c}{dl} = -2g_\rho g_c, \quad \frac{dg_\rho}{dl} = -2g_c^2, \]
\[ \frac{dg_s}{dl} = -2g_\sigma g_s, \quad \frac{dg_\sigma}{dl} = -2g_s^2, \]
where the running dimensionless coupling constants \( \tilde{g}_i(l) = \frac{g_i(l)}{\sqrt{\beta(l)}} \) (\( i \) denotes \( c, \rho, \sigma, \) or \( s \)). The solutions to the two pairs of scaling equations establish the RG flows illustrated in Fig. 2. The separatrices \( \tilde{g}_s = \pm \tilde{g}_\sigma \) and \( \tilde{g}_c = \pm \tilde{g}_\rho \) divide the flow diagrams into two qualitatively different sectors, respectively for the spin and charge modes:

1. For \( \tilde{g}_\sigma \geq |\tilde{g}_s| (\tilde{g}_\rho \geq |\tilde{g}_c|), \) the system is dominated in the weak-coupling regime. The perturbative mass \( \tilde{g}_s (\tilde{g}_c) \) connecting with the cosine term in the Hamiltonian (8) flows
FIG. 2: Renormalized flow diagrams of Eqs. (12)–(13) corresponding to (a) spin channel, and (b) charge channel. The direction of flow upon the increasing length scales is denoted by arrows.

to 0 and the spin (charge) excitation is gapless. In this situation the system reduces to the Gaussian model describing a free scalar field.

(ii) For \( \tilde{g}_\sigma < |\tilde{g}_s| \) (\( \tilde{g}_\rho < |\tilde{g}_c| \)), the system flows to the strong-coupling regime and the mass \( \tilde{g}_s(\tilde{g}_c) \to \pm \infty \). A relevant perturbation is typically the source of a finite gap in the low-energy excitation spectrum, so the excitation is gapped. In this case, depending on the sign of initial couplings \( g_c(s) \), the bosonic fields \( \phi_s (\phi_c) \) get ordered with the vacuum expectation values

\[
\langle \phi_s(c) \rangle = \begin{cases} 
\pi/2 & (g_{s(c)} > 0), \\
0 & (g_{s(c)} < 0).
\end{cases}
\]

The RG analysis tells that the length scale is increased until a fixed point or singularity is reached. Fixed-point physics helps to understand what happens in the vicinity of the fixed point.

We begin with the RG scaling behavior of the spin sector. When \( \tilde{g}_\sigma < |\tilde{g}_s| \), the spin excitation goes to the strong-coupling (SC) regime, and \( \tilde{g}_s \) tends either to +\( \infty \) or to −\( \infty \) relying on the sign of the initial coupling \( g_s(0) \). This leads to the dynamical generation of an energy gap in the spin excitation spectrum. The low symmetry pushes the RG flows away from the the separatix \( \tilde{g}_s = \tilde{g}_c \) with SU(2)-symmetry. The phase field \( \phi_s \) is locked at the average amplitude \( \pi/2 \) (mod \( \pi \)) for \( V_\perp < \min\{V_\parallel; \frac{U}{2}\} \), and thus we obtain the Triplet\(^0\) behavior. While for \( \frac{U}{2} < \min\{V_\parallel; V_\perp\} \), the phase field \( \phi_s \) gets the expectation value \( \langle \varphi_s \rangle = 0 \) (mod \( \pi \)), and the spin ordering is Singlet-like [see Fig. 2 (a)]. In the other cases, \( V_\perp \in (V_\parallel, \frac{U}{2}) \) or \( \frac{U}{2} \in (V_\parallel, V_\perp) \), the RG flows scale to zero, \( \tilde{g}_s \to 0 \). Consequently, the spin-gap transitions have two branches. One is

\[
V_\perp = V_\parallel
\]

for \( g_s \geq 0 \), and the other is

\[
V_\parallel = \frac{U}{2}
\]
for \( g_s < 0 \). In these two cases, one obtains a free spin field for the gapless spin mode \((\Delta_s = 0)\). The corresponding physics at a large distance is equivalent to a free scalar Gaussian mode, where the fixed-point theory completely describes the infrared behavior in the spin sector [24–26].

The above RG analysis can be repeated in the charged part. If the parameter \( V_\perp \) satisfies the following condition,

\[
\frac{U}{2} < V_\perp < V_\parallel,
\]

or \( U \) fulfills

\[
2V_\perp < U < -(2V_\perp + 2V_\parallel),
\]

