Simulations of Spectral Asymmetries of Pure Two-Stream Waves in the Equatorial Electrojet

C. L. Fern, S. Y. Chou and F. S. Kuo

Institute of Space Science, National Central University,
Chung-Li, Taiwan 320, R.O.C.

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The nonlinear quasi-steady state of the Farley-Buneman two-stream wave observed in the E region ionosphere at altitudes in the range of 95-110 km is studied by numerical simulation. The behavior of this two-stream wave in the plane perpendicular to the Earth’s magnetic field is simulated with a two-dimensional two-fluid code in which the electron inertia is neglected while the ion inertia is retained. We found that, for the waves with wavelength well above (smaller than) 3 meters, the most intense waves propagate in the \( \mathbf{k} \cdot \mathbf{E}_0 < 0 ( \mathbf{k} \cdot \mathbf{E}_0 > 0 ) \) region. The simulation result of up-down asymmetry for the 3-meter wave is consistent with the radar observations. From the simulations, we conclude that the background polarization electric field driving the electrojet is responsible for the spectral asymmetries.

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PACS. 94.20.Rr – Interactions between waves and particles.

I. Introduction

At the altitude range of 95 \( \leftrightarrow \) 110 km in the E-region ionosphere, a large current in the magnetic equator called the equatorial electro-jet is created by a dynamic process involving atmospheric tides [1-3]. If the electron drift velocity relative to the ions exceeds a certain critical limit, a streaming instability called the Farley-Buneman or two-stream instability develops [4-6]. In nature this instability saturates via some nonlinear processes. So, nonlinear features necessarily play an important role in the behavior of these waves and their spectra. There have been several attempts to understand the nonlinear aspects by theoretical means [7-13], and by numerical simulations [14-22].

From the past studies of VHF radar observations [23-31], HF radar observations [32-34], and rocket in situ measurements [35-40], two types of radar echoes have been confirmed in the E region equatorial electro-jet, Type 1 radar echoes have a narrow spectrum with a Doppler shift corresponding to the ion acoustic velocity, and was first proposed to result from the modified two-stream plasma instability [4-6]. Type 2 radar echoes have broad and variable spectra peaking at a lower velocity near zero Doppler shift, and were considered to be due to the gradient drift plasma instability [41, 42]. There are plenty of results that the linear theory cannot account for, for example: (1) Type 1 waves (two-stream waves) were observed to travel at velocities close to the acoustic speed at all elevation angles [24, 26, 27, 35, 43], rather than proportional to the electron drift velocity times the cosine of the elevation angle as predicted by linear theory; (2) The two-stream waves observed by oblique radar beams [24, 44] as well as by vertical radar beams
[29, 30, 45-47] show asymmetries in spectra not predicted by the linear theory. In the daytime electro-jet, the upward irregularities are stronger than the downward irregularities, and the reverse is true in the nighttime electro-jet. This is the main subject of this study. (3) Power received from type 1 echoes drops 0.3 dB/degree as the elevation angle increases from 0° (horizontal) for 3 m waves [48].

There are two primary sources of type 1 echoes. First, in the upper electro-jet (105-115 km), where no appreciable density gradient exists, when the electron drift speed exceeds the threshold of the Farley-Buneman instability, a horizontally traveling pure two-stream wave will develop. These horizontally traveling two-stream waves can be observed by obliquely directed radar. Second, in the lower electro-jet, plasma gradients enable kilometer scale gradient-drift waves to develop at an electron drift speed far below the threshold of the Farley-Buneman instability, and the nonlinear development of these horizontally propagating large scale primary waves could excite the vertically traveling two-stream waves [49]. These secondary two-stream waves exist on the extrema of the gradient-drift wave, and can be observed by vertically directed radar.

The up-down spectral asymmetry of two-stream waves is a consistent feature of radar echoes obtained from the electro-jet at both vertical incidence and oblique incidence. It is an upward asymmetry in the daytime electro-jet and a downward asymmetry in the nighttime electro-jet. Cohen and Bowels [24] were the first, followed by Fejer et al. [29], to report the daytime asymmetry of the equatorial electro-jet echoes. That the nighttime asymmetry existed in the opposite direction was later shown in the high-resolution studies of the altitudinal structure of the equatorial electro-jet turbulence [30, 50] and in the radar observations of the Condor electro-jet campaign [45]. The vertical incidence radar echoes possibly come from the vertical traveling secondary two-stream waves or the vertical components of horizontally traveling pure two-stream waves. The vertical secondary two-stream waves, being probably generated from the two-step process [7], involve the large-scale horizontally traveling gradient-drift waves where the generated perturbed electric fields are large enough to drive the two-stream instability. Kudeki [49] had presented a theory involving nonlinear distortion of large-scale primary waves to explain the spectral asymmetry of the vertically traveling two-stream waves. One should not confuse the vertically traveling two-stream waves with the horizontally traveling two-stream waves: the former is a secondary wave while the latter is a primary wave. Janhunen [19] was the first, followed by Oppenheim and Otani [22], to obtain the spectral asymmetry of pure two-stream waves by numerical simulation.

There are a number of numerical simulations associated with pure two-stream waves. The earliest nonlinear simulations of pure two-stream waves were conducted by Newman and Ott [20], using pure fluid code and adopting wavelength dependent viscosity to model the kinetic Landau damping effect. Janhunen [19] used a two-dimensional explicit particle code to study the stabilization and quasi-steady state of the E region Farley-Buneman wave, and found the spatial power spectral asymmetry: that the most intense waves propagate in the linearly unstable sector $k \cdot E < 0$. Oppenheim and Otani [21] simulate the behavior of the Farley-Buneman instability in the plane perpendicular to the Earth's magnetic field, using a two-dimensional hybrid code, which models the electron dynamics as a fluid and the ion dynamics with a particle-in-cell approach. They confirmed Janhunen's result [19], that the most intense waves propagate at the linearly unstable sector $k \cdot E < 0$. Based on a 2-D simulation, Oppenheim and Otani [21] suggested that the nonlinear effects would firstly generate the distortion of primary waves, and then bend to develop a vertical component similar to the small vertical secondary waves, and finally cause the
primary waves to turn. This nonlinear development would produce the asymmetric propagation of pure two-stream waves. However, the spectral asymmetry obtained by the simulations of [19, 21] is opposite in direction to the radar observation. It is pointed out that the scale sizes of the principal two-stream waves in these simulations are too small to be resolved by radar. Therefore, the issue of spectral asymmetry requires further study.

