Review

CP Violation in the Standard Model: Quantum Subtleties and the Insights of Yogi Berra

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Our knowledge of flavour dynamics has undergone a ‘quantum jump’ since just before the turn of the millennium: direct CP violation has been firmly established in $K_L \to \pi\pi$ decays in 1999; the first CP asymmetry outside $K_L$ decays was discovered in 2001 in $B_d \to \psi K_S$, followed by $B_d \to \pi^+\pi^-$, $\eta' K_S$, and $B \to K^\pm\pi^\mp$ establishing direct CP violation also in the beauty sector. Counterintuitive, yet central features of quantum mechanics, like meson-antimeson oscillations and EPR correlations, have been crucial in making such effects observable. The CKM dynamics of the Standard Model (SM) of HEP allows, a description of CP insensitive and sensitive $B$, $K$, and $D$ transitions that is impressively consistent even on the quantitative level. We know now that at least the lion’s share of the observed CP violation is provided by the SM. Yet these novel successes do not invalidate the theoretical arguments for it being incomplete. We have also more direct evidence for New Physics, namely neutrino oscillations, the observed baryon number of the Universe, dark matter, and dark energy. While the New Physics anticipated at the TeV scale is not likely to shed any light on the SM’s mysteries of flavour, detailed and comprehensive studies of heavy flavour transitions will be essential in diagnosing salient features of that New Physics. Strategic principles for such studies are outlined.

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Prologue

The pronouncements of athletic coaches carry little weight in academic circles. That is not always justified. Take Yogi Berra for example, a well-known former catcher and baseball manager. He could be considered the founder of the most popular American school of philosophy, at least the most quoted one. He once stated: “If the world were perfect, it wouldn’t be.” This can be seen as a memorable formulation of the principle underlying baryogenesis: if there were perfect balance between matter and antimatter in our universe, it would bear no similarity to our universe. On another occasion he declared: “When you come to a fork in the road, take it.” I know of no more concise formulation of one of quantum mechanics most counter-intuitive features that underlies the interference pattern observed in a double-slit experiment with particle beams: even a single electron can pass through both slits.

These two quotes refer to two central topics in this review: CP violation and its connection to baryogenesis on one hand, and, on the other, the essential role played by intrinsically quantum mechanical effects to make CP violation observable. The third central topic is my judgment that none of the impressive successes of the SM invalidate the arguments for it being incomplete, that New Physics has to exist probably at scales as low as $\sim 1$ TeV, that whereas this nearby New Physics is unlikely to shed any light on the flavour structure of the SM, its impact on flavour transitions will reveal essential features of it unlikely to be obtained any other way; yet I have to add the caveat that this potential impact will probably be at most of modest strength numerically thus requiring accuracy on the experimental as well as theoretical side for proper interpretation.

In Sect. I I will provide a brief evaluation of the SM, sketch the discovery of CP violation in kaon decays and introduce the salient features of CKM dynamics leading to the ‘SM’s Paradigm of Large CP Violation in $B$ Decays’ described in Sect. II, as it had emerged by the end of the last millenium; in Sect. III I describe the validation of this paradigm since the turn of the millenium; in Sect. IV I outline strategies for searches for New Physics in flavour dynamics and sketch the future landscape of High Energy Physics (HEP), which in my vision should contain a Super-Flavour Factory, before concluding with an Epilogue.

I. THE SM AND CKM DYNAMICS BEFORE 2000

I-1. On the Uniqueness of the SM

A famous American football coach once declared: “Winning is not the greatest thing – it is the only thing!” This quote provides some useful criteria for sketching the status of the different components of the Standard Model (SM). It can be characterized by the carriers of its strong and electroweak forces that are described by gauge dynamics and the
mass matrices for its quarks and leptons as follows:

\[ \text{SM}^* = SU(3)_C \times SU(2)_L \times U(1) \oplus \text{CKM}(\oplus \text{PMNS}). \]  

(1)

I have attached the asterisks to ‘SM’ to emphasize the SM contains a very peculiar pattern of fermion mass parameters that is not illuminated at all by its gauge structure. Next I will address the status of these components. My goal is to emphasize the intrinsic connection of CKM dynamics, which is behind the observed CP violation, with central mysteries of the SM.

1. QCD — the ‘Only’ Thing

While it is important to subject QCD again and again to quantitative tests as the theory for the strong interactions, one should note that these serve more as tests of our computational control over QCD dynamics than of QCD itself. For its features can be inferred from a few general requirements and basic observations. A simplified list reads as follows: Our understanding of chiral symmetry as a spontaneously realized one, the measured values for \( R = \sigma(e^+e^- \to \text{had})/\sigma(e^+e^- \to \mu^+\mu^-) \) (and likewise the branching ratios for \( \pi^0 \to \gamma\gamma, \tau^- \to e^-\bar{\nu}_e\nu_\tau \) and \( B \to l\nu X \)), the need to deal with massless spin-one fields, and the requirement of combining confinement with asymptotic freedom lead us quite uniquely to a local nonabelian gauge theory with three colours. A true failure of QCD would thus create a genuine paradigm shift, for one had to adopt an intrinsically non-local description. It should be remembered that string theory was first put forward for describing the strong interactions.

A theoretical problem arises for QCD from an unexpected quarter that is relevant for our context: QCD does not automatically conserve \( P, T \), and \( \text{CP} \). To reflect the nontrivial topological structure of QCD’s ground state one employs an effective Lagrangian containing an additional term to the usual QCD Lagrangian [1]:

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} + \theta \frac{g^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}, \quad \tilde{G}_{\mu\nu} = \frac{i}{2} \epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}. \]  

(2)

Since \( G_{\mu\nu} \tilde{G}^{\mu\nu} \) is a gauge invariant operator, its appearance in general cannot be forbidden, and what is not forbidden has to be considered allowed in a quantum field theory. It represents a total divergence, yet in QCD — unlike in QED — it cannot be ignored due to the topological structure of the ground state. Since under parity \( P \) and time reversal \( T \)

\[ G_{\mu\nu} \tilde{G}^{\mu\nu} \overset{P,T}{\Rightarrow} -G_{\mu\nu} \tilde{G}^{\mu\nu}, \]  

(3)

the last term in Eq. (2) violates \( P \) as well as \( T \). Being flavour-diagonal \( G_{\mu\nu} \tilde{G}^{\mu\nu} \) generates an electric dipole moment (EDM) for the neutron. From the upper bound on the latter \( d_N < 0.63 \cdot 10^{-25} \text{ e cm} \) one infers [1] \( \theta < 10^{-9} \). Being the coefficient of a dimension-four operator \( \theta \) can be renormalized to any value, even zero. Yet the modern view of renomalization is more demanding: requiring the renormalized value to be smaller than its
'natural' one by orders of magnitude is frowned upon, since it requires fine tuning between the loop corrections and the counterterms. This is what happens here. For purely within QCD the only intrinsically 'natural' scale for $\theta$ is unity. If $\theta \sim 0.1$ or even $0.01$ were found, one should not be overly concerned. Yet a bound like $\theta < 10^{-9}$ is viewed with great alarm as very unnatural — unless a symmetry can be called upon. If any quark were massless — most likely the $u$ quark — chiral rotations representing symmetry transformations in that case could be employed to remove $\theta$ contributions. Yet a considerable phenomenological body rules against such a scenario.

A much more attractive solution would be provided by transforming $\theta$ from a fixed parameter into the manifestation of a dynamical field — as is done for gauge and fermion masses through the Higgs-Kibble mechanism, see below — and imposing a Peccei-Quinn symmetry that would lead naturally to $\theta \ll \mathcal{O}(10^{-9})$. Alas — this attractive solution does not come ‘for free’: it requires the existence of axions. Those have not been observed despite great efforts to find them.

This is a purely theoretical problem. Yet I consider the fact that it remains unresolved a significant chink in the SM’s armour. I still have not given up hope that ‘victory can be snatched from the jaws of defeat’: establishing a Peccei-Quinn-type solution would be a major triumph for theory.

2. $SU(2)_L \times U(1)$ — not even the Greatest Thing

The requirements of unitarity, which is nonnegotiable, and of renormalizability, which is to some degree, significantly restrict possible theories of the electroweak interactions. There are other strong points as well among them:
⊕ Since there is a single $SU(2)_L$ group, there is a single set of gauge bosons. Their self-coupling also controls how they couple to the fermion fields. As explained later in more detail, this implies the property of ‘weak universality’.

The generation of masses for gauge bosons and fermions is highly nontrivial, yet can be achieved through a feat of theoretical engineering that should be — although rarely is — referred to as the Higgs-Brout-Englert-Guralnik-Hagen-Kibble mechanism. Intriguingly enough the Higgs doublet field can generate masses for gauge bosons and fermions alike. Those masses are controlled by a single vacuum expectation value (VEV) $\langle 0 | \phi | 0 \rangle$ and, in the case of fermions, their Yukawa couplings — a point we will return to.

Despite all the impressive, even amazing successes of the SM the community is not happy with it for several reasons: Unlike for the strong interactions there is no uniqueness about the electroweak gauge group, it provides merely the minimal solution. With it being $SU(2)_L \times U(1)$, only partial unification has been achieved. And then there is the whole issue of family replication.
3. The Family Mystery

The twelve known quarks and leptons are arranged into three families. Those families possess identical gauge couplings and are distinguished only by their mass terms, i.e. their Yukawa couplings. We do not understand this family replication or why there are three families. It is not even clear whether the number of families represents a fundamental quantity or is due to the more or less accidental interplay of complex forces as one encounters when analyzing the structure of nuclei. The only hope for a theoretical understanding we can spot on the horizon is superstring or M theory — which is merely a euphemistic way of saying we hardly have a clue.

Yet the circumstantial evidence that we miss completely a central element of Nature’s ‘Grand Design’ is even stronger in view of the strongly hierarchical pattern in the masses for up- and down-type quarks, charged leptons and neutrinos, and the CKM parameters as discussed later. In any case mass generation in particular for fermions — including neutrinos — and family replication constitute central mysteries of the SM, upon which the known gauge dynamics shed no light. It is for this reason that I had attached an * to the term SM.

I-2. Basics of C, T, CP, and CPT

Charge conjugation exchanges particles P and antiparticles $\bar{P}$ and thus flips the sign of all charges like electric charge, hyper-charge, etc. It is described by a linear operator C. CP transformations include a parity operation as well.

