A New Physical Picture of Superconductivity

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In this article, it is argued that superconductivity originates from the Coulomb correlation effects between electrons rather than the electron-phonon interaction. The authors have already shown \cite{1} that the factor 2e in the magnetic flux quantization in a superconductor or Josephson effect has nothing to do with Cooper’s pairing. Instead, it is a natural consequence of the gauge invariance for electro-magnetic waves and the London equation for a superconductor. Moreover, it was found that the BCS Hamiltonian can be easily deduced from the fermion Hamiltonian in the second quantization formalism using the grouping condition for momentum $k + k' = 0$, which is usually termed the Cooper condition, (also, for spin $\sigma + \sigma' = 0$), and that the interaction $v(k', k)$ in the Hamiltonian is nothing else, but the interaction between an electron and a hole in momentum (k) space, and should be naturally attractive according to the Coulomb law. In contrast, $U$ in the Hubbard model must be positive because the Hubbard model describes the behavior of particles in real space. With the re-interpretation of the interaction, the results regarding a gap in the electron energy spectrum and the isotope effect from the BCS theory remain valid. With the new physical picture in mind, one can thus unify the theory for both low- and high-temperature superconductivity \cite{2}.

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I. Introduction

The concept of Cooper’s pairing of electrons has rooted in superconductivity. \cite{3, 4} and seems to have become a criterion to assess the validity of a theory or model in the study of superconductivity. A large variety of attempts, since 1986, were made to find out the pairing mechanism and mediator of high-temperature superconductivity, but none of them appears to be successful. With the concept of Cooper’s pairing, it is natural to conclude that two electrons are bound to form a boson-like entity and thus the Bose condensation applies as well as the unit of carrier charge is $2e$. Moreover, $v(k^0, k)$ in the BCS Hamiltonian has to be interpreted as an attractive interaction between two electrons due to the phonon exchange.

The authors have carefully examined these fundamental issues and found that the concept of Cooper’s pairing in its original sense - two electrons being paired above a quiescent Fermi sea -(Fig. 1(b)) does not lead to metal-superconductor, but a metal-insulator transition \cite{5}. In this article, to correctly understand and interpret the BCS Hamiltonian a new concept, electron grouping, instead of Cooper’s pairing, is introduced to simplify the Hamiltonian formalism of second quantization and thus deduce the BCS Hamiltonian. Correspondingly, the physical significance of $v(k^0, k)$ has to be reinterpreted as given below.
II. Re-interpretation of the BCS Hamiltonian

In general, the formalism of the fermion Hamiltonian in second quantization is given as

\[ H = \sum_{\mathbf{p}, \sigma} \varepsilon_{\mathbf{p}} c_{\mathbf{p}, \sigma}^{\dagger} c_{\mathbf{p}, \sigma} + \frac{1}{2k_{0}} \sum_{\mathbf{q} \neq \mathbf{0}, \sigma, \sigma'} \varepsilon_{\mathbf{q}} c_{\mathbf{k} + \mathbf{q}, \sigma}^{\dagger} c_{\mathbf{k}, \sigma'} c_{\mathbf{q}, \sigma} c_{\mathbf{q}, \sigma'}^{\dagger} c_{\mathbf{k}, \sigma}. \]  

(1)

As shown by the authors in the work [6], in terms of the grouping (pairing, in the statistical sense) condition for momentum, \( k_{0} + k = 0 \), and for spin, \( \sigma^{\dagger} \), Eq. (1) can be simplified as below.

\[ H = \sum_{k, \sigma} \varepsilon_{k, \sigma} c_{k, \sigma}^{\dagger} c_{k, \sigma} + \frac{1}{2k_{0}} \sum_{k} V(k_{0}, k) c_{k, 0}^{\dagger} c_{-k, 0} c_{-k, 0}^{\dagger} c_{k, 0}. \]  

(2)