In this paper, we will extend the simulations of pure two-stream waves into the larger-scale wavelength regime and a longer time scale compatible with the radar observation. We also consider the interactions of two primary waves. In our two-fluid simulations, the flux-corrected transport (FCT) algorithm is applied to prevent numerical errors in solving the continuity equation. The results of our 1D and 2D testing simulations are in good agreement with the dispersion relation of linear theory. The 2D simulations result in a similar nonlinear phenomenon as reported in [19, 21], and account for the relevant problem associated with the opposite asymmetry of pure two-stream waves. Our simulations show that the asymmetry propagation of shorter wavelength modes including the 3-meter wave is opposite to the longer wavelength modes. In addition, the simulation can be applied to check the theory of the resonant nonlinear coupling process of pure two-stream waves. The detailed results and discussions are presented in the 3rd and 4th sections following the numerical model and analysis methods presented in the section 2. A brief summary is given in section 5.

II. Numerical model and method of data analysis

II-1. Numerical model

Our numerical model is similar to that of [15] and [20]. The x-axis of the rectangular coordinate system points to the east, the y-axis points to the north and the z-axis points upward. The set of equations that are solved is a subset of the full governing equations. They consist of the continuity equation of the plasma,

$$\frac{\partial n}{\partial t} + \mathbf{r} \cdot (n \mathbf{v}_e) = 0;$$

(1)

the electron and ion velocity equations,

$$\mathbf{v}_e = \left(3 \frac{e}{m_e} \frac{e \mathbf{E}_0}{B_0} + \frac{K_B T_e}{e B_0} \frac{n}{n} + \frac{g_{en}^e}{e} \mathbf{V}_D \right)^{\frac{1}{3}} \left(3 \frac{e}{m_e} \frac{e \mathbf{E}_0}{B_0} + \frac{K_B T_e}{e B_0} \frac{n}{n} + \frac{g_{en}^e}{e} \mathbf{V}_D \right)^{\frac{1}{3}};$$

(2)

and the equation for the quasi-neutrality condition,

$$-\mathbf{d} \cdot \mathbf{j} = 0;$$

(4)

Here \( \mathbf{j} = n \mathbf{v}_e \mathbf{V}_D = n \mathbf{v}_i \mathbf{V}_D + \mathbf{V}_D \) is the current density and \( n \) is the number density of the plasma; \( K_B \) is the Boltzmann constant; \( g_{in} \), \( g_{en} \), \( i \), \( e \), \( T_i, T_e \), \( B \), \( \mathbf{E} \), \( \mathbf{V}_D \), \( \mathbf{v}_e \), \( \mathbf{v}_i \) are
the ion-neutral collision frequency, the electron-neutral collision frequency, ion gyro-frequency, electron gyro-frequency, ion temperature, electron temperature, magnetic field, first-order electric field, zero-order electron drift velocity, first-order electron and ion velocity respectively; \( \mathbf{v}_i \) and \( \mathbf{v}_e \) can be taken as \( \mathbf{v}_i = \mathbf{v}_f \); \( \mathbf{v}_e = \mathbf{v}_D + \mathbf{v}_e \). Substituting all these physical quantities into (4), we obtain

\[
\frac{1}{2} \dot{\mathbf{v}}_j + \frac{1}{2} \rho_j \mathbf{g} + \frac{q_j}{m_j} (E + \mathbf{v}_i \cdot \mathbf{B}) \cdot \mathbf{v}_j = \frac{1}{2} \rho_j n \mathbf{v}_j \cdot \mathbf{U} \tag{6}
\]

where \( \mathcal{A}(\mathbf{x}; z) \) is the potential function for the perturbed electric field \( \mathbf{E}^0 \), i.e., \( \mathbf{E}^0 = \mathcal{A} \).

Equations (2) and (3) are derived respectively from the momentum equation (6) by neglecting the gravitational term and the neutral wind effect, and setting the time derivative term of the electron species equal to zero, but retaining the time derivative term of the ion species,

\[
\frac{d\mathbf{v}_j}{dt} = \mathcal{A} \cdot \mathbf{p}_j + \frac{q_j}{m_j} (E + \mathbf{v}_j \cdot \mathbf{B}) \cdot \mathbf{v}_j = \frac{1}{2} \rho_j n \mathbf{v}_j \cdot \mathbf{U} \tag{6}
\]

The subscript \( j \) stands for the plasma species, \( \frac{1}{2} = n M_j \) is the mass density of species \( j \), \( M_j \) and \( q \) are the particle mass and charge respectively, and \( p_j \) is the pressure of species \( j \). Because the collision frequency of electrons and ions \( \gamma_{in} \) and \( \gamma_{en} \) given in the \( \mathbf{E} \) region are not very small, both \( \gamma_{en} = \gamma_{in} = i \) can’t be neglected. Also, in order to simplify the variables of our numerical model, we neglect the neutral wind and gravitational effects.

The zero-order eastward drift velocity \( \mathbf{v}_D \) of the electrons is driven by a vertically downward zero order polarization electric field \( \mathbf{E}_0 = \mathbf{E}^0 \cdot \mathbf{B}_0 \), where \( \mathbf{B}_0 = B_0 \mathbf{\mathbf{e}}_z \) is a constant magnetic field, of magnetude \( B_0 = 0.28 \) G, pointing to the north. The ion-neutral collision frequency \( \gamma_{in} \) and electron-neutral collision frequency \( \gamma_{en} \) are assumed to be constant in our simulation range with \( \gamma_{in} = 2.5 \times 10^3 \) s\(^{-1} \) and \( \gamma_{en} = 4.0 \times 10^4 \) s\(^{-1} \). The ion and electron temperatures are assumed to be \( 230 \) K. The background plasma can be regarded as uniform with a number density of \( 1.0 \times 10^{11} \) m\(^{-3} \). These background parameters are similar to those used in the past studies of the equatorial electrojet [7, 15].