Time reversal is operationally defined as a reversal of motion

$$\langle \vec{p}, \vec{l} \rangle \overset{T}{\longrightarrow} -\langle \vec{p}, \vec{l} \rangle,$$

which follows from $(\vec{r}, t) \overset{T}{\longrightarrow} (\vec{r}, -t)$. While the Euclidean scalar $\vec{l}_1 \cdot \vec{l}_2$ is invariant under the time reversal operator T, the triple correlations of (angular) momenta are not:

$$\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) \overset{T}{\longrightarrow} -\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) \text{ with } \vec{v} = \vec{p}, \vec{l}.$$  

The expectation value of such triple correlations accordingly are referred to as T odd moments.

In contrast to P or C the T operator is antilinear:

$$T(\alpha|a\rangle + \beta|b\rangle) = \alpha^* T|a\rangle + \beta^* T|b\rangle.$$  

This property of T is required to have the commutation relation $[X, P] = i\hbar$ invariant under T:

$$T^{-1}[X, P]T = -[X, P] \text{ and } T^{-1}i\hbar T = -i\hbar.$$  

The anti-linearity of T implies two important properties:
• T violation manifests itself through complex phases. CPT invariance then implies that also CP violation enters through complex phases in the relevant couplings. For T or CP violation to become observable in a decay transition one thus needs the contribution from two different, yet coherent amplitudes.

• While a non-vanishing P odd moment establishes unequivocally P violation, this is not necessarily so for T odd moments; i.e., even T invariant dynamics can generate a non-vanishing T odd moment. T being antilinear comes into play when the transition amplitude is described through second (or even higher) order in the effective interaction, i.e. when final state interactions are included denoted symbolically by

\[
T^{-1}(L_{eff} \Delta t + \frac{i}{2} (L_{eff} \Delta t)^2 + ...) T = L_{eff} \Delta t - \frac{i}{2} (L_{eff} \Delta t)^2 + ... \neq L_{eff} \Delta t + \frac{i}{2} (L_{eff} \Delta t)^2 + ...
\]

(8) even for \([T, L_{eff}] = 0\).

I-3. The Special Role of CP Invariance and its Violation

While the discovery of P violation in the weak dynamics in 1957 caused a well documented shock in the community, even the theorists quickly recovered. Why then was the discovery of CP violation in 1964 through \(K_L \to \pi^+\pi^-\) not viewed as a ‘deja vue all over again’ in the language of Yogi Berra? I know of only one ‘heretic’, namely Okun, who in his 1963 text book [2] explicitly listed the search for \(K_L \to \pi^+\pi^-\) as a priority, i.e. one year before its discovery.

To see what moves physicists, one should not focus on what they say (rarely a good indicator for scientists in general), but on what they do. Point in case: How much this discovery shook the HEP community is best gauged by noting the efforts made to reconcile the observation of \(K_L \to \pi^+\pi^-\) with CP invariance:

• To infer CP violation from \(K_L \to \pi\pi\) one has to invoke the superposition principle of quantum mechanics. One can introduce [3] nonlinear terms into the Schrödinger equation in such a way as to allow \(K_L \to \pi^+\pi^-\) with CP invariant dynamics. While completely ad hoc, it is possible in principle. Such efforts were ruled out by further data, most decisively by \(\Gamma(K^0(t) \to \pi^+\pi^-) \neq \Gamma(\bar{K}^0(t) \to \pi^+\pi^-)\).

• One can try to emulate the success of Pauli’s neutrino hypothesis. An apparent violation of energy-momentum conservation had been observed in \(\beta\) decay \(n \to p e^-\), since the electron exhibited a continuous momentum spectrum. Pauli postulated that the reaction actually was

\[
n \to p e^- \bar{\nu},
\]

(9) with \(\bar{\nu}\) a neutral and light particle that had escaped direct observation, yet led to a continuous spectrum for the electron. That is, Pauli postulated a new particle — and
a most whimsical one at that — to save a symmetry, namely the one under translations in space and time responsible for the conservation of energy and momentum. Likewise it was suggested that the real reaction was

\[ K_L \rightarrow \pi^+\pi^- U , \]

(10)

with \( U \) a neutral and light particle with odd intrinsic \( CP \) parity; i.e., a hitherto unseen particle was introduced to save \( CP \) symmetry. This attempt at evasion was also soon rejected experimentally, as explained later. This represents an example of the ancient Roman saying:

"Quod licet Jovi, non licet bovi."

"What is allowed [the supreme god] Jupiter, is not allowed a bull."

That is, we mere mortals cannot get away with speculations like ‘Jupiter’ Pauli.

There are several reasons itemized below for the physicists’ reluctance to part with \( CP \) symmetry. For \( P \) violation being maximal (in the weak sector) — no right-handed neutrinos couple to the weak interactions — and likewise for \( C \) violation, yet with their combined \( CP \) transformation describing an exact symmetry one had a natural ‘fall back’ position, as described by Oscar Wilde: “... people are attracted to men with a future and women with a past ...”. However the discovery of \( CP \) violation shattered this balanced picture. Furthermore even Luther’s redemption of last resort “peccate fortiter” (“sin boldly”) could not be invoked, since \( CP \) violation announced its arrival with a mere whimper: characterized by \( \text{BR}(K_L \rightarrow \pi^+\pi^-) \approx 0.0023 \) or \( \text{Im}M_{12}/M_K = 2.2 \cdot 10^{-17} \) it appears as the feeblest observed violation of any symmetry. \( CP \) symmetry as a ‘near-miss’ is rather puzzling in view of its fundamental consequences listed next.

Let me start with an analogy from politics. In my days as a student — at a time long ago and a place far away — politics was hotly debated. One of the subjects drawing out the greatest passions was the questions of what distinguished the ‘left’ from the ‘right’. If you listened to it, you quickly found out that people almost universally defined ‘left’ and ‘right’ in terms of ‘positive’ and ‘negative’. The only problem was they could not quite agree who the good guys and the bad guys are.

There arises a similar conundrum when considering decays like \( \pi \rightarrow e\nu \). When saying that a pion decay produces a left handed charged lepton one had \( \pi^- \rightarrow e_L^-\bar{\nu} \) in mind. However \( \pi^+ \rightarrow e_R^+\nu \) yields a right handed charged lepton. ‘Left’ is thus defined in terms of ‘negative’. No matter how much \( P \) is violated, \( CP \) invariance imposes equal rates for these \( \pi^\pm \) modes, and it is untrue to claim that nature makes an absolute distinction between ‘left’ and ‘right’. The situation is analogous to the saying that the thumb is left on the right hand — a correct, yet useless statement, since circular. \( CP \) violation is required to define ‘matter’ vs. ‘antimatter’, ‘left’ vs. ‘right’, ‘positive’ vs. ‘negative’ in a convention independent way.
• Due to the almost unavoidable CPT symmetry, violation of CP implies one of T.

• It is one of the key ingredients in the Sakharov conditions for baryogenesis [6]: to obtain the observed baryon number of our Universe as a dynamically generated quantity rather than an arbitrary initial condition, one needs baryon number violating transitions with CP violation to occur in a period where our Universe had been out of thermal equilibrium.

I-4. On the Observability of CP Violation

Since CPT symmetry confines CP violation to the emergence of complex phases, one needs two coherent, yet different amplitudes contributing to the same transition. The most successful realization of this requirement have been meson-antimeson oscillations, as described in Sect. II-1. The requirement can be met in a different way that can be implemented also for charged mesons P (and baryons), when one has two amplitudes contribute with both different weak phases and different strong phase shifts:

\[ T(P \to f) = e^{i\phi_1,w}e^{i\alpha_1,s}|M_1| + e^{i\phi_2,w}e^{i\alpha_2,s}|M_2|, \]
\[ T(\bar{P} \to \bar{f}) = e^{-i\phi_1,w}e^{i\alpha_1,s}|M_1| + e^{-i\phi_2,w}e^{i\alpha_2,s}|M_2|, \]
where I have factored out the weak and strong phases \( \phi_{i,w} \) and \( \alpha_{i,s} \). Then one finds for the direct CP asymmetry

\[ \Gamma(\bar{P} \to \bar{f}) - \Gamma(P \to f) \propto \sin(\phi_{1,w} - \phi_{2,w})\sin(\alpha_{1,s} - \alpha_{2,s})|M_1||M_2|. \]

I-5. The Heroic Era — CP Violation in \( K_L \) Decays

1. Basic Phenomenology

The discussion here will be given in terms of strangeness \( S \), yet can be generalized to any other flavour quantum number \( F \) like beauty, charm, etc.

Weak dynamics can drive \( \Delta S = 1 \& 2 \) transitions, i.e. decays and oscillations. While the underlying theory has to account for both, it is useful to differentiate between them on the phenomenological level. The interplay between \( \Delta S = 1 \& 2 \) affects also CP violation and how it can manifest itself. Consider \( K_L \to \pi\pi \): while \( \Delta S = 2 \) dynamics transform the flavour eigenstates \( K_0 \) and \( \bar{K}_0 \) into mass eigenstates \( K_L \) and \( K_S \), \( \Delta S = 1 \) forces produce the decays into pions.

\[ [K^0 \xrightarrow{\Delta S=2} \bar{K}^0] \Rightarrow K_L \xrightarrow{\Delta S=1} \pi\pi. \]

Both of these reactions can exhibit CP violation, which is usually expressed as follows:

\[ \eta_{+\rightarrow00} \equiv \frac{T(K_L \to \pi^+\pi^-[\pi^0,\pi^0])}{T(K_S \to \pi^+\pi^-[\pi^0,\pi^0])}, \]
\[ \eta_{+\rightarrow} \equiv \epsilon_K + \epsilon', \quad \eta_{00} \equiv \epsilon_K - 2\epsilon'. \]

Both \( \eta_{+\rightarrow}, \eta_{00} \neq 0 \) signal CP violation; \( \epsilon_K \) is common to both observables and reflects the CP properties of the state mixing that drives oscillations, i.e. in \( \Delta S = 2 \) dynamics; \( \epsilon' \) on
the other hand differentiates between the two final states and parametrizes CP violation in $\Delta S = 1$ dynamics. With an obvious lack in Shakespearean flourish $\epsilon_K \not= 0$ is referred to as ‘indirect’ or ‘superweak’ CP violation and $\epsilon' \not= 0$ as ‘direct’ CP violation. As long as CP violation is seen only through a single mode of a neutral meson — in this case either $K_L \to \pi^+\pi^-$ or $K_L \to \pi^0\pi^0$ — the distinction between direct and indirect CP violation is somewhat arbitrary, as explained later for $B_d$ decays.