the RG flows scale toward \( \tilde{g}_c \to 0 \), where the fixed-point theory completely describes the infrared behavior in the charge sector [24–26], and the effect of the cosine term in Eq. (9) may be safely neglected. The charge mode is gapless, which denotes the behavior of the Doublet-like ordering. When the parameter \( V_\perp \) satisfies the condition \( V_\perp > \max\{-V_\parallel; \frac{U}{2}\} \), and \( \tilde{g}_c(l) \) flows to the SC fixed point, \( \tilde{g}_c(l) \to +\infty \). One obtains the Néel behavior and a phase field \( \varphi_c \) is locked at the expectation value \( \langle \varphi_c \rangle = \pi/2 \). When the parameter \( U \) satisfies the condition \( U > \max\{-2(V_\parallel + V_\perp); 2V_\perp\} \), \( \tilde{g}_c(l) \) flows to the opposite SC fixed point, \( \tilde{g}_c(l) \to -\infty \). One obtains the Dimer-type charge ordering and a phase fields \( \varphi_c \) ordered with \( \langle \varphi_c \rangle = 0 \) [see Fig. 2(b)]. The charge-gap transition also has two branches in the channel. One corresponding to \( g_c > 0 \) is

\[
V_\perp + V_\parallel = 0.
\]

The other corresponding to \( g_c < 0 \) is

\[
2V_\perp + 2V_\parallel + U = 0.
\]

**IV. WEAK-COUPLING GROUND-STATE PHASE DIAGRAM**

Now we are in the position to analyze the ground-state phase diagram of the model (1) in the weak-coupling regime. In order to characterize the dominated phases, a set of order parameters are introduced in the same way as in the works [6, 20, 25–30, 36, 37]. Parameters \( \mathcal{U}_{SDW^\pm} \) and \( \mathcal{U}_{SDW^z} \), are used to describe transverse and longitudinal site-located spin density waves, respectively,

\[
\mathcal{U}_{SDW^\pm} \propto \cos \phi_c e^{\pm i\theta_s},
\]

\[
\mathcal{U}_{SDW^z} \propto \cos \phi_c \sin \phi_s.
\]

Parameters \( \mathcal{U}_{bd-SDW^\pm} \) and \( \mathcal{U}_{bd-SDW^z} \) are used to represent transverse and longitudinal bond-located spin density waves, respectively,

\[
\mathcal{U}_{bd-SDW^\pm} \propto \sin \phi_c e^{\pm i\theta_s},
\]

\[
\mathcal{U}_{bd-SDW^z} \propto \sin \phi_c \sin \phi_s.
\]
Parameters $\tilde{U}_{CDW}$ and $\tilde{U}_{bd-CDW}$ describe the site- and bond-located charge density waves accordingly:

$$\tilde{U}_{CDW} \propto \sin \phi_c \cos \phi_s,$$

$$\tilde{U}_{bd-CDW} \propto \cos \phi_c \cos \phi_s.$$  \hfill (25)

Besides the above insulating phases, there are superconducting order parameters,

$$\tilde{U}_{TS\pm} \propto \exp(i\theta_c) \exp(i\theta_s),$$

$$\tilde{U}_{TS0} \propto \exp(i\theta_c) \sin \phi_s,$$

$$\tilde{U}_{SS} \propto \exp(i\theta_c) \cos \phi_s,$$

which are closely related to triplet- and singlet-superconductivity correlations.

With the above order-parameters and flows of Eqs. (12)–(13), we can make predictions for the weak-coupling model (1) based on effective field theory. Combining the spin-gap transitions Eqs. (15)–(16) with the charge-gap transition Eqs. (19)–(20), one obtains the following regimes of behavior. [see Fig. 3].

![Diagram](image-url)  
**FIG. 3:** Weak-coupling ground-state phase diagram of the half-filled 1D $t-U-V^\perp-V^\parallel$ model. (a) on-site repulsion $U > 0$, (b) on-site attraction $U < 0$.

**Sector A ($\frac{U}{2} < \min\{V^\perp, V^\parallel\}$, $V^\perp > -V^\parallel$):** Both charge and spin modes are gapped ($\Delta_c \neq 0$, $\Delta_s \neq 0$). The phase fields $\varphi_c$ and $\varphi_s$ take the vacuum expectation values $\langle \varphi_c \rangle = \frac{\pi}{2}$, $\langle \varphi_s \rangle = 0$ (mod $\pi$), respectively. The behavior is marked by the CDW ordering in the ground state. The true long-range correlation with a constant power-law decay at a large distance is displayed particularly for conventional Coulomb repulsion $V^\parallel=V^\perp=V > \frac{U}{2}$.

**Sector B ($V^\parallel < \min\{-V^\perp; \frac{U}{2}\}$, $V^\perp > \frac{U}{2}$):** $\langle \varphi_c \rangle = \frac{\pi}{2}$ and $K^+_s > 1$. In this sector, the charge excitation is massive ($\Delta_c \neq 0$), and the charge phase-field is locked at $\langle \varphi_c \rangle = \frac{\pi}{2}$ due to initial-value $g_c(0) > 0$, while the spin excitation is gapless ($\Delta_s = 0$). According to Eq. (23), the system is dominated in the bd-SDW$^\pm$ ordering, and all the others are completely suppressed in the ground state. With the fixed-point theory, the low-energy
physics in the massless spin channel is controlled by an finite effective value \( K^*_s > 1 \). The bd-SDW± correlation shows a power-law decay at a large distance,

\[
\langle \tilde{\mathcal{O}}_{\text{bd-SDW}^\pm}(y)\tilde{\mathcal{O}}_{\text{bd-SDW}^\pm}(y') \rangle \sim |y - y'|^{-\frac{1}{K^*_s}}.
\]  