The numerical computations were performed on a two-dimensional Cartesian mesh using 121 points in the \( x \) direction (east-west) and 61 points in the \( z \) direction (vertical). Periodic boundary conditions are imposed on both the electron density \( n \) and electric potential \( \mathcal{A} \) in the \( x \) and \( z \) direction. The flux-corrected transport (FCT) technique [51, 52] has been applied to carry out the time integration of the continuity equation (1). A detailed discussion of the application of the FCT technique to study ionospheric irregularities can be found in [53]. At \( t = 0 \), the electrons are set to move uniformly at drift speed \( \mathbf{v}_D \), and the ions assume a constant velocity \( \mathbf{v}_0 = -i \mathbf{E}_0 \mathbf{e}_0 \mathbf{B}_0 \), which is the steady state velocity of ions under the combined force from the electric field \( \mathbf{E}_0 \) and the ion-neutral collision (note that \( \gamma_{in} \mathbf{A} - i \)). Then a density perturbation with amplitude \( \pm n = n_0 \sin(kx) \) is superposed on the background density \( n_0 \), where \( k \) is the wave-number to be assigned. At each time step of the computation, the electron velocity at each grid
point is calculated from equation (2), and the ion velocity at each grid is obtained by solving the differential equation (3) using the 2nd order Runge-Kutta scheme. These velocities and densities at each grid point are substituted into (5) to solve for the electric potential \( \phi(x; z) \) using the successive-over-relaxation (SOR) technique. Then, the plasma density distribution \( n(x; z) \) at time \( t + \Delta t \) is calculated by the FCT scheme to complete one cycle of the computation. In order to guarantee the numerical accuracy, we set the relative error limit in the potential solver as small as \( 10^{-6} \). The simulation is called to stop whenever the relative error of any grid point fails to converge to within this error limit within 10000 steps of SOR iteration. This criterion sets a limit for the density gradient, beyond which the off-diagonal terms become so much larger than the diagonal terms in equation (5) that the SOR calculation fails to converge. That means that the simulation will be stopped when the density gradient becomes so sharp that the small waves with scale-length comparable to the grid size start to grow.

II-2. Method of data analysis

By a series of computations, we obtain the plasma density at each grid point at every simulation step. Then, we make a temporal- and spatial-Fourier analysis of the plasma density to obtain the information of different wave modes. The results of the Fourier analysis may be shown in mode power maps (or spectra maps). The method of analysis is as follows: First, let’s consider the spatial-Fourier analysis of the 1-D plasma density \( n(x; t) \),

\[
f_n(x; t) = \sum_k f A_k(t) \cos(kx) + B_k(t) \sin(kx)
\]

followed by the temporal-Fourier analysis of the time series of \( A_k(t) \) and \( B_k(t) \),

\[
f A_k(t) = \sum_! f A_k^{(1)}(t) \cos(\!t) + B_k^{(1)}(t) \sin(\!t)
\]

\[
f B_k(t) = \sum_! f A_k^{(2)}(t) \cos(\!t) + B_k^{(2)}(t) \sin(\!t)
\]

Then substituting the expressions of \( A_k(t) \) and \( B_k(t) \) back into \( n(x; t) \) to obtain

\[
X
\sum_k f A_k(t) \cos(kx) + B_k(t) \sin(kx)
\]

\[
= \sum_k \cos(kx) \cdot f A_k^{(1)}(t) \cos(\!t) + B_k^{(1)}(t) \sin(\!t)
\]

\[
+ \sum_k \sin(kx) \cdot f A_k^{(2)}(t) \cos(\!t) + B_k^{(2)}(t) \sin(\!t)
\]

\[
= \sum_{k;1} \frac{1}{2}(A_k^{(1)} + B_k^{(2)}) \cos(\!t + kx) + \frac{1}{2}(B_k^{(1)} - A_k^{(2)}) \sin(\!t + kx)
\]

\[
+ \sum_{k;1} \frac{1}{2}(A_k^{(1)} - B_k^{(2)}) \cos(\!t - kx) + \frac{1}{2}(B_k^{(1)} + A_k^{(2)}) \sin(\!t - kx)
\]
Consequently, the amplitude $C_{k!}^+$ ($C_{k!}^i$) of the wave mode with wave-number $k$ and frequency $\omega$ propagating in the $+\hat{x}$ direction is expressed as follows:

$$C_{k!}^+ = \frac{1}{2} q \left( A_{k!}^{(1)} + B_{k!}^{(2)} \right)^2 + \left( B_{k!}^{(1)} i A_{k!}^{(2)} \right),$$

$$C_{k!}^i = \frac{1}{2} q \left( A_{k!}^{(1)} i B_{k!}^{(2)} \right)^2 + \left( B_{k!}^{(1)} + A_{k!}^{(2)} \right).$$

This procedure of separation between the oppositely propagating waves is fundamentally similar to the procedure presented in [54]. The Fourier analysis of the 2-D plasma density distribution $n(x; z; t)$ can be expressed as:

$$fn(x; z; t) = \sum A_k(z; t) \cos(kx) + B_k(z; t) \sin(kx) \int \frac{X}{k!} \cos(kx) \cos(\omega t) (A_{k!}^{(1)} (t) \cos(\omega t) + B_{k!}^{(2)} (t) \sin(\omega t))$$

$$+ \sin(kx) (A_{k!}^{(1)} (t) \cos(\omega t) + B_{k!}^{(2)} (t) \sin(\omega t))$$

$$\cdot \cos(\omega t) \cos(\omega t) (A_{k!}^{(1)} (t) \cos(\omega t) + B_{k!}^{(2)} (t) \sin(\omega t))$$

$$+ \sin(kx) \cos(\omega t) \int \frac{X}{k!} \cos(kx) \cos(\omega t) (A_{k!}^{(1)} (t) \cos(\omega t) + B_{k!}^{(2)} (t) \sin(\omega t))$$

$$+ \sin(kx) \cos(\omega t) \int \frac{X}{k!} \cos(kx) \cos(\omega t) (A_{k!}^{(1)} (t) \cos(\omega t) + B_{k!}^{(2)} (t) \sin(\omega t))$$

$$+ \sin(kx) \sin(\omega t) (A_{k!}^{(1)} (t) \cos(\omega t) + B_{k!}^{(2)} (t) \sin(\omega t));$$

where,

$$A_k(z; t) = \int (A_{k!}^{(1)} (t) \cos(\omega t) + B_{k!}^{(2)} (t) \sin(\omega t));$$

$$B_k(z; t) = \int (A_{k!}^{(1)} (t) \cos(\omega t) + B_{k!}^{(2)} (t) \sin(\omega t));$$

$$A_{k!}(t) = \int (A_{k!}^{(1)} (t) \cos(\omega t) + B_{k!}^{(1)} (t) \sin(\omega t));$$