Five types of CP violating observables have emerged reflecting the fact that $K_L$ is not an exact mass eigenstate and involving $K^0-\bar{K}^0$ oscillations in one way or another:

- **Existence** of a transition: $K_L \to \pi^+\pi^-, \pi^0\pi^0$;
- An **asymmetry** due to the initial state: $K^0 \to \pi^+\pi^-$ vs. $\bar{K}^0 \to \pi^+\pi^-$;
- An **asymmetry** in the final state: $K_L \to l^+\nu\pi^-$ vs. $\bar{K}_L \to l^-\bar{\nu}\pi^+$, $K_L \to \pi^+\pi^-$ vs. $\bar{K}_L \to \pi^0\pi^0$;
- A **microscopic** $T$ asymmetry: rate($K^0 \to \bar{K}^0$) \not= rate($\bar{K}^0 \to K^0$);
- A $T$ **odd correlation** in the final state: $K_L \to \pi^+\pi^-e^+\bar{e}^-$.\

(i) Using today’s numbers [50],

$$\text{BR}(K_L \to \pi^+\pi^-) = (1.976 \pm 0.008) \cdot 10^{-3}, \quad (16)$$

one derives

$$|\eta_{+-}| = (2.236 \pm 0.007) \cdot 10^{-3}; \quad (17)$$

It allows one to describe also the asymmetry in semileptonic $K_L$ decays

$$\delta_l \equiv \frac{\Gamma(K_L \to l^+\nu\pi^-) - \Gamma(K_L \to l^-\bar{\nu}\pi^+)}{\Gamma(K_L \to l^+\nu\pi^-) + \Gamma(K_L \to l^-\bar{\nu}\pi^+)} = (3.32 \pm 0.06) \cdot 10^{-3}, \quad (18)$$

since

$$\delta_l|_{KL \to 2\pi} \simeq 2\text{Re} \epsilon = (3.16 \pm 0.01) \cdot 10^{-3}. \quad (19)$$

(ii) CPT invariance — an (almost) inescapable property of relativistic local quantum field theories — tells us that for every violation of CP symmetry there has to be a commensurate one for $T$ invariance. Verifying this statement experimentally is far from straightforward though. For in a decay process $A \to B + C$ practical considerations prevent one from creating the time reversed sequence $B + C \to A$.

Meson-antimeson oscillations (and likewise neutrino oscillations) provide unique opportunities to probe $T$ violations. For one can compare directly the rates for $K^0 \to \bar{K}^0$ and $\bar{K}^0 \to K^0$, which is referred to as the ‘Kabir test’ [4], listed as the fourth item above. For that purpose one has to determine the flavour of the final state — $K^0$ or $\bar{K}^0$ — as well as tag the flavour of the initial one. Semileptonic channels can achieve the former through the
SM $\Delta S = \Delta Q$ selection rule. For the latter one can rely on associated production in, say, proton-antiproton annihilation: $p\bar{p} \rightarrow K^+K^0\pi^-$. Thus one compares the sequences $p\bar{p} \rightarrow K^+K^0\pi^- \rightarrow K^+(l^+\bar{\nu}\pi^-)\pi^-$ and $p\bar{p} \rightarrow K^-K^0\pi^+ \rightarrow K^-(l^-\nu\pi^-)\pi^-$. Using this technique the CPLEAR collaboration found [5]:

$$A_T = \frac{\text{rate}(\bar{K}^0 \rightarrow K^0) - \text{rate}(K^0 \rightarrow \bar{K}^0)}{\text{rate}(K^0 \rightarrow \bar{K}^0) + \text{rate}(\bar{K}^0 \rightarrow K^0)} = (6.6 \pm 1.6) \cdot 10^{-3},$$  \hspace{1cm} (20)

in full agreement with what is inferred from $\text{BR}(K_L \rightarrow \pi^+\pi^-)$:

$$A_T |_{K_L \rightarrow 2\pi} \simeq 4\text{Re} \epsilon = (6.32 \pm 0.02) \cdot 10^{-3}.$$

(iii) The fifth item in the list above has a novel aspect to it: it reflects a CP asymmetry in a final state distribution. Consider $K_L \rightarrow \pi^+\pi^-e^+e^-$ and define $\phi$ as the angle between the planes spanned by the two pions and the two leptons in the $K_L$ restframe:

$$\phi \equiv \angle (\vec{n}_l, \vec{n}_\pi), \quad \vec{n}_l = \vec{p}_{e^+} \times \vec{p}_{e^-}/|\vec{p}_{e^+} \times \vec{p}_{e^-}|, \quad \vec{n}_\pi = \vec{p}_{\pi^+} \times \vec{p}_{\pi^-}/|\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}|. \tag{22}$$

One analyzes the decay rate as a function of $\phi$:

$$\frac{d\Gamma}{d\phi} = \Gamma_1\cos^2\phi + \Gamma_2\sin^2\phi + \Gamma_3\cos\phi\sin\phi. \tag{23}$$

Since

$$\cos\phi\sin\phi = (\vec{n}_l \times \vec{n}_\pi) \cdot (\vec{p}_{\pi^+} + \vec{p}_{\pi^-})(\vec{n}_l \cdot \vec{n}_\pi)/|\vec{p}_{\pi^+} + \vec{p}_{\pi^-}| \tag{24}$$

one notes that

$$\cos\phi\sin\phi \xrightarrow{T, CP} - \cos\phi\sin\phi \tag{25}$$

under both $T$ and $CP$ transformations; i.e. the observable $\Gamma_3$ represents a $T$- and $CP$-odd correlation. It can be projected out by comparing the $\phi$ distribution integrated over two quadrants:

$$A = \frac{\int_0^{\pi/2} d\phi \frac{d\Gamma}{d\phi} - \int_{\pi/2}^{\pi} d\phi \frac{d\Gamma}{d\phi}}{\int_0^{\pi} d\phi \frac{d\Gamma}{d\phi}} = \frac{2\Gamma_3}{\pi(\Gamma_1 + \Gamma_2)}. \tag{26}$$

It was first measured by KTEV and then confirmed by NA48 [50]:

$$A = (13.7 \pm 1.5)\%. \tag{27}$$

$A \neq 0$ is induced by $\epsilon_K$, the CP violation in the $K^0 - \bar{K}^0$ mass matrix, leading to the prediction [52]

$$A = (14.3 \pm 1.3)\%. \tag{28}$$
The observed value for the $T$ odd moment $A$ is fully consistent with $T$ violation. Yet $A \neq 0$ by itself does not establish $T$ violation [49].

It is actually easy to see how this sizable forward-backward asymmetry is generated from the tiny quantity $|\eta_{+-}| \approx 0.0023$. For $K_L \rightarrow \pi^+\pi^-e^+e^-$ is driven by the two subprocesses

$$K_L \xrightarrow{CP \, k\Delta S=1} \pi^+\pi^- \rightarrow E_1 \pi^+\pi^-\gamma^* \rightarrow \pi^+\pi^-e^+e^-, \quad (29)$$

$$K_L \xrightarrow{M1&\Delta S=1} \pi^+\pi^-\gamma^* \rightarrow \pi^+\pi^-e^+e^-, \quad (30)$$

where the first reaction is suppressed, since it requires CP violation in $K_L \rightarrow 2\pi$, and the second one, since it involves an $M1$ transition. Those two a priori very different suppression mechanisms happen to yield comparable amplitudes, which thus generate sizable interference. The price one pays is the small branching ratio, namely $BR(K_L \rightarrow \pi^+\pi^-e^+e^-) = (3.32 \pm 0.14 \pm 0.28) \cdot 10^{-7}$. I will revisit the issue of CP violation in final state distributions.

(iv) The fact that $K^0 - \bar{K}^0$ oscillations are involved in an essential way can most directly be established by analyzing $K^0(t) \rightarrow \pi^+\pi^-$ — i.e. the decay rate evolution as a function of the (proper) time of decay $t$ of a beam that initially contained only $K^0$ mesons, see Fig. 1. Comparing it with its CP conjugate $\bar{K}^0(t) \rightarrow \pi^+\pi^-$, as listed as the second item in the list above, one finds clear $t$ dependent asymmetries in both the pure $K_L$ as well as the $K_S - K_L$ interference domains, though not in the pure $K_S$ domain. Studying just $K^0(t) \rightarrow \pi^+\pi^-$ is actually sufficient to establish CP violation due to the following

**Theorem:**
If one finds that the evolution of the decays of an arbitrary linear combination of neutral mesons into a CP eigenstate as a function of (proper) time of decay cannot be described by a single exponential, then CP invariance must be broken. Or formulated more concisely for the case at hand:

$$\frac{d}{dt} e^{\Gamma t} \Gamma(K^{\text{neut}} \to \pi^+ \pi^-) \neq 0 \text{ for all real } \Gamma \implies \text{CP violation!} \quad (31)$$

The proof is elementary: With CP being conserved, mass eigenstates have to be either even or odd CP eigenstates as well and can decay only into final states of the same CP parity. Their decay rate evolution thus has to be given by a single exponential in time; q.e.d.

The fact that the curve in Fig. 1 shows an interference region between the practically pure $K_S$ and $K_L$ regimes provided the conclusive evidence against one of the aforementioned attempts to maintain CP symmetry by postulating that $K_L \to \pi^+ \pi^- U$ was occurring, see Eq. (II-3): for there can be no interference between $K_S \to \pi^+ \pi^-$ and $K_L \to \pi^+ \pi^- U$.

2. Completion of the Heroic Era: Direct CP Violation

In the decades after 1964 dedicated searches for direct CP violation were undertaken. Measurements launched in the 1980’s yielded intriguing, though not conclusive evidence:

$$\frac{\epsilon'}{\epsilon_K} = \begin{cases} (2.30 \pm 0.65) \cdot 10^{-3} & \text{NA31} \\ (0.74 \pm 0.59) \cdot 10^{-3} & \text{E731} \end{cases} \quad (32)$$

Direct CP violation has been unequivocally established in 1999. The present world average dominated by the data from NA48 and KTeV reads as follows [21]:

$$\langle \frac{\epsilon'}{\epsilon_K} \rangle = (1.63 \pm 0.22) \cdot 10^{-3}. \quad (33)$$

Quoting the result in this way does not do justice to the experimental achievement, since $\epsilon_K$ is a very small number itself. The sensitivity achieved becomes more obvious when quoted in terms of actual widths [21]:

$$\frac{\Gamma(K^0 \to \pi^+ \pi^-) - \Gamma(\bar{K}^0 \to \pi^+ \pi^-)}{\Gamma(K^0 \to \pi^+ \pi^-) + \Gamma(\bar{K}^0 \to \pi^+ \pi^-)} = (5.04 \pm 0.82) \cdot 10^{-6}! \quad (34)$$

This represents a discovery of the very first rank.1 Its significance does not depend on whether the SM can reproduce it or not — which is the most concise confirmation of how important it is. The HEP community can take pride in this achievement; the tale behind it is a most fascinating one about imagination and perseverance. The two groups and their predecessors deserve our respect; they have certainly earned my admiration.