(30)

Sector C (\(-\frac{U}{2} < V_\| < V_\perp < -V_\|\)): \( \Delta_c = 0, \Delta_s = 0, K^*_c > 1 \), and \( K^*_s > 1 \). The regime with both a charge- and spin-excitation gapless spectrum corresponds to the appearance of a Luttinger liquid state (TS±), showing a power-law decay at a large distance,

\[
\langle \tilde{\mathcal{O}}_{\text{TS}^\pm}(y)\tilde{\mathcal{O}}_{\text{TS}^\pm}(y') \rangle \sim |y - y'|^{-\frac{1}{K^*_s}}.
\]  

(31)

Sector D (\( V_\perp < \min\{V_\|; \frac{U}{2}\}, V_\| < -\frac{U}{2}\)): \( \Delta_c \neq 0, \Delta_s \neq 0, \langle \varphi_c \rangle = 0 \) and \( \langle \varphi_s \rangle = \frac{\pi}{2} \). One finds the order parameter \( \tilde{\mathcal{O}}_{\text{SDW}^s} \) takes the maximal value, and the ground state is characterized by a true long-range SDW± correlation with a power-law \( |y - y'| \sim \text{constant} \) decay at large distance. It is noted that the Hubbard chain is extended by turning on a spin-dependent \( V_\perp \) repulsion in this regime, which is analogous to an extension of the Hubbard chain (t-U-Jz).

Sector E (\( V_\perp < \min\{V_\|; \frac{U}{2}\}, V_\| > -\frac{U}{2}\)): \( \Delta_c \neq 0, \Delta_s = 0, \langle \varphi_c \rangle = 0 \) and \( \langle \varphi_s \rangle = 0 \). This sector is dual to Sector B, where the ordering of \( \langle \varphi_c \rangle \) with the average value \( \langle \varphi_c \rangle = 0 \) (mod \( \pi \)) leads to the realization of the dominated SDW± correlations showing a power-law decay at a large distance,

\[
\langle \tilde{\mathcal{O}}_{\text{SDW}^s}(y)\tilde{\mathcal{O}}_{\text{SDW}^s}(y') \rangle \sim |y - y'|^{-\frac{1}{K^*_s}}.
\]  

(33)

It should be noted that this phase occurs only for the dominating on-site repulsion, where the Hubbard chain is extended by turning on spin-dependent \( V_\perp \) repulsion, analogous to an extension of the Hubbard chain (t-U-Jz).

Next we examine the influence of the on-site attraction (\( U < 0 \)) on the ground-state phase diagram. We observe in Fig. 3(b) that apart from the same sectors as the case of on-site repulsion \( U > 0 \) denoted by A, B, C, D and E, there is a new Sector G replacing Sector F in the ground state phase diagram.

Sector G (\( \frac{-U}{2} < V_\| < V_\perp < -\frac{U}{2}, U < 0\)): \( \Delta_c = 0, \Delta_s \neq 0, \langle \varphi_s \rangle = 0 \) and \( K^*_c > 1 \). This regime is characterized by a gapped spin excitation and a gapless charge excitation, corresponding to the existence of singlet superconducting (SS) ordering with a power law decay at a large distance,

\[
\langle \tilde{\mathcal{O}}_{\text{SS}}(y)\tilde{\mathcal{O}}_{\text{SS}}(y') \rangle \sim |y - y'|^{-\frac{1}{K^*_c}}.
\]  

(34)

in the ground state. Such an ordering is only realized in the system with on-site attraction.
V. CONCLUSIONS AND DISCUSSION

In this paper we have studied a half-filled 1D $t$-$U$-$V_{\perp}$-$V_{\parallel}$ mode including a general interaction. The small values of the interaction parameters make it reasonable to perform the bosonization and RG calculation. The result further confirms that the nearest-neighbor spin-dependent interaction, irrespective of repulsion and attraction, gives rise to a non-critical behavior in the spin channel. This leads to the collapse of degenerate correlations corresponding to TS and SDW ordering, and hence TS$^\pm$ (LL) and SDW$^\pm$, TS$^0$ (LE) and SDW$^{\parallel}$ (Néel) can survive in the ground state. The considered model displays rich phase structures denoted by TS$^\pm$, bd-SDW$^\pm$, SDW$^\pm$, and SS orderings in the phase diagram, but the SDW$^\pm$ phase appears for $U > 0$ and the SS phase exists for $U < 0$.

Compared to the repulsive case ($U, V_{\perp}, V_{\parallel} > 0$), the nearest-neighbor spin-dependent interaction with $V_{\perp}$ and $V_{\parallel}$ attraction has important impacts on the topological phase diagram of the 1D extended Hubbard model. Our study will provide a useful understanding of the strongly correlated electron system.

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