$$B_{k!}(t) = \int (A_{k!}^{(1)} (t) \cos(\omega t) + B_{k!}^{(2)} (t) \sin(\omega t));$$

$$A_{k!}^{(1)}(t) = \int (A_{k!}^{(1)} (t) \cos(\omega t) + B_{k!}^{(1)} (t) \sin(\omega t));$$

$$B_{k!}^{(1)}(t) = \int (A_{k!}^{(1)} (t) \cos(\omega t) + B_{k!}^{(4)} (t) \sin(\omega t)).$$
Finally,
\[
\begin{align*}
    f_n(x; z; t) g &= \frac{1}{4} \left( A_{kl!}^{(1)} + B_{kl!}^{(2)} + B_{kl!}^{(3)} + A_{kl!}^{(4)} \right) g \cos(\lambda t + kx + lz) \\
    &+ \frac{1}{4} \left( A_{kl!}^{(1)} + B_{kl!}^{(2)} + B_{kl!}^{(3)} + A_{kl!}^{(4)} \right) g \sin(\lambda t + kx + lz) \\
    &+ \frac{1}{4} \left( A_{kl!}^{(1)} + B_{kl!}^{(2)} + B_{kl!}^{(3)} + A_{kl!}^{(4)} \right) g \cos(\lambda t + kx + lz) \\
    &+ \frac{1}{4} \left( A_{kl!}^{(1)} + B_{kl!}^{(2)} + B_{kl!}^{(3)} + A_{kl!}^{(4)} \right) g \sin(\lambda t + kx + lz) \\
    &+ \frac{1}{4} \left( A_{kl!}^{(1)} + B_{kl!}^{(2)} + B_{kl!}^{(3)} + A_{kl!}^{(4)} \right) g \cos(\lambda t + kx + lz) \\
    &+ \frac{1}{4} \left( A_{kl!}^{(1)} + B_{kl!}^{(2)} + B_{kl!}^{(3)} + A_{kl!}^{(4)} \right) g \sin(\lambda t + kx + lz) \\
    &+ \frac{1}{4} \left( A_{kl!}^{(1)} + B_{kl!}^{(2)} + B_{kl!}^{(3)} + A_{kl!}^{(4)} \right) g \cos(\lambda t + kx + lz) \\
    &+ \frac{1}{4} \left( A_{kl!}^{(1)} + B_{kl!}^{(2)} + B_{kl!}^{(3)} + A_{kl!}^{(4)} \right) g \sin(\lambda t + kx + lz)
\end{align*}
\]
(12)

Therefore, the amplitude of the different space and frequency modes propagating in the two-dimensional space involving four different quadrants can be expressed as follows:

\[
\begin{align*}
    C_{kl!}^{(1)} &= \frac{1}{4} q \left( A_{kl!}^{(1)} + B_{kl!}^{(2)} + B_{kl!}^{(3)} + A_{kl!}^{(4)} \right)^2; \text{ (first quadrant)} \\
    C_{kl!}^{(2)} &= \frac{1}{4} q \left( A_{kl!}^{(1)} + B_{kl!}^{(2)} + B_{kl!}^{(3)} + A_{kl!}^{(4)} \right)^2; \text{ (second quadrant)} \\
    C_{kl!}^{(3)} &= \frac{1}{4} q \left( A_{kl!}^{(1)} + B_{kl!}^{(2)} + B_{kl!}^{(3)} + A_{kl!}^{(4)} \right)^2; \text{ (third quadrant)} \\
    C_{kl!}^{(4)} &= \frac{1}{4} q \left( A_{kl!}^{(1)} + B_{kl!}^{(2)} + B_{kl!}^{(3)} + A_{kl!}^{(4)} \right)^2; \text{ (fourth quadrant)}
\end{align*}
\]

III. Numerical results

III-1. Test of numerical models

Since the 1-D theory of the Farley-Buneman instability is exact, a convenient way of examining the numerical model is to compare the result of the 1-D simulation with the analytical theory. Two classes of 1-D simulations with common parameters as listed in Table I have been carried out. All the simulations of the first class have the same initial perturbation wavelength of \( \lambda = 15 \text{ m} \) and different drift velocity ranging from 100 m/s to 800 m/s; the simulations of the second class have the same drift velocity of \( V_D = 500 \text{ m/s} \) and different initial perturbation wavelength ranging
TABLE I. Parameters of the Background Plasma

<table>
<thead>
<tr>
<th>Parameter(s)</th>
<th>Physical Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static magnetic field, $B_0$</td>
<td>0.28</td>
<td>G</td>
</tr>
<tr>
<td>Polarization E field, $E_0$</td>
<td>0.015</td>
<td>V/m</td>
</tr>
<tr>
<td>Mean plasma density, $n_0$</td>
<td>$1 \times 10^{11}$</td>
<td>m$^{-3}$</td>
</tr>
<tr>
<td>Temperature $T_e, T_i$</td>
<td>230</td>
<td>K</td>
</tr>
<tr>
<td>Effective ion mass</td>
<td>$5.0 \times 10^{26}$</td>
<td>Kg</td>
</tr>
<tr>
<td>$e_i i_n$ collision frequency, $\varrho_{en}$</td>
<td>$4.0 \times 10^{4}$</td>
<td>s$^{-1}$</td>
</tr>
<tr>
<td>Ion-n collision frequency, $\varrho_{in}$</td>
<td>$2.5 \times 10^{3}$</td>
<td>s$^{-1}$</td>
</tr>
</tbody>
</table>

FIG. 1. (a) Time variation for the 1D-spatial amplitude $A_k(t)$ of the primary mode. (b) Time average of the wave amplitude $\sqrt{A_k^2 + B_k^2}$ averaging over a time interval of one-quarter-wave period ($\varphi = 4 = 2\pi/4! = r$).