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1 As a consequence of Eq. (34) I am not impressed by CPT tests falling short of the $10^{-6}$ level.
I-6. CKM Dynamics — an ‘Accidental Miracle’

The existence of three quark-lepton families that differ only in their mass related parameters — and within the SM thus only in their Yukawa couplings — is one of the profound puzzles about the SM. The latter’s Yukawa sector is indeed its most unsatisfactory feature. Yet this three family structure is an observed fact, and it gives rise to a very rich phenomenology in weak dynamics based on a huge body of data — including CP violation — that so far is fully consistent with the SM’s predictions.

CP violation was discovered in 1964 through the observation of $\mathcal{K}_L \to \pi^+\pi^-$, yet it was not realized for a number of years that dynamics known at that time could not generate it. We should not be too harsh on our predecessors for that oversight: as long as one did not have a renormalizable theory for the weak interactions and thus had to worry about infinities in the calculated rates, one can be excused for ignoring a seemingly marginal rate with a branching ratio of $2 \cdot 10^{-3}$. Yet even after the emergence of the renormalizable Glashow-Salam-Weinberg model its phenomenological incompleteness was not recognized right away. There is a short remark by Mohapatra in a 1972 paper invoking the need for right-handed currents to induce CP violation.

It was the 1973 paper by Kobayashi and Maskawa \cite{8} that fully stated the inability of even a two-family SM to produce CP violation and that explained what had to be added to it: right-handed charged currents, extra Higgs doublets — or (at least) a third quark family. Of the three options Kobayashi and Maskawa listed, their name has been attached only to the last one as the CKM description. They were helped by the ‘genius loci’ of Nagoya University:

- Since it was the home of the Sakata school, and in the Sakata model of elementary particles quarks were viewed as physical degrees of freedom from the start.
- It was also the home of Prof. Niu who in 1971 had observed \cite{9} a candidate for a charm decay in emulsion exposed to cosmic rays and actually recognized it as such. The existence of charm, its association with strangeness, and thus of two complete quark families were thus taken for granted at Nagoya.

Their argument went as follows. The six quark flavours of the SM are arranged in three up-type and three down-type quarks fields that can be written as vectors $U^F = (u,c,t)^F$ and $D^F = (d,s,b)^F$, respectively, in terms of the flavour eigenstates denoted by the superscript $F$. One can form two $3 \times 3$ mass matrices

$$\mathcal{L}_M \propto \bar{U}^F_L \mathcal{M}_U U_R^F + \bar{D}^F_L \mathcal{M}_D D_R^F.$$  \hfill (35)

There is no a priori reason why the matrices $\mathcal{M}_{U/D}$ should be diagonal. Applying bi-unitary rotations $\mathcal{J}_{U/D,L}$ will allow to diagonalize them

$$\mathcal{M}_{U/D}^{\text{diag}} = \mathcal{J}_{U/D,L} \mathcal{M}_{U/D} \mathcal{J}_{U/D,R}^\dagger$$  \hfill (36)

and obtain the mass eigenstates of the quark fields:

$$U^m_{L/R} = \mathcal{J}_{U,L/R} U^F_{L/R}, \quad D^m_{L/R} = \mathcal{J}_{D,L/R} D^F_{L/R}.$$  \hfill (37)
That is the flavour eigenstates ‘mix’ to form the mass eigenstates. The eigenvalues of $\mathcal{M}_{U/D}$ represent the masses of the quark fields. The measured values exhibit a very peculiar hierarchical pattern for up- and down-type quarks, charged and neutral leptons that hardly appears to be accidental.

There is much more to it. The neutral current coupling keeps its form when expressed in terms of the mass eigenstates:

$$\mathcal{L}_{NC}^{U/D} \propto g_Z \bar{U} F \gamma_\mu U^F [D^F] Z^\mu$$

i.e., there are no tree-level flavour changing neutral currents. This important property is referred to as the ‘generalized’ GIM mechanism [7].

The charged currents do change their form when going from flavour to mass eigenstates:

$$\mathcal{L}_{CC} \propto g_W \bar{U}_L \gamma_\mu U^F [W^F] Z^\mu = \bar{U}_m \gamma_\mu V_{CKM} D^m W^\mu , \quad V_{CKM} = J_{U,L} J^t_{D,L}.$$ (39)

While the matrix $V_{CKM}$ has to be unitary within the SM, there is no reason why it should be the identity matrix or even diagonal. It means that the charged current couplings of the mass eigenstates will be modified in an observable way. In which way and by how much this happens requires further analysis, since the phases of fermion fields are not necessarily observables. An $N \times N$ unitary matrix contains $N^2$ independent real parameters. Since the phases of quark fields like other fermion fields can be rotated freely, $2N - 1$ phases can be removed from $\mathcal{L}_{CC}$ reducing the number of independent physical parameters to $(N - 1)^2$. An $N \times N$ orthogonal matrix has $N(N - 1)/2$ angles; thus we can conclude that an $N \times N$ unitary matrix contains $(N - 1)(N - 2)/2$ physical phases in addition. Accordingly for $N = 2$ families we have only the Cabibbo angle and no phases, while for $N = 3$ we obtain three angles and one irreducible phase; i.e., a three family ansatz can support $CP$ violation with a single source — the ‘CKM phase’.

For three families the unitarity of the CKM matrix,

$$V_{CKM} = \begin{pmatrix} V(ud) & V(us) & V(ub) \\ V(cd) & V(cs) & V(cb) \\ V(td) & V(ts) & V(tb) \end{pmatrix},$$ (40)

yields three universality relations

$$\sum_{j=d,s,b} |V(ij)|^2 = 1 , \quad i = u, c, t$$ (41)

as well as six orthogonality conditions

$$\sum_{j=u,c,t} V^*(ji)V(jk) = 0 , \quad i \neq k = d, s, b.$$ (42)

Even if some speculative dynamics were to enforce an alignment between the $U$ and $D$ quark fields at some high scales causing their mass matrices to get diagonalized by the same bi-unitary transformation, this alignment would probably get upset by renormalization down to the electroweak scales.
Eqs. (42) represent triangle relations in the complex plane, a point I will repeatedly return to. Changing the phase conventions for the quark fields will change the orientations of these triangles in the complex plane, but not their internal angles. Those represent the relative phases of the elements of $V_{CKM}$, which in turn can give rise to observable CP asymmetries.

This graphic interpretation also makes it transparent, why the charged currents cannot generate CP violation with two families. In that case the orthogonality relations of the corresponding $2 \times 2$ matrix are trivial stating that two products of a priori complex matrix elements had to add to zero, i.e. cannot exhibit a nontrivial relative phase.

For the three families of the SM there are six triangles. They can and do vary greatly in their shapes, yet have to possess the same area — a consequence of there being just a single CKM phase, as stated above. PDG suggests the following parametrization:

$$V_{CKM} = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{13}s_{23} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} - c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{13}c_{23}
\end{pmatrix}, \tag{43}$$

where

$$c_{ij} \equiv \cos \theta_{ij}, \quad s_{ij} \equiv \sin \theta_{ij} \tag{44}$$

with $i, j = 1, 2, 3$ label the families.

This is a completely general, yet not unique representation: a different set of Euler angles could be chosen; the phases can be shifted around among the matrix elements by using a different phase convention.

The CKM implementation of CP violation depends on the form of the quark mass matrices $M_{U,D}$, not so much on how those are generated. Nevertheless something can be inferred about the latter: within the SM all fermion masses are driven by a single vacuum expectation value of a neutral Higgs field (VEV); to obtain an irreducible relative phase between different quark couplings thus requires such a phase in quark Yukawa couplings; this means that in the SM CP violation arises in dimension-four couplings, i.e., is ‘hard’ in the language of quantum field theory.

I-7. ‘Maximal’ CP Violation?

As already mentioned charged current couplings with their $V - A$ structure break parity and charge conjugation maximally. Since due to CPT invariance CP violation is expressed through couplings with complex phases, one might say that maximal CP violation is characterized by complex phases of $90^\circ$. However this would be fallacious: for by changing the phase convention for the quark fields one can change the phase of a given CKM matrix element and even rotate it away; it will of course re-appear in other matrix elements. For example $|s\rangle \rightarrow e^{i\delta_s} |s\rangle$ leads to $V_{qs} \rightarrow e^{i\delta_s}V_{qs}$ with $q = u, c, t$. In that sense the CKM phase is like the ‘Scarlet Pimpernel’: "Sometimes here, sometimes there, sometimes everywhere."

One can actually illustrate with a general argument, why there can be no straightforward definition for maximal CP violation. Consider neutrinos: Maximal CP violation
means there are $\nu_L$ and $\bar{\nu}_R$, yet no $\nu_R$ or $\bar{\nu}_L$. Likewise there are $\nu_L$ and $\bar{\nu}_R$, but not $\bar{\nu}_L$ or $\nu_R$. One might then suggest that maximal CP violation means that $\nu_L$ exists, but $\bar{\nu}_R$ does not. Alas — CPT invariance already enforces the existence of both.

Similarly — and maybe more obviously — it is not clear what maximal T violation would mean although some formulations have entered daily language like the ‘no future generation’, the ‘woman without a past’ or the ‘man without a future’.

II. THE SM’S PARADIGM OF LARGE CP VIOLATION IN $B$ DECAYS BEFORE 2000

II-1. Basics

Since $\text{CP}$ violation enters the dynamics through complex couplings, one needs two different amplitudes contributing coherently to a reaction for an observable $\text{CP}$ asymmetry to emerge. In 1979 it was pointed out that $B^0 - \bar{B}^0$ oscillations are well suited to satisfy this requirement for final states $f$ that can be fed both by $B^0$ and $\bar{B}^0$ decays, in particular since those oscillation rates were expected to be sizable [11]:

$$B^0 \Rightarrow \bar{B}^0 \to f \leftarrow B^0 \quad \text{vs.} \quad \bar{B}^0 \Rightarrow B^0 \to \bar{f} \leftarrow \bar{B}^0.$$  \hspace{1cm} (45)

In 1980 it was predicted [12] that in particular $B_d \to \psi K_S$ should exhibit a $\text{CP}$ asymmetry larger by two orders of magnitude than the corresponding one in $K^0 \to 2\pi$ vs. $\bar{K}^0 \to 2\pi$, if CKM theory provides the main driver of $K_L \to \pi^+\pi^-$; even values close to 100 % were suggested as conceivable. The analogous mode $B_s \to \psi\phi$ should however show an asymmetry not exceeding the few percent level.