This is nothing else, but the BCS Hamiltonian. It deserves a mention that here no Cooper’s pairing is mentioned or used. Instead, one just groups particles statistically pair by pair. Each pair contains two particles that momentarily possess the opposite momenta and spins. If one focus attention on \( k \)-space, the interaction of \( v(k_{0}, k) \) clearly connects \( k_{0} \) with \( k \), which are momenta of an electron, related to the creation operator \( c_{k, 0}^{\dagger} \), and a hole, related to the annihilation operator \( c_{k} \), respectively, as shown in the diagram (Fig. 1(a)). Therefore, \( v(k_{0}, k) \) actually describes the interaction between an electron and a hole. In other words, it is the interaction in the particle-hole channel. It can be seen that two electrons and two holes in Eq. (2) are respectively “paired” in the statistical sense, but not bound. The word of “pairing” here is by no means the same as Cooper’s pairing. Naturally, \( v(k_{0}, k) \) in Eq. (2) is attractive without invoking phonons, whereas the interaction between electrons or the interaction in the particle-particle channel is still repulsive as given by the Coulomb law. This is just the case of two electrons in the Hubbard model where the description is given in real space.

Cooper’s pairing in its original definition is for two electrons put above a quiescent Fermi sea [7]. It means that no quasi-particle-hole excitations are considered. Further, BCS introduced into the Hamiltonian two new operators \( b_{k} \) and \( b_{k}^{\dagger} \), where \( b_{k} = c_{k, \sigma} c_{k, \sigma}^{\dagger} \), and \( b_{k}^{\dagger} = c_{k, \sigma}^{\dagger} c_{k, \sigma} \) [3, 4]. This replacement further simplifies the Hamiltonian expression and should retain the original physical significance: removing two electrons with opposite momenta and spins below the Fermi level and creating two electrons with opposite momenta and spins above the Fermi level. \( v(k_{0}, k) \) should be logically thought of as the interaction between an electron and a hole.

With this understanding, the procedure of solving Eq. (2) would be the same as done by BCS. The BCS treatment and solution to the BCS Hamiltonian are mathematically sound. Meanwhile, the results obtained in the original work of BCS remain valid for conventional superconductors even though a completely new physical picture is proposed for the interaction.

III. Clarification of the experimental evidences of coopper’s pairing

In 1961, it was found experimentally [8, 9] that the quantum of the magnetic flux \( \Phi \) in a bored cylindrical superconductor is \( hc/2e \), which is only one half of the value calculated by London [10] based on the gauge invariance, and consistent with Onsager’s suggestion [11]. Meanwhile, the discussions and calculations by Byers and Yang [12] showed a result of \( \Phi = (n + 1/2)(hc/e) \). In summary, Schrieffer [4] made the same conclusion as Onsager to favor Cooper’s
pairing mechanism of electrons. However, this is obviously contrary to the gauge invariance [9]. In 1992, Cheng [13, 14] argued that in a superconductor placed in the arrangement of the Aharonov-Bahr effect, [15] the factor $2e$ has nothing to do with Cooper’s pairing. The authors have also carefully examined this important issue and rigorously derived the factor $2e$ from the gauge invariance, as London emphasized, and the London equation [1]. Therefore, the factor $2e$ cannot be thought of as an experimental evidence of Cooper’s pairing.

In addition, as mentioned above, in the case of electron-phonon interaction with a given attractive potential between two electrons a metal-insulator rather than metal-superconductor phase transition will occur [5]. Therefore, it is impossible to have phonon- or any type of boson-mediated attractive electron-electron interaction to get superconductivity. One has in fact recognized in the study of high-temperature superconductivity that it is impossible to have a mechanism for high-temperature superconductivity from phonon and has turned to the Coulomb interaction. However, the constraint of the concept of Cooper’s pairing seems to push one to find a mechanism of Cooper’s pairing with Coulombic interaction. This is perhaps why one has to invoke the so-called $d$-wave. If one could discard the concept of Cooper’s pairing, the controversy would immediately disappear, and there would no such intense disputes between $s$- and $d$-waves.

IV. The origin of superconductivity - coulombic correlations

Thus, one understands that $v(k^0, k)$ in the BCS Hamiltonian represents the interaction between an electron and a hole or in the particle-hole channel, and is naturally attractive as given by the Coulomb law, while the interaction between electrons, or in the particle-particle channel, is still repulsive. The following arguments clarify and support this picture.