from 3 m to 30 m. At the beginning of each simulation, a small 0.1% perturbation with given wavelength is superposed on the uniform density background with given drift velocity $V_D$. At each time step of the simulation, the density distribution over space is Fourier analyzed to obtain $A_k$ and $B_k$ as equations (7, 8a, 8b) indicated. Fig. 1 demonstrates one example of the 1-D simulation with $V_D = 500$ m/s and $\lambda = 15$ m. The time variation of $A_k(t)$ of the primary mode corresponding to the wavelength of 15 m is shown in Fig. 1(a), which clearly displays a sinusoidal oscillation, whose frequency $! r$, can be obtained precisely by measuring the peak-to-peak time interval. The time average of the wave amplitude $\sqrt{A_k^2 + B_k^2}$ over a time interval of one-quarter wave period ($\varphi = 4 = 2\pi/4! = r$) is shown in Fig. 1(b) as a function of time on a logarithmic scale.
It reveals an exponential growing during the early stage of simulation until saturation; the linear growth rate $\lambda_1$ is estimated by least square fitting the earliest 5 points to a straight line. The results of the 1-D simulations are summarized (dark circles) in Fig. 2. The dark circles in Fig. 2(a) show the $\lambda_r$ vs $V_D$ relation (left panel) and $\lambda_i$ vs $V_D$ relation (right panel) obtained from the first class simulations at a fixed initial perturbation wavelength of $\lambda = 15$ m; The dark circles in Fig. 2(b) show the $\lambda_r$ vs $\chi$ relation (left panel) and $\lambda_i$ vs $\chi$ relation (right panel) obtained from the second class simulations at a fixed drift velocity of $V_D = 500$ m/s.

Rogister and D’Angelo [55] gave the dispersion relation of the two-stream wave theory in terms of the frequency $\omega = \omega_r + i \omega_i$ and wave vector $k(= 2\pi n)$ as follows:

$$\omega_r = \frac{k \chi V_D}{(1 + A_0)};$$

$$\omega_i = \frac{A_0}{(1 + A_0)} \left[ 1 - \frac{1}{9} (\frac{2}{r_i} k^2 C_s^2) + \frac{\omega_r e^{-k L}}{k n^e} \right].$$

FIG. 2. The linear dispersion relation plots of two-stream waves with respect to the frequency $\omega = \omega_r + i \omega_i$, where the solid line represents the theoretical value and the dark circles represent the 1D-simulation results. (a) $\omega_r$ (left panel) and $\omega_i$ (right panel) versus drift velocity with fixed wavelength (15 m). (b) $\omega_r$ (left panel) and $\omega_i$ (right panel) versus wavelength for fixed drift velocity (500 m/sec).
For the pure two-stream wave in the uniform background density, the second term of equation (14) can be neglected, then equation (14) becomes

\[ \!i = \frac{\tilde{A}_0}{\left(1 + A_0\right)} \cdot \frac{1}{\omega_{in}} \left(\! r \cdot \! i \cdot k^2 C_s^2\right)^{\cdot} \]

(15)

where \( C_s \) is the ion acoustic speed, and \( \omega_{0} = \omega_{in} \omega_{en} = \omega_{i} \). The theoretical values of the real part \( \! r \) (wave-frequency) and the imaginary part \( \! i \) (growth rate) of the complex frequency versus drift velocity \( V_D \) for fixed wavelength (\( \lambda_0 = 15 \text{ m} \)), obtained from equations (13) and (15), are plotted as solid lines in the left panel and right panel respectively of Fig. 2(a); the relations of \( \! r : \omega_{in} : \omega_{en} \cdot \omega_{i} \) and \( \! i : \omega_{en} : \omega_{i} \) for fixed drift velocity (\( V_D = 500 \text{ m/s} \)) are plotted respectively on the left panel and right panel of Fig. 2(b) along with the results of simulations. Fig. 2 clearly shows that our 1-D simulations are consistent with the predictions of the linear theory of pure two-stream waves. We must emphasize that such a close match between theory and simulation is obtained when the time step size of simulation is equal to or smaller than \( \Delta t = 10000 \) (\( \Delta t \) is the period of the first wave mode \( k = 1 \)). When the time step size is raised to \( \Delta t = 1000 \), the result of the simulation is only qualitatively consistent with the theory.

We have also tested the two-dimensional model by superposing a single wave perturbation on the uniform density background at a different drift velocity \( V_D \), using a simulation box of \( X \times Z = 27 \text{ m} \times 54 \text{ m} \) with grid resolution of 121 \( \times \) 61. The threshold velocity of the Farley-Buneman instability is given by

\[ V_{th} = \frac{\mu}{1 + \omega_{in} \omega_{en}} \cdot \omega_{i} \cdot C_s; \]

where \( C_s = \sqrt{K_B (T_i + T_e)/(m_i + m_e)} \) is the ion acoustic velocity. Substituting the background parameters shown in Table I, we obtain \( C_s = 356 \text{ m/s} \) and \( V_{th} = 437 \text{ m/s} \). The wave number (\( 2/3 k = X, 2/3 \lambda = Z \)) of the initial perturbation wave is given by \( k = 9, \lambda = 0 \), and the initial amplitude is 0.1% of the background density \( n_0 \). The time step size of the simulations is \( \Delta t = 1000 \). Examples of the development of \( A_k(t) \) with \( k = 9 \) and \( \lambda = 0; 1; 2 \) of the perturbation wave as a function of time are shown in Fig. 3. When \( V_D = 550 \text{ m/s} \) (Fig. 3a) and 500 m/s (Fig. 3b), being greater than the Farley-Buneman threshold, the wave amplitude is seen to grow with time; and for the cases with \( V_D = 400 \text{ m/s} \) (Fig. 3c) and 350 m/s (Fig. 3d), which are below the Farley-Buneman threshold, the wave amplitudes decay with time, as expected from the theory. The oscillation period of each case can be precisely determined by measuring the peak-to-peak interval (see Fig. 3), therefore the propagation velocity can be precisely determined for each case. Fig. 4 shows that the linear relation between the phase velocity \( V_p \) of the two-stream wave and the drift velocity \( V_D \) of electrons with a relation \( V_D = V_p = 1:24 \). We notice that this ratio is very close to the value \( \mu = 1:227 \) predicted by Eq. (13). It is fair to say that our numerical code is very accurate as long as the time step size is fine enough.
FIG. 3. Time variations of the spatial Fourier mode amplitude $A_{k'}(t)$ for the two-dimensional testing simulations. The number pair $(k';\lambda)$ denoted on the top of each panel represents the normalized wave-number, e.g., $(9,1)$ represents the 9th horizontal and the 1st vertical Fourier mode. From top to bottom, the electron drift velocities are (a) 550 m/s, (b) 500 m/s, (c) 400 m/s, and (d) 350 m/s successively as indicated above the panels of each case.
FIG. 4. The phase velocities of the two-stream waves propagating in the electrojet as a function of the electron drift velocities obtained by two-dimensional testing simulations (dark circles).