It was also suggested that in rare modes like $\bar{B}_d \to K^-\pi^+$ sizable direct $\text{CP}$ violation could emerge due to the intervention of ‘Penguin’ operators [13].

We now know that these predictions were rather prescient. It should be noted that at the time of these predictions very little was known about $B$ mesons. While their existence had been inferred from the discovery of the $\Upsilon(1S - 4S)$ family at FNAL in 1977ff, none of their exclusive decays had been identified, and their lifetimes were unknown as were a fortiori their oscillation rates. While very little was thus known about the values of the contributing CKM parameters the relevant formalism for $\text{CP}$ asymmetries involving $B^0 - \bar{B}^0$ oscillations was already fully given, inspired by what we had learnt from the $K^0 - \bar{K}^0$ complex.

To describe oscillations in the presence of $\text{CP}$ violation one applies the Weisskopf-Wigner approximation [10] and turns to solving a nonrelativistic Schrödinger equation, which I formulate for the general case of a pair of neutral mesons $P^0$ and $\bar{P}^0$ with flavour quantum number $F$; it can denote a $K^0$, $D^0$ or $B^0$:

$$\frac{d}{dt}\begin{pmatrix} P^0 \\ \bar{P}^0 \end{pmatrix} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix} \begin{pmatrix} P^0 \\ \bar{P}^0 \end{pmatrix},$$  \hspace{1cm} (46)

$^3$ To be more precise: $\nu_L$ and $\bar{\nu}_R$ couple to weak gauge bosons, $\nu_R$ or $\bar{\nu}_L$ do not.
CPT invariance imposes
\[ M_{11} = M_{22} \quad \text{and} \quad \Gamma_{11} = \Gamma_{22}. \] (47)

The subsequent discussion might strike the reader as overly technical, yet I hope (s)he will bear with me since these remarks will lay important groundwork for a proper understanding of CP asymmetries in B decays.

The mass eigenstates obtained through diagonalising this matrix are given by (for details see [1])
\[ |P_{A|B}\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \left( p|P^0\rangle + [-]q|\bar{P}^0\rangle \right) \] (48)
with eigenvalues
\[ M_{A|B} - \frac{i}{2} \Gamma_{A|B} = M_{11} - \frac{i}{2} \Gamma_{11} + [-]\frac{1}{p} \left( M_{12} - \frac{i}{2} \Gamma_{12} \right), \] (49)
as long as
\[ \left( \frac{q}{p} \right)^2 = \frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}} \] (50)
holds. I am using letter subscripts A and B for labeling the mass eigenstates rather than numbers 1 and 2 as is usually done, for I want to avoid confusing them with the matrix indices 1, 2 in \( M_{ij} - \frac{i}{2} \Gamma_{ij} \).

Eqs. (49) yield for the differences in mass and width
\[ \Delta M \equiv M_B - M_A = -2 \text{Re} \left[ \frac{q}{p} \left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \right], \] (51)
\[ \Delta \Gamma \equiv \Gamma_A - \Gamma_B = -2 \text{Im} \left[ \frac{q}{p} \left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \right]. \] (52)
Note that the subscripts \( A, B \) have been swapped in going from \( \Delta M \) to \( \Delta \Gamma \). This is done to have both quantities positive for kaons.

In expressing the mass eigenstates \( P_A \) and \( P_B \) explicitly in terms of the flavour eigenstates — Eq. (48) — one needs \( \frac{q}{p} \). There are two solutions to Eq. (50):
\[ \frac{q}{p} = \pm \sqrt{\frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}} \] (53)
There is actually a more general ambiguity than this binary one, for antiparticles are defined up to a phase only:
\[ \text{CP}|P^0\rangle = \eta|\bar{P}^0\rangle \quad \text{with} \quad |\eta| = 1. \] (54)
Adopting a different phase convention will change the phase for $M_{12} - \frac{i}{2} \Gamma_{12}$ as well as for $q/p$:

\[ |\bar{P}^0\rangle \rightarrow e^{i\xi} |\bar{P}^0\rangle \implies (M_{12}, \Gamma_{12}) \rightarrow e^{i\xi} (M_{12}, \Gamma_{12}) \& \frac{q}{p} \rightarrow e^{-i\xi} \frac{q}{p}, \tag{55} \]

yet leave $(q/p)(M_{12} - \frac{i}{2} \Gamma_{12})$ invariant — as it has to be since the eigenvalues, which are observables, depend on this combination, see Eq. (49). Also $\left|\frac{q}{p}\right|$ is an observable; its deviation from unity is one measure of CP violation in $\Delta F = 2$ dynamics.

By convention most authors pick the positive sign in Eq. (53)

\[ \frac{q}{p} = \sqrt{M_{12}^* - \frac{i}{2} \Gamma_{12}^*} \] \[ \frac{M_{12} - \frac{i}{2} \Gamma_{12}}{M_{12}^* - \frac{i}{2} \Gamma_{12}^*} \tag{56} \]

Up to this point the two states $|P_{A,B}\rangle$ are merely labelled by their subscripts. Indeed $|P_A\rangle$ and $|P_B\rangle$ switch places when selecting the minus rather than the plus sign in Eq. (53).

One can define the labels $A$ and $B$ such that $\Delta M \equiv M_B - M_A > 0$ is satisfied. Once this convention has been adopted, it becomes a sensible question whether $\Gamma_B > \Gamma_A$ or $\Gamma_B < \Gamma_A$ holds, i.e. whether the heavier state is shorter or longer lived.

One can write the general mass eigenstates in terms of the CP eigenstates as well:

\[ |P_A\rangle = \frac{1}{\sqrt{1 + |\bar{\epsilon}|^2}} (|P_+\rangle + \bar{\epsilon}|P_-\rangle), \quad CP|P_\pm\rangle = \pm|P_\pm\rangle, \tag{57} \]

\[ |P_B\rangle = \frac{1}{\sqrt{1 + |\bar{\epsilon}|^2}} (|P_-\rangle + \bar{\epsilon}|P_+\rangle); \tag{58} \]

$\bar{\epsilon} = 0$ means that the mass and CP eigenstates coincide, i.e. CP is conserved in $\Delta F = 2$ dynamics driving $P^0 - \bar{P}^0$ oscillations. With the phase between the orthogonal states $|P_+\rangle$ and $|P_-\rangle$ arbitrary, the phase of $\bar{\epsilon}$ can be changed at will and is not an observable; $\bar{\epsilon}$ can be expressed in terms of $\frac{q}{p}$, yet in a way that depends on the convention for the phase of antiparticles. For $CP|P^0\rangle = |ar{P}^0\rangle$ one has

\[ \bar{\epsilon} = \frac{1 - \frac{q}{p}}{1 + \frac{q}{p}}. \tag{59} \]

Later I will describe how to evaluate $M_{12}$ and thus also $\Delta M$ within a given theory for the $P^0 - \bar{P}^0$ complex. The examples just listed illustrate that some care has to be applied in interpreting such results. For expressing mass eigenstates explicitly in terms of flavour eigenstates involves some conventions. Once adopted we have to stick with a convention; yet our original choice cannot influence observables.

Decay rates for CP conjugate channels of B mesons specifically can be expressed as follows:

\[ \text{rate}(B^0|\bar{B}^0|(t) \rightarrow f[\bar{f}]) = e^{-\Gamma_B t} G_f(t)[\bar{G}_{\bar{f}}], \tag{60} \]
where \textbf{CPT} invariance has been invoked to assign the same lifetime $\Gamma^{-1}_B$ to $B$ and $\bar{B}$ hadrons. Obviously if

$$\frac{G_f(t)}{\bar{G}_f(t)} \neq 1$$

(61)

is observed, CP violation has been found. Yet one should keep in mind that this can manifest itself in two (or three) qualitatively different ways:

1.

$$\frac{G_f(t)}{G_f(t)} \neq 1 \text{ with } d\frac{G_f(t)}{dt} = 0 ;$$

(62)

i.e., the asymmetry is the same for all times of decay. This is true for \textit{direct} CP violation; yet, as explained later, it also holds for CP violation \textit{in the oscillations}.

2.

$$\frac{G_f(t)}{\bar{G}_f(t)} \neq 1 \text{ with } d\frac{G_f(t)}{dt} \neq 0 ;$$

(63)

here the asymmetry varies as a function of the time of decay. This can be referred to as CP violation \textit{involving} oscillations.

A straightforward application of quantum mechanics with its linear superposition principle yields [1] for $\Delta \Gamma = 0$, which holds for $B^\pm$ and $\Lambda_b$ exactly and for $B_d$ to a good approximation:\footnote{Later I will address the scenario with $B_s$, where $\Delta \Gamma$ presumably reaches a measurable level.}

$$G_f(t) = |T_f|^2 \left[ \left( 1 + \left| \frac{e^2}{q} \right|^2 |\bar{\rho}_f|^2 \right) + \left( 1 - \left| \frac{e^2}{q} \right|^2 |\bar{\rho}_f|^2 \right) \cos \Delta m_B t - 2(\sin \Delta m_B t) \Im \frac{\bar{\rho}_f}{T_f} \right],$$

$$\bar{G}_f(t) = |\bar{T}_f|^2 \left[ \left( 1 + \left| \frac{\bar{e}^2}{q} \right|^2 |\rho_f|^2 \right) + \left( 1 - \left| \frac{\bar{e}^2}{q} \right|^2 |\rho_f|^2 \right) \cos \Delta m_B t - 2(\sin \Delta m_B t) \Im \frac{\rho_f}{\bar{T}_f} \right].$$

(64)

The amplitudes for the instantaneous $\Delta B = 1$ transition into a final state $f$ are denoted by $T_f = T(B \rightarrow f)$ and $\bar{T}_f = T(\bar{B} \rightarrow f)$ and

$$\bar{\rho}_f = \frac{\bar{T}_f}{T_f}, \quad \rho_f = \frac{T_f}{\bar{T}_f}.$$ 

(65)

Staring at the general expression is not always very illuminating; let us therefore consider three limiting cases:
For a flavour-specific mode one has in general considers a transition that requires oscillations to take place.