The Hamiltonian, after the second quantization, has become a formalism in k-space. For fermions, it is supposed to be interpreted based on the concept of particle-hole excitations, as done by Landau [16] and represented by the diagram in Fig. 1(a). At $T = 0$, all of states under the Fermi surface are occupied and there is no room for two electrons to change to two new states under the Fermi surface. In other words, no final states after scattering can be found under the Fermi surface. Owing to interactions between particles, there must exist particle-hole excitations. After the simplifying treatment of grouping (pairing), the Hamiltonian takes on the form given in Eq. (2). Now, its physical meaning is clear enough as presented in Fig. 1(a). Pauli’s exclusion principle furnishes this marvelous feature for fermions: two electrons are excited to the two states above the Fermi surface, where all of the states were previously unoccupied, and simultaneously leave two holes below the Fermi surface. It can be seen that Eq. (2) may serve as the ground of the concept of particle-hole excitations. Fig. 1(a) may also be viewed as the picture of Landau’s idea related to the description of the physical process given by Eq. (2). It turns out that the two similar formalisms of the Hamiltonian for bosons and fermions, respectively, should hold two different physical pictures of scattering. This distinction is apparently due to Pauli’s exclusion principle that fermions must obey. The historical misunderstanding seems to be due to the influence of the concept of Cooper’s problem, which may be illustrated by the diagram in Fig. 1(b), where two electrons stay above the Fermi surface as if they were put there by ‘hand.’ This is actually an artificial problem.

Although earlier efforts with the Coulomb interaction were made [17], no appreciable progress was made in this direction. The authors think that the reason perhaps lies in the constraint of the concept of Cooper’s pairing. In addition, the mean field approximation may also be the
main obstacle for one to reach superconductivity with the Coulombic interaction.

V. A unified theory of superconductivity

With the new physical picture in mind, one is able to establish a unified theory to describe both high- and low-temperature superconductivity, [2] in which a unified model was used in the framework of quantum field theory in terms of Green’s function technique and Feynman diagrams. A breakthrough in the work is that by means of iterations a method accounting for high-order
corrections of the vertex part to the two-particle interaction kernel or the single-particle self-energy has been developed. The results are obtained based on the full summation over all the possible diagrams of interactions avoiding the misleading of the partial summations. Due to the page limitation there is no space to talk about it in detail here. However, a couple of the major results can be illustrated in Fig. 2. Interested readers may consult the reference [2].

In the work of [18], transition temperature $T_c$ for layered compounds like cuprates was found to be related to physical, chemical and structural factors, such as the dielectric constant $\varepsilon$ of the background lattice, effective band mass of electron $m_b$, and interlayer spacing $c$. Also, a bell-shape phase diagram of $T_c$ vs. the density of carriers was obtained with a variety of the combinations of those physical, chemical and structural factors mentioned above (Fig. 2(a)). This provides a good clue for experimentalists to search for samples with higher transition temperature. Moreover, the validity of the theory can be examined and verified by its predictions. For example, for cuprates, a bell-shape relation of $T_c$ to the dielectric constant was predicted and an optimum of the dielectric constant exists for any peculiar sample to reach its highest transition temperature (Fig. 2(b)). In addition, it is of interest to note that a sample with higher $T_c$ is more sensitive than that with lower $T_c$ to the dielectric constant. Therefore, it is logically concluded that the sample with high transition temperature is not stable under the ambient condition and is not easily reproducible, either. This result may be used to explain why the La$_{2-x}$Sr$_x$CuO$_4$ compound can be well and easily reproduced everywhere, but other samples with $T_c$ as high as 250 K [19] or above could hardly be reproduced and kept for too long time.

In summary, from the analyses above, the original concept of Cooper’s pairing appears to be inadequate for superconductivity. Instead, “pairing” is just grouping. one way to simplify the Hamiltonian of second quantization. The BCS theory is mathematically sound. With the reinterpretation as given above, low- and high-temperature superconductivity can be unified in the same framework to provide a inherent description and understanding.

References