### III-2. The up-down spectral asymmetry

In order to study the possible controlling factors of the spectral asymmetry of pure two-stream waves, five cases of the two-dimensional model were simulated starting with small perturbations with electrons drifting at $V_D = 500$ m/s: one case (Case A) starting with a single primary wave perturbation, and four cases (Cases N1 » N4) starting from multiple primary wave perturbations. The wave numbers and their initial amplitudes of the initial perturbations are given in Table II, and the relevant background conditions are listed in Table I, which roughly correspond to the conditions of the ionosphere at the altitude around 105 km [56]. The simulation box and its grid resolution are the same as the testing runs, namely, the horizontal length $X$ and the vertical length $Z$ of the simulations are 27 m and 54 m respectively, and the corresponding grid resolution is 121 £ 61. The time step size of the simulation is controlled to be so small that the simulation will execute 10000 steps (Case A) or 100000 steps (Case N1 » N4) in one cycle period $\xi_1$ (0.0674s) of the largest wave mode ($k = 1; \lambda = 0$).

<table>
<thead>
<tr>
<th>Wave modes ($k; \lambda$)</th>
<th>Initial wave amplitudes</th>
<th>Time step</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A</td>
<td>(9,0): 1 wave</td>
<td>$0.0005 n_0$</td>
</tr>
<tr>
<td>Case N1</td>
<td>(2,0) » (10,0): 9 waves</td>
<td>$0.001 n_0$</td>
</tr>
<tr>
<td>Case N2</td>
<td>(8,0) » (15,0): 8 waves</td>
<td>$0.001 n_0$</td>
</tr>
<tr>
<td>Case N3</td>
<td>(2,0) » (12,0): 11 waves</td>
<td>$0.001 n_0$</td>
</tr>
<tr>
<td>Case N4</td>
<td>(1,0) » (15,0): 15 waves</td>
<td>$0.001 n_0$</td>
</tr>
</tbody>
</table>
Fig. 5. Gray scale maps of the time average of 2-D spatial mode powers over 36 oscillation periods of the principal mode (9,0) for the simulation Case A, at three consecutive time stages. The time center of each time stage is denoted on the top of each panel. Downward asymmetry is clearly revealed by the right panel.

By multiple-Fourier analysis as described in section 2.2, we obtain the long time-average of the 2D spatial spectral powers over 36 primary-wave-periods ($36T_0 = 4T_1$) for Case A at three different time stages, as shown in the gray-scale maps in Fig. 5, where the time center of each time stage is denoted on the top of each panel. The wave vector $(2\pi k = 2.7 \text{ m}, 2\pi \lambda/54 \text{ m})$ with $(k > 0, \lambda > 0)$ stands for the right-upward propagating wave, $(k > 0, \lambda < 0)$ for the right-downward wave, $(k < 0, \lambda > 0)$ for the left-upward wave, and $(k < 0, \lambda < 0)$ for the left-downward wave. The time averages of the spatial mode powers shown in Fig. 5 reveal that the horizontally propagating primary waves at an early time stage (left and middle panels), travel obliquely to the electron drift direction and eventually develop a downward asymmetry (right panel) at a later time stage. The result that the dominant modes travel obliquely to the electron drift direction, does confirm the results of [19-21]. But our result that the magnitude of vertical component of spatial mode powers displays a downward asymmetry distribution does not agree with the results presented in [19, 21]. The results of the simulation of [19, 21] show an upward asymmetry in the nighttime condition, being opposite to the radar observation, while our result of downward asymmetry is consistent with the radar observation. Notice that the horizontal wavelength of the principal mode in the simulation of [21] is 0.86 m, and that of [19] is about 0.5 m. These are too small to be resolved by the radar observations, and their directions of up-down asymmetries are all opposite to the radar experiments.

The development of up-down asymmetry of two-stream waves deserves more discussion since it cannot be explained by linear theory. The horizontal gradient density contour map Fig. 6 (case A at 0.067s and 0.539s) shows that the vertical small-scale waves do develop from the distortion of the principal waves and cause the dominant waves to turn. The turning of the primary wave was at first reported by Oppenheim et al. [22]. We believe that, and will explain later, why the polarization electric field $E_0$, which drives the electro-jet current, is the dominant factor causing the turning of the wave.
FIG. 6. The gray scale maps of the horizontal-gradient density for simulation Case A, at two time stages with the time center denoted on the top of each panel.

FIG. 7. The averaged spectral powers as a function of the horizontal wave number. Dark circles denote the right-downward propagating component, and the open triangles denote the right-upward component. (a) Case N1; (b) Case N2; (c) Case N3; (d) Case N4. The parameters of each case are listed in Table II.
Further study of the spectral asymmetry was carried out by the simulations of Case N1 » N4. Notice that the 3-meter wave corresponds to the 9\textsuperscript{th} k-mode in this study. Because of its sensitivity to the radar observation, we include this mode in every case of the simulation. The time series covering a total time of \(6_4\) starting from the beginning of each case is analyzed to obtain the averaged spatial spectral power. The spectral power as a function of the horizontal wave number is shown in Fig. 7, where the dark circles denote the averaged power of the right-downward propagating wave, and the open triangles denote the averaged power of the right-upward waves. The results clearly show that the larger wavelength waves have developed an upward asymmetry, while the smaller wavelength waves including the 3-meter wave \((k_z = 2.09 = \text{m})\) become downward asymmetric. Based on these results, we expect that the longer wavelength component of the white noise will grow into upward asymmetry while the short wavelength component will grow into downward asymmetry in the nighttime electro-jet. The important point of this result is that the downward asymmetry of the 3-meter wave in the nighttime condition is consistent with the radar observation. We believe that the opposite asymmetry between the large wave component and the small wave component is due to the existence of the background electric field \(E_0\), which points downward in the nighttime- and upward in the daytime-electro-jet. Because the field \(E_0\) tends to attract the electron parcels upward (downward) during the nighttime (daytime) leading to a charge separation between the ions and the electrons and creating a polarization electric field \(E^0\) pointing upward (downward). Wherever the local polarization field \(E^0\) is stronger than the background field \(E_0\), the smaller electron parcels will be attracted downward (upward). That will result in
FIG. 9. The power spectra of Case A obtained from the density perturbations. The horizontal axis shows the phase velocity, \( v_{ph} = \frac{\omega}{k} = \frac{j}{k} \), in units of the sound speed, \( C_s = \left[ k_B (T_i + 0.1 T_e) \right]^{1/2} = 356 \text{ m/s} \), while the vertical axis shows the power on a linear scale in arbitrary units. The labels across the top give \( k_x \) for that column, while the labels on the left give \( k_z \) for that row. The top number in the upper corner of each plot shows the reduction in that mode’s maximum density perturbation with respect to the mode with the largest density perturbation found in the simulation in decibels. The bottom number in the upper corner shows the total power contained in the mode compared to the total power of the mode containing the most power (the principal mode) in decibels.