This situation can therefore be denoted also in the following way:

$$\Delta m_B = 0, \text{ i.e. } no \ B^0 - \bar{B}^0 \text{ oscillations:}$$

$$G_f(t) = 2|T_f|^2, \quad \bar{G}_f(t) = 2|\bar{T}_f|^2 \sim \frac{\bar{G}_f(t)}{G_f(t)} = \left| \frac{\bar{T}_f}{T_f} \right|^2, \quad \frac{d}{dt} G_f(t) \equiv 0 \equiv \frac{d}{dt} \bar{G}_f(t). (66)$$

This is explicitly what was referred to above as direct CP violation.

\[ \Delta m_B \neq 0 \text{ and } f \text{ a flavour-specific final state with no direct CP violation; i.e., } T_f = 0 = \bar{T}_f \text{ and } \bar{T}_f = T_f; \]

$$G_f(t) = \left| \frac{q}{p} \right|^2 |T_f|^2 (1 - \cos \Delta m_B t), \quad \bar{G}_f(t) = \left| \frac{q}{p} \right|^2 |\bar{T}_f|^2 (1 - \cos \Delta m_B t) \sim \frac{\bar{G}_f(t)}{G_f(t)} = \left| \frac{q}{p} \right|^4, \quad \frac{d}{dt} G_f(t) \equiv 0, \quad \frac{d}{dt} \bar{G}_f(t) \neq 0 \neq \frac{d}{dt} G_f(t). (67)$$

This constitutes CP violation in the oscillations, for the CP conserving decay into the flavour-specific final state is used merely to track the flavour identity of the decaying meson. This situation can therefore be denoted also in the following way:

$$\frac{\text{Prob}(B^0 \Rightarrow \bar{B}^0; t) - \text{Prob}(\bar{B}^0 \Rightarrow B^0; t)}{\text{Prob}(B^0 \Rightarrow \bar{B}^0; t) + \text{Prob}(\bar{B}^0 \Rightarrow B^0; t)} = \frac{|q/p|^2 - |p/q|^2}{|q/p|^2 + |p/q|^2} = 1 - \frac{|p/q|^4}{1 + |p/q|^4}. (68)$$

$$\Delta m_B \neq 0 \text{ with } f \text{ now being a flavour-nonspecific final state — a final state common to } B^0 \text{ and } \bar{B}^0 \text{ decays — of a special nature, namely a CP eigenstate — } |\tilde{f}⟩ = CP|f⟩ = ±|f⟩ \text{ — without direct CP violation — } |\bar{\rho}_f⟩ = 1 = |\rho_f⟩:$$

$$G_f(t) = 2|T_f|^2 \left[ 1 - (\sin \Delta m_B t) \cdot \text{Im} \frac{q}{p} \tilde{\rho}_f \right],$$

$$\bar{G}_f(t) = 2|\bar{T}_f|^2 \left[ 1 + (\sin \Delta m_B t) \cdot \text{Im} \frac{q}{p} \tilde{\rho}_f \right],$$

$$\sim \frac{d}{dt} \bar{G}_f(t) \neq 0,$$

$$\frac{G_f(t) - \bar{G}_f(t)}{G_f(t) + \bar{G}_f(t)} = (\sin \Delta m_B t) \cdot \text{Im} \frac{q}{p} \tilde{\rho}_f,$$

is the concrete realization of what was called CP violation involving oscillations.

For \( f \) still denoting a CP eigenstate, yet with \( |\bar{\rho}_f| \neq 1 \) one has the more complex asymmetry expression

$$\frac{\bar{G}_f(t) - G_f(t)}{G_f(t) + \bar{G}_f(t)} = S_f \cdot (\sin \Delta m_B t) - C_f \cdot (\cos \Delta m_B t) \quad (70)$$

with

$$S_f = \frac{2 \text{Im} \frac{q}{p} \bar{\rho}_{\pi^+ \pi^-}}{1 + \left| \frac{q}{p} \bar{\rho}_{\pi^+ \pi^-} \right|^2}, \quad C_f = \frac{1 - \left| \frac{q}{p} \bar{\rho}_{\pi^+ \pi^-} \right|^2}{1 + \left| \frac{q}{p} \bar{\rho}_{\pi^+ \pi^-} \right|^2}. \quad (71)$$

\[ \text{For a flavour-specific mode one has in general } T_f \cdot T_f = 0; \text{ the more intriguing case arises when one considers a transition that requires oscillations to take place.} \]
An obvious, yet still useful criterion for CP observables is that they must be ‘rephasing’ invariant under $|\bar{B}_0\rangle \to e^{-i\xi}|\bar{B}_0\rangle$. The expressions above show that there are three classes of such observables:

- An asymmetry in the instantaneous transition amplitudes for CP conjugate modes: 
  $$|T(B \to f)| \neq |T(\bar{B} \to \bar{f})| \iff \Delta B = 1.$$  
  (72)
  It reflects pure $\Delta B = 1$ dynamics and thus amounts to direct CP violation. Those modes are most likely to be nonleptonic; in the SM they practically have to be.

- CP violation in $B^0 - \bar{B}^0$ oscillations: 
  $$|q| \neq |p| \iff \Delta B = 2.$$  
  (73)
  It requires CP violation in $\Delta B = 2$ dynamics. The theoretically cleanest modes here are semileptonic ones due to the SM $\Delta Q = \Delta B$ selection rule.

- CP asymmetries involving oscillations: 
  $$\text{Im} \frac{q}{p} S_f \neq 0, \quad S_f = \frac{T(\bar{B} \to f)}{T(B \to f)} \iff \Delta B = 1\&2.$$  
  (74)

Such an effect requires the interplay of $\Delta B = 1\&2$ forces.

While $C_f \neq 0$ unequivocally signals direct CP violation in Eq. (70), the interpretation of $S_f \neq 0$ is more complex. (i) As long as one has measured $S_f$ only in a single mode, the distinction between direct and indirect CP violation — i.e. CP violation in $\Delta B = 1$ and $\Delta B = 2$ dynamics — is convention dependent, since a change in phase for $B^0 (\sim |B_0^0 \to e^{-i\xi}|B_0^0\rangle)$ leads to $\rho_f \to e^{-i\xi}\bar{\rho}_f$ and $(q/p) \to e^{i\xi}(q/p)$, i.e. one can shift any phase from $(q/p)$ to $\rho_f$ and back while leaving $(q/p)\rho_f$ invariant. However once $S_f$ has been measured for two different final states $f$, then the distinction becomes truly meaningful independent of theory: $S_{f_1} \neq S_{f_2}$ implies $(q/p)\rho_{f_1} \neq (q/p)\rho_{f_2}$ and thus $\rho_{f_1} \neq \rho_{f_2}$, i.e. CP violation in the $\Delta B = 1$ sector. One should note that this direct CP violation might not generate a $C_f$ term, see Sect. I-4. For $\rho_{f_1} = e^{i\phi_1,w}$ and $\rho_{f_2} = e^{i\phi_2,w}$ causing $S_{f_1} \neq S_{f_2}$ would both lead to $C_{f_1} = 0 = C_{f_2}$.

In summary: to observe such CP asymmetries in $B^0$ decays, one hopes for three conditions to be satisfied to a sufficient degree:

- With the asymmetry parameter given by $\text{Im} \frac{q}{p} \rho_f$, one needs weak complex phases to enter through $\Delta B = 2$ and/or $\Delta B = 1$ dynamics — $\frac{q}{p}$ and/or $\rho_f$, respectively.

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6 This condition is formulated for the simplest case of $f$ being a CP eigenstate.
The coefficient of the asymmetry parameter — $\sin\Delta M_B t$ — reflects the presence of oscillations needed to provide the second amplitude. With $\Delta M_B$ denoting the oscillation rate one wishes for $\Delta M_B \simeq \Gamma_B$ as the optimal scenario: while $\Delta M_B \ll \Gamma_B$ would suppress the signal, $\Delta M_B \gg \Gamma_B$ would provide greater experimental challenges in resolving the signal. This can be read off from the time-integrated asymmetry obtained from Eqs. (60,69), which carries the factor $x/(1 + x^2)$ with $x = \Delta M_B/\Gamma_B$: it is maximal for $x = 1$, vanishes like $x$ for $x \rightarrow 0$ and like $1/x$ for $x \rightarrow \infty$.

While time integrated $\text{CP}$ asymmetries can be extracted (from data well above threshold), being able to measure the peculiar time dependence predicted adds greatly to the experimental sensitivity and provides an excellent tool to reject background; i.e., one wishes not only $x \sim 1$, but also $\Gamma_B$ and thus $\Delta M_B$ sufficiently small so that decay and oscillation times can be resolved experimentally.

Nature provided us with a generous gift in this context. Neutral charm mesons had been observed with lifetimes around $4 \cdot 10^{-13}$ sec in the late 1970’s. If the three times heavier $B$ mesons had a lifetime around $10^{-12}$ sec, then one could rely on the microvertex detectors developed for charm lifetime measurements to track the decay rate evolution of $B$ mesons as well.

II-2. The First Central Pillar of the Paradigm: Long Lifetimes

Beauty, the existence of which had been telegraphed by the discovery of the $\tau$ as the third charged lepton, was indeed observed exhibiting a surprising feature: starting in the early 1980’s its lifetime was found to be about $10^{-12}$ sec. This was considered ‘long’, for one can get an estimate for $\tau(B)$ by relating it to the muon lifetime:

$$\tau(B) \simeq \tau_\mu \sim \tau(\mu) \left(\frac{m(\mu)}{m(b)}\right)^5 \frac{1}{9} \frac{1}{|V(cb)|^2} \simeq 3 \cdot 10^{-14} \left|\frac{\sin \theta_C}{|V(cb)|}\right|^2 \sec . \quad (75)$$

One had expected $|V(cb)|$ to be suppressed, since it represents an out-of-family coupling. Yet one had assumed without deeper reflection that $|V(cb)| \sim \sin \theta_C$ — what else could it be? The measured value for $\tau(B)$ however pointed to $|V(cb)| \sim |\sin \theta_C|^2$. By the end of the millenium one had obtained a rather accurate value: $\tau(B_d) = (1.55 \pm 0.04) \cdot 10^{-12}$ s. Now the data have become even more precise:

$$\tau(B_d) = (1.530 \pm 0.009) \cdot 10^{-12} \sec , \quad \tau(B^\pm)/\tau(B_d) = 1.071 \pm 0.009 . \quad (76)$$

The lifetime ratio, which reflects the impact of hadronization, had been predicted [14] successfully well before data of the required accuracy had been available.