III-3. Individual Spectra

Fig. 9 shows the spectra of a few important \((k_x; k_z)\) values of Case A obtained from a time series lasting an interval of 425 s with time centered at \( t = 0:275 \text{ s} \). The labels across the top give the value of \( k_x \) for that column, while the labels on the left give the value of \( k_z \) for that row, where \( k_x \) and \( k_z \) are respectively the horizontal- and the vertical- wave numbers. The initial

\[
K_x = \{-2.56, -2.09, -1.63, -1.16, 0, 1.16, 1.63, 2.09, 2.56\}
\]

\[
K_z = \{0, 0.12, 0.23, 0.35, 0.58\}
\]
modes, while \((8,0)\) and \((2,0)\) represents the two secondary modes in this figure. The two primary modes of the initial perturbation are denoted by \(\mathbf{k}_1\) and \(\mathbf{k}_2\) with \(k_{1x} = 0.7 \text{ m}^{-1}\), \(k_{2x} = 1.16 \text{ m}^{-1}\) and \(k_{1z} = k_{2z} = 0\). Notice that \(\mathbf{k}_1\) and \(\mathbf{k}_2\) are the wave vectors of the 3rd and 5th horizontal Fourier modes respectively. The initial perturbation amplitudes of \(\mathbf{k}_1\) and \(\mathbf{k}_2\) are taken to be \(0.02 n_0\) and \(0.003 n_0\) and the time step of simulation is \(10^{-4} \tau_1\). We expect that the two primary waves will grow due to the Farley-Buneman instability. Meanwhile, two secondary waves will be generated through resonant coupling: \((\mathbf{k}_{3x}; \mathbf{k}_{3z})\) and \((\mathbf{k}_{4x}; \mathbf{k}_{4z})\) with \(k_{3x} = k_{1x} + k_{2x} = 1.86 \text{ m}^{-1}\), \(k_{4x} = k_{2x}\), \(k_{1z} = 0.46 \text{ m}^{-1}\), and \(k_{3z} = k_{4z} = 0\). Figs. 10 and Fig. 11 show the time variations of \(A_{\mathbf{k}}(t)\) (defined in Eq. 11c) of the various spatial Fourier modes. The pair of numbers \((\mathbf{k}; \lambda)\) on the top of each diagram represents the wave vector normalized to the basic wave vector unit of this simulation: \(k = k_x = 0.23 \text{ m}^{-1}\), and \(\lambda = k_z = 0.116 \text{ m}^{-1}\). For instance \((3,0)\) and \((5,0)\) denotes the two primary modes, while \((8,0)\) and \((2,0)\) represents the second secondary modes in this figure.

III-4. The resonant coupling

Saturation of the Farley-Buneman instability is a nonlinear process involving wave-wave interactions. Dougherty and Farley [57], Volosevich et al. [58], and Kudeki [45] suggest that a resonant interaction satisfying the following relations may take place in the equatorial electrojet region:

\[
\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_s \quad \text{and} \quad !_1 + !_2 = !_s;
\]

where \(\mathbf{k}_1, \mathbf{k}_2\) are the wave vectors of the two primary waves, and \(!_1, !_2\) are their corresponding frequencies; \(\mathbf{k}_s\), and \(!_s\), stand for the secondary wave. In order to study the resonant coupling process of pure two-stream waves, we simulate the nighttime two-stream waves by exciting two primary modes instead of one in the initial perturbation.

The two primary modes of the initial perturbation are denoted by \(\mathbf{k}_1 = (k_{1x}; k_{1z})\) and \(\mathbf{k}_2 = (k_{2x}; k_{2z})\) with \(k_{1x} = 0.7 \text{ m}^{-1}\), \(k_{2x} = 1.16 \text{ m}^{-1}\) and \(k_{1z} = k_{2z} = 0\). Notice that \(\mathbf{k}_1\) and \(\mathbf{k}_2\) are the wave vectors of the 3rd and 5th horizontal Fourier modes respectively. The initial perturbation amplitudes of \(\mathbf{k}_1\) and \(\mathbf{k}_2\) are taken to be \(0.02 n_0\) and \(0.003 n_0\) and the time step of simulation is \(10^{-4} \tau_1\). We expect that the two primary waves will grow due to the Farley-Buneman instability. Meanwhile, two secondary waves will be generated through resonant coupling: \((\mathbf{k}_{3x}; \mathbf{k}_{3z})\) and \((\mathbf{k}_{4x}; \mathbf{k}_{4z})\) with \(k_{3x} = k_{1x} + k_{2x} = 1.86 \text{ m}^{-1}\), \(k_{4x} = k_{2x}\), \(k_{1z} = 0.46 \text{ m}^{-1}\), and \(k_{3z} = k_{4z} = 0\). Figs. 10 and Fig. 11 show the time variations of \(A_{\mathbf{k}}(t)\) (defined in Eq. 11c) of the various spatial Fourier modes. The pair of numbers \((\mathbf{k}; \lambda)\) on the top of each diagram represents the wave vector normalized to the basic wave vector unit of this simulation: \(k = k_x = 0.23 \text{ m}^{-1}\), and \(\lambda = k_z = 0.116 \text{ m}^{-1}\). For instance \((3,0)\) and \((5,0)\) denotes the two primary modes, while \((8,0)\) and \((2,0)\) represents the second secondary modes in this figure.
FIG. 10. The time variations of the spatial Fourier mode amplitude $A_{k\ell}(t)$ for primary and secondary waves developed from the 2D simulations. The labels on the top of each plot shows the spatial mode number $(k; \ell)$, where $(3,0)$ and $(5,0)$ represent the two primary modes, $(2,0)$ and $(8,0)$ are the resonant coupling modes.
FIG. 11. The same as Fig. 10, except that the spatial wave modes include obliquely propagating waves ($\theta \neq 0$).

We can see from Fig. 10 that the growth rate of the resonant coupling mode $k_3(8,0)$ is significantly larger than that of any other secondary modes as well as the two primary modes. The other resonant coupling mode $k_4(2,0)$ also reveals a growth rate significantly larger than the neighboring secondary modes. The wave mode of $(6,0)$ results from the self-coupling of the $k_3(3,0)$ mode, and the wave mode $(10,0)$ results from the self-coupling of the $k_2(5,0)$ mode. Both self-coupling modes also show significant growth rates. Resonant coupling is really an effective process to cascade the energy into different scale sizes. From a careful inspection of the oscillation curves in Fig. 11, we find that all the different waves identified by different $\theta$ with the same $k$ have exactly the same wave period; and all the different waves identified by different $k$ have...
different wave periods satisfying the following coupling equations:

\[
\begin{align*}
k_1 + k_2 &= k_3; \quad \lambda_1 + \lambda_2 = \lambda_3; \quad k_2 \parallel k_1 = k_4; \quad \lambda_2 \parallel \lambda_1 = \lambda_4; \\
\frac{\lambda_1}{k_1} &= \frac{\lambda_2}{k_2} = \frac{\lambda_3}{k_3} = \frac{\lambda_4}{k_4} = \frac{V_D}{1:24} \left( \frac{404}{4} \right) \text{m/s};
\end{align*}
\]

(17a) (17b)

Notice that the frequency of each spatial mode is independent of its vertical wave number. This is very interesting, because the wave-wave interaction seems to have very little effect on the vertical wave number. We suggest again from this result that the development of the vertical structure of the two-stream wave is dominated by the vertical polarization electric field \( E_0 \). Since \( E_0 \) is a constant field, the change of the vertical wave number by \( E_0 \) can not change the wave frequency, i.e., \( E_0 \) is responsible for the turning of both the primary and secondary two stream-waves away from the electron drift direction. This is another evidence that the up-down asymmetry in the horizontally traveling two-stream wave is most probably caused by the vertical polarization electric field in the electro-jet. Also, the fact that \( E_0 \) turns the two-stream wave away from the electron drift direction by increasing the vertical wave number, without changing the frequency and the horizontal wave number, will result in reducing the phase velocity to below the value predicted by the linear theory as shown in the \( \lambda \) spectra of Fig. 9.

IV. Discussion

Janhunen [19] was the first, followed by Oppenheim and Otani [21], to obtain the spectral asymmetry of pure two-stream waves by simulation. However, their results that the most intense waves propagate at the linearly unstable sector \( k \cdot E < 0 \), are opposite to the radar observations. The most probable reason for this inconsistency is that the scale lengths of the primary two-stream waves in their simulations are much smaller than the 3-meter wave observed by radar: the wavelength of the dominant mode in the simulation of [21] is about 0.86 m and is about 0.5 m in the simulation of [19]. In this paper, we have presented the detailed results of a series of simulations studying the controlling factors of the asymmetry mechanism of pure two-stream waves.

We believe that the zero order polarization electric field \( E_0 \) driving the electrojet current is responsible for driving the spectral asymmetry of the pure two-stream waves, since the radar observations indicated that the up-down spectral asymmetries are always in favor of the direction parallel to \( E_0 \) and the vertical current driven by \( E_0 \) might cause the asymmetry. Farley et al. [30] disagrees with this idea. They argued that the vertical current driven by \( E_0 \) flows upward during daytime (\( E_0 \) is pointing upward), meaning that the electrons move downward relative to the ions, favoring excitation of the down-going waves, just the opposite of the observed daytime asymmetry. This argument is only partially right. In addition to driving the plasma drift westward (\( E \perp B \) drift), the daytime \( E_0 \) will also drive (large) electron parcels downward (Pederson current), generating a space charge field \( E_{01} \) directed in the opposite direction of \( E_0 \). This space charge field \( E_{01} \) will tend to drive the electron parcels to move in the same direction as \( E_0 \) (opposite to the direction of \( E_{01} \)). As a net effect of the combined field \( E_0 \) and \( E_{01} \), wherever \( E_{01} \) is stronger than \( E_0 \), the smaller electron parcels will be driven to move in the same direction as \( E_0 \), favoring excitation of the upward wave. And the reverse is true in the nighttime electro-jet. This is exactly the result revealed by our simulation, that the large wave component in the night-time (day-time)
electro-jet develops upward (downward) asymmetry while the smaller wave components including the 3-meter wave develop downward (upward) asymmetry.

Another interesting result of our simulation lies in the spectra (see Fig. 9). The spectra of the horizontally traveling \((k_z = 0)\) primary two-stream waves are very sharp with phase velocity slightly higher than the ion acoustic speed. For the wave modes with \(k_z \neq 0\), the wave frequency depends only on \(k_x\) (i.e., \(\omega\) being independent of \(k_z\)), so the phase velocity decreases with increasing \(k_z\), and the phase velocities of many obliquely propagating two-stream waves are below the value predicted by linear theory. This is a consequence of the turning of the pure two-stream waves away from the electro-jet drift direction. These results are consistent with the simulation results of [21]. However, the spectra widths of our \(\omega\) spectra are much narrower than theirs, i.e., our simulations on the pure two-stream waves do not produce the type II-like spectra as claimed by the simulation of [21].

V. Summary

We have obtained the following results for the saturation of pure two-stream waves. (1) The spectral asymmetry depends on the scale size of the two-stream wave. For the waves with scale size well above 3 meters, the excitation of upward (downward) asymmetry is favored in the nighttime (daytime) electrojet over downward (upward) asymmetry, and the opposite is true for the smaller waves including the 3 meter wave. (2) On the way to saturation, the pure two-stream wave gradually turns away from the electro-jet drift direction through the increase of its vertical wave number without changing its frequency and horizontal wave number, resulting in the decrease of its propagation phase speed. (3) Resonant interaction between two pure two-stream waves will create two new waves, one larger and one smaller than the primary waves, propagating at the same speed as the primary waves. The secondary waves will also turn away from the electron drift direction. (4) Nonlinear processes among the pure two-stream waves do not create type II-like waves. (5) The results in (1) and (2) are controlled by the zero-order polarization electric field \(\mathbf{E}_0\), which is the driving force of the electrojet.

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