II-3. The Second Central Pillar: $B^0 - \bar{B}^0$ Oscillations

Many lessons learnt from $K^0 - \bar{K}^0$ oscillations have been applied profitably to $B^0 - \bar{B}^0$ oscillations. The generalized mass matrix introduced in Sect. II-1 is generated from $\Delta B = 2$ dynamics:

$$\mathcal{M}_{12} = M_{12} + \frac{i}{2} \Gamma_{12} = \langle B^0|\mathcal{L}_{eff}(\Delta B = 2)|\bar{B}^0 \rangle . \quad (77)$$
In the SM $\mathcal{L}_{\text{eff}}(\Delta B = 2)$ is produced by iterating two $\Delta B = 1$ operators:

$$\mathcal{L}_{\text{eff}}(\Delta B = 2) = \mathcal{L}(\Delta B = 1) \otimes \mathcal{L}(\Delta B = 1).$$

(78)

This leads to the well known quark box diagrams, which generate a local [short-distance] $\Delta B = 2$ operator for $M_{12} [\Gamma_{12}]$. The contributions that do not depend on the mass of the internal quarks cancel against each other due to the GIM mechanism, which leads to highly convergent diagrams. The situation is actually simpler and under better theoretical control than for the $\Delta S = 2$ case: (i) The matrix $M$ is dominated by a single contribution, namely from $t \bar{t}$ internal quarks. (ii) The matrix $\Gamma$ receives its leading contribution from $c \bar{c}$ internal quarks, and it is a reasonable ansatz (albeit not a guaranteed one) that also $\Delta \Gamma$ can be evaluated using the quark box diagram.

The overall leading contribution is obtained by evaluating the quark box diagram with internal $W$ and top quark lines corresponding to integrating those heavy degrees of freedom out in a straightforward way leading to

$$\mathcal{L}_{\text{eff}}(\Delta B = 2, \mu) \simeq \left( \frac{G_F}{4\pi} \right)^2 M_W^2 \cdot \xi_i^2 \mathcal{E}(x_i) \eta_{tt}(\bar{q}\gamma\mu(1-\gamma_5)b)^2 + \text{h.c.},$$

(79)

with $q = d, s$, $\xi_i = V(is)V^*(id)$, $\mu = c, t$, $\eta_{tt} \simeq 0.57 \pm 0.01$ denoting the QCD radiative correction, $x_i = m_t^2/M_W^2$, and $\mathcal{E}(x_i)$ reflecting the box loop with internal $W$ and top quarks:

$$\mathcal{E}(x_i) = x_i \left( \frac{1}{4} + \frac{9}{4(1-x_i)} - \frac{3}{2(1-x_i)^2} \right) - \frac{3}{2} \left( \frac{x_t}{1-x_t} \right)^3 \log x_t. \quad (80)$$

Evaluating $\Delta M_B$ as a function of the top quark mass one obtains from Eq. (80)

$$\Delta M_B \propto \left( \frac{m_t}{M_W} \right)^2 \text{ for } m_t \gg M_W. \quad (81)$$

The factor on the right hand side reflects the familiar GIM suppression for $m_t \ll M_W$. Yet for $m_t \gg M_W$ it represents a huge enhancement that would seem to contradict the expected decoupling, for it would mean that the low energy observable $\Delta M_B$ is controlled more and more by a field, namely that for the top quark, at asymptotically high scales. The resolution of this seeming paradox arises in an intriguing way: the massive $W$ boson has ‘ancestors’, namely the original massless gauge boson forming the transverse components and the charged scalar component of the Higgs doublet field introduced to drive electroweak symmetry breaking, which is re-incarnated as the longitudinal $W$ component. The latter, for which there is no decoupling theorem, generates the $(m_t/M_W)^2$ contribution.

There are two types of $B^0$ mesons that can oscillate, namely the $B_d$ and $B_s$ mesons, with the reliable SM predictions

$$\Delta M_{B_d} \ll \Delta M_{B_s}, \quad \Delta \Gamma_{B_d} \ll \Delta \Gamma_{B_s}, \quad \Delta \Gamma_B \ll \Delta M_B,$$

(82)

and accordingly (since $\Gamma_{B_d} \simeq \Gamma_{B_s}$)

$$x_d \equiv \frac{\Delta M_{B_d}}{\Gamma_{B_d}} \ll x_s \equiv \frac{\Delta M_{B_s}}{\Gamma_{B_s}}. \quad (83)$$

(83)
Due to the SM selection rule $\Delta B = \Delta Q$, $x$ can be extracted from the ratio of ‘wrong-sign’ vs. ‘right-sign’ leptons in semileptonic $B^0$ decays:

$$r_B \equiv \frac{\Gamma(\bar{B}^0 \to l^+\nu X^-)}{\Gamma(B^0 \to l^-\nu X^+)} = \frac{x^2}{2 + x^2}, \quad \chi_B \equiv \frac{\Gamma(\bar{B}^0 \to l^+\nu X^-)}{\Gamma(B^0 \to l^-\nu X^+)} = \frac{r_B}{1 + r_B}. \quad (84)$$

1. The Discovery of $B_d - \bar{B}_d$ Oscillations

When ARGUS discovered $B_d - \bar{B}_d$ oscillations in 1986 it caused quite a stir, for their observation of $x \sim 0.7$ was much larger than the quantitative theoretical predictions given before. Yet in all fairness one should understand the main reason behind this underestimate: $\Delta M_{B_d}$ is very sensitive to the value of the top quark mass $m_t$. In the early 1980’s there had been an experimental claim by UA1 to have discovered top quarks in $p\bar{p}$ collisions with $m_t = 40 \pm 10$ GeV. To their later chagrin theorists by and large had accepted this claim. Yet after the ARGUS discovery theorists quickly concluded that top quarks had to be much heavier than previously thought, namely $m_t > 100$ GeV [1]; this was the first indirect evidence for top quarks being ‘super-heavy’ before they were discovered in $p\bar{p}$ collisions at Fermilab. Present data tell us [50]

$$x_d = \frac{\Delta M_{B_d}}{\Gamma_{B_d}} = 0.776 \pm 0.008, \quad (85)$$
$$m_t = 174.2 \pm 3.3 \text{ GeV}. \quad (86)$$

The measured values of $\Delta M_{B_d}$ and $m_t$ are completely compatible with the SM.

The important consequence for the future was that the observed value for $x_d$ is close to optimal for studying CP violation, as explained before. This gave great impetus to the plans for building a $B$ factory.

2. The ‘hot’ news: $B_s - \bar{B}_s$ oscillations

Nature has actually provided us with an ‘encore’. As explained above, within the SM one predicts $\Delta M(B_s) \gg \Delta M(B_d)$, i.e. that $B_s$ mesons oscillate much faster than $B_d$ mesons. Those rapid oscillations have been resolved now [15, 16]:

$$\Delta M_{B_s} = \begin{cases} (19 \pm 2) \text{ ps}^{-1} = (1.25 \pm 0.13) \cdot 10^{-2} \text{ eV} & \text{D0} \\ (17.77 \pm 0.12) \text{ ps}^{-1} = (1.17 \pm 0.01) \cdot 10^{-2} \text{ eV} & \text{CDF} \end{cases}, \quad (87)$$
$$x_s = \frac{\Delta M_{B_s}}{\Gamma_{B_s}} \simeq 25, \quad (88)$$

to be compared with the theoretical prediction:

$$\Delta M_{B_s} = (18.3^{+6.5}_{-1.5}) \text{ ps}^{-1} = (1.20^{+0.43}_{-0.10}) \cdot 10^{-2} \text{ eV} \quad \text{CKM fit}. \quad (89)$$

This finding represents another triumph of CKM theory even more impressive than a mere comparison of the observed and predicted values of $\Delta M_{B_s}$, as explained later.
There is also marginal evidence for $\Delta \Gamma_{B_s} \neq 0$ [17],

$$\frac{\Delta \Gamma_{B_s}}{\Gamma_{B_s}} = 0.31 \pm 0.13.$$  \hfill (90)

II-4. Large CP Asymmetries in $B$ Decays Without ‘Plausible Deniability’

With both the lifetime and oscillation rate falling into the aforementioned practically optimal range, and thus satisfying two of the conditions listed at the end of Sect. II-1, one turns to the third one concerning large weak phases.

The above mentioned observation of a long $B$ lifetime pointed to $|V_{cb}| \sim \mathcal{O}(\lambda^2)$ with $\lambda = \sin \theta_C$. Together with the observation $|V_{ub}| \ll |V_{cb}|$ and coupled with the assumption of three-family unitarity this allows expanding the CKM matrix in powers of $\lambda$, which yields the following most intriguing result through order $\lambda^5$, as first recognized by Wolfenstein:

$$V_{CKM} = \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i \eta + \frac{i}{2} \eta \lambda^2) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 - i A^2 \lambda^4 & A \lambda^2 (1 + i \eta \lambda^2) \\
A \lambda^3 (1 - \rho - i \eta) & A \lambda^2 (1 + i \eta \lambda^2) & 1
\end{pmatrix}. \hfill (91)
$$

The three Euler angles and one complex phase of the representation given in Eq. (43) are taken over by the four real quantities $\lambda$, $A$, $\rho$, and $\eta$; $\lambda$ is the expansion parameter with $\lambda \ll 1$, whereas $A$, $\rho$, and $\eta$ are a priori of order unity. The ‘long’ lifetime of beauty hadrons of around 1 psec together with beauty’s affinity to transform itself into charm, and the assumption of only three quark families tell us that the CKM matrix exhibits a very peculiar hierarchical pattern in powers of $\lambda$:

$$V_{CKM} = \begin{pmatrix}
1 & \mathcal{O}(\lambda) & \mathcal{O}(\lambda^3) \\
\mathcal{O}(\lambda) & 1 & \mathcal{O}(\lambda^2) \\
\mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & 1
\end{pmatrix}, \quad \lambda = \sin \theta_C. \hfill (92)
$$

We know within the SM this matrix has to be unitary. Yet in addition it is almost the identity matrix, almost symmetric and the moduli of its elements shrink with the distance from the diagonal. It has to contain a message from nature — albeit in a highly encoded form.

Among the six unitarity triangles is a special one, shown in Fig. 2. The sides of this triangle are given by $\lambda \cdot V_{cb}$, $V_{ub}$, and $V^*(td)$. Therefore their lengths are of the same order, namely $\lambda^3$ and their angles thus naturally large, i.e. $\sim 10$ degrees. The sides control the rates for CKM favoured and disfavoured $B_{u,d}$ decays and $B_d - \bar{B}_d$ oscillations, and the angles their CP asymmetries.

Some comments on notation might be useful. The BABAR collaboration and its followers refer to the three angles of the CKM unitarity triangle as $\alpha$, $\beta$, and $\gamma$; the BELLE collaboration instead has adopted the notation $\phi_1$, $\phi_2$, and $\phi_3$. While it poses no problem to be conversant in both languages, the latter has not only historical priority on its side [19], but is also more rational. For the angles $\phi_i$ in the ‘bd’ triangle of Fig.(2) are always
opposite the side defined by $V^*(id)V(ib)$. Furthermore this classification scheme can readily be generalized to all six unitarity triangles; those triangles can be labeled by $kl$ with $k \neq l = d, s, b$ or $k \neq l = u, c, t$. Its 18 angles can then be unambiguously denoted by $\phi_{kl}^i$: it is the angle in triangle $kl$ opposite the side $V^*(ik)V(il)$ or $V^*(ki)V(li)$, respectively. Therefore I view the notation $\phi_{1}^{(kl)}$ as the only truly Cartesian one.

The discovery of $B_d$ oscillations defined the ‘CKM Paradigm of Large CP Violation in $B$ Decays’ that had been anticipated in 1980:

- A host of nonleptonic $B$ channels has to exhibit sizable CP asymmetries.
- For $B_d$ decays to flavour-nonspecific final states (like CP eigenstates) the CP asymmetries depend on the time of decay in a very characteristic manner; their size should typically be measured in units of 10% rather than 0.1%.
- There is no plausible deniability for the CKM description, if such asymmetries are not found.
- For $m_t \geq 150$ GeV the SM prediction for $\epsilon_K$ is dominated by the top quark contribution like $\Delta M_{B_d}$. It thus drops out from their ratio, and $\sin2\phi_1$ can be predicted within the SM irrespective of the (superheavy) top quark mass. In the early 1990’s, i.e., before the direct discovery of top quarks, it was predicted [20] that

$$\frac{\epsilon_K}{\Delta M_{B_d}} \propto \sin2\phi_1 \sim 0.6 - 0.7,$$

(93)

with values for $B_B f_B^2$ inserted as now estimated by Lattice QCD.
• The CP asymmetry in the Cabibbo favoured channels $B_s \to \psi\phi/\psi\eta$ is Cabibbo suppressed, i.e. below 4%, for reasons very specific to CKM theory, as pointed out already in 1980 [12].

II-5. CKM Theory at the End of the 2nd Millenium

It is indeed true that large fractions of the observed values for $\Delta M_K$, $\epsilon_K$, and $\Delta M_B$ and even most of $\epsilon'$ could be due to New Physics, given the limitations in our theoretical control over hadronic matrix elements. Equivalently, constraints from these and other data translate into 'broad' bands in plots of the unitarity triangle, see Fig. 3.

The problem with this statement is that it is not even wrong — it misses the real point. Observables like $\Gamma(B \to \ell\nu X_{c,u})$, $\Gamma(K \to \ell\nu\pi)$, $\Delta M_K$, $\Delta M_B$, $\epsilon_K$ and $\sin2\phi_1$, etc. represent very different dynamical regimes that proceed on time scales that span several orders of magnitude. The very fact that CKM theory can accommodate such diverse observables always within a factor two or better and relate them in such a manner that its parameters can be plotted as meaningful constraints on a triangle is highly nontrivial and — in my view — must reflect some underlying, yet unknown dynamical layer. Furthermore the CKM parameters exhibit an unusual hierarchical pattern — $|V(ud)| \sim |V(cs)| \sim |V(tb)| \sim 1$, $|V(us)| \simeq |V(cd)| \simeq \lambda$, $|V(cb)| \sim |V(ts)| \sim \mathcal{O}(\lambda^2)$, $|V(ub)| \sim |V(td)| \sim \mathcal{O}(\lambda^3)$ — as do the quark masses culminating in $m_t \simeq 175$ GeV. Picking such values for these parameters would have been seen as frivolous at best — had they not been forced upon us by (independent) data. Thus I view it already as a big success for CKM theory that the experimental constraints on its parameters as they existed in 2000 can be represented through triangle plots in a meaningful way.
Interlude: Singing the Praise of Hadronization

Hadronization and nonperturbative dynamics in general are usually viewed as unwelcome complications, if not outright nuisances. A case in point was already mentioned: while I view the CKM predictions for $\Delta M_K$, $\Delta M_B$, $\epsilon_K$ to be in remarkable agreement with the data, significant contributions from New Physics could be hiding there behind the theoretical uncertainties due to lack of computational control over hadronization. Yet without hadronization bound states of quarks and antiquarks will not form; without the existence of kaons $K^0 - \bar{K}^0$ oscillations obviously cannot occur. It is hadronization that provides the ‘cooling’ of the (anti)quark degrees of freedom, which allows subtle quantum mechanical effects to add up coherently over macroscopic distances. Otherwise one would not have access to a super-tiny energy difference $\text{Im} M_{12} \sim 10^{-8} \text{eV}$, which is very sensitive to different layers of dynamics, and indirect CP violation could not manifest itself. The same would hold for $B$ mesons and $B^0 - \bar{B}^0$ oscillations.

To express it in a more down to earth way:

- Hadronization leads to the formation of kaons and pions with masses greatly exceeding (current) quark masses. It is the hadronic phase space that suppresses the CP conserving rate for $K_L \rightarrow 3\pi$ by a factor $\sim 500$, since the $K_L$ barely resides above the three pion threshold.

- It awards ‘patience’; i.e. one can ‘wait’ for a pure $K_L$ beam to emerge after starting out with a beam consisting of $K^0$ and $\bar{K}^0$.

- It enables CP violation to emerge in the existence of a reaction, namely $K_L \rightarrow 2\pi$ rather than an asymmetry; this greatly facilitates its observation.

For these reasons alone we should praise hadronization as the hero in the tale of CP violation rather than the villain it is all too often portrayed.

End of Interlude

By the end of the second millenium a rich and diverse body of data on flavour dynamics had been accumulated, and CKM theory provided a surprisingly successful description of it. This prompted some daring spirits to perform detailed fits of the CKM triangle to infer a rather accurate prediction for the CP asymmetry in $B_d \rightarrow \psi K_S$ [22]:

$$\sin 2\phi_1 = 0.72 \pm 0.07.$$  \hfill (94)

III. THE ‘EXPECTED’ TRIUMPH OF A PECULIAR THEORY

III-1. Establishing the CKM Ansatz as a Theory

The three angles $\phi_{1,2,3}$ in the CKM unitarity triangle (see Fig. 2 for notation) can be determined through CP asymmetries in $B_d(t) \rightarrow \psi K_S, \pi^+ \pi^-$, and $B_d \rightarrow K^+ \pi^- \ldots$
in principle. In practice the angle $\phi_3$ can be extracted from $B^\pm \to D^{\text{neut}}K^\pm$ with better theoretical control, and $B \to 3\pi$, $4\pi$ offer various experimental advantages over $B \to 2\pi$. These issues will be addressed in five acts plus two interludes.

1. Act I: $B_d(t) \to \psi K_S$ and $\phi_1$ (a.k.a. $\beta$)

The first published result on the CP asymmetry in $B_d \to \psi K_S$ was actually obtained by the OPAL collaboration at LEP I [23], followed by the first value measured by CDF inside the physical range [24]:

$$\sin^2 \phi_1 = 0.79 \pm 0.44.$$  \hspace{1cm} (95)

In 2001 the two $B$ factory collaborations BABAR and BELLE established the first CP violation outside the $K^0 - \bar{K}^0$ complex:

$$\sin^2 \phi_1 = \begin{cases} 0.59 \pm 0.14 \pm 0.05 & \text{BABAR '01} \\ 0.99 \pm 0.14 \pm 0.06 & \text{BELLE '01} \end{cases}.$$  \hspace{1cm} (96)

By 2003 the numbers from the two experiments had converged well allowing one to state just the world averages, which is actually a BABAR/BELLE average [17]:

$$\sin^2 \phi_1 = 0.675 \pm 0.026 \text{ \ WA '06}.$$  \hspace{1cm} (97)

The CP asymmetry in $B_d \to \psi K_S$ is there, is huge, and as expected even quantitatively. For CKM fits based on constraints from $|V(ub)/V(cb)|$, $B^0 - \bar{B}^0$ oscillations and — as the only CP sensitive observable — $\epsilon_K$ yield [25]

$$\sin^2 \phi_1|_{\text{CKM}} = 0.755 \pm 0.039.$$  \hspace{1cm} (98)

The measured value also fully agrees with the predictions from the last millenium, Eqs. (93,94). The CKM prediction has stayed within the $0.72 - 0.75$ interval for the last several years. Through 2005 it has been in impressive agreement with the data. In 2006 a hint of a deviation has emerged. It is not more than that, since it is not (yet) statistically significant. This is illustrated by Fig. 3 showing these constraints. This figure actually obscures another impressive triumph of CKM theory: the CP insensitive observables $|V(ub)/V(cb)|$ and $\Delta M_{B_d}/\Delta M_{B_s}$ — i.e. observables that do not require CP violation for acquiring a non-zero value — imply

- a non-flat CKM triangle and thus CP violation, see the left of Fig. 4
- that is fully consistent with the observed CP sensitive observables $\epsilon_K$ and $\sin^2 \phi_1$, see the right of Fig. 4.
2. CP violation in K and B decays — exactly the same, only different

There are several similarities between $K^0 - \bar{K}^0$ and $B_d - \bar{B}_d$ oscillations even on the quantitative level. Their values for $x = \Delta M / \Gamma$ and thus for $\chi$ are very similar. It is even more intriguing that also their pattern of CP asymmetries in $K^0(t)/\bar{K}^0(t) \rightarrow \pi^+\pi^-$ and $B_d(t)/\bar{B}_d(t) \rightarrow \psi K_S$ is very similar. Consider the two lower plots in Fig. 5, which show the asymmetry directly as a function of $\Delta t$: it looks intriguingly similar qualitatively and even quantitatively. The lower left plot shows that the difference between $K^0 \rightarrow \pi^+\pi^-$ and $\bar{K}^0 \rightarrow \pi^+\pi^-$ is actually measured in units of 10% for $\Delta t \sim (8-16)\tau_{K_S}$, which is the $K_S - K_L$ interference region.

Clearly one can find domains in $K \rightarrow \pi^+\pi^-$ that exhibit a truly large CP asymmetry. Nevertheless it is an empirical fact that CP violation in B decays is much larger than in K decays. For the mass eigenstates of neutral kaons are very well approximated by CP eigenstates, as can be read off from the upper left plot: it shows that the vast majority of $K \rightarrow \pi^+\pi^-$ events follow a single exponential decay law that coincides for $K^0$ and $\bar{K}^0$ transitions. This is in marked contrast to the $B_d \rightarrow \psi K_S$ and $\bar{B}_d \rightarrow \psi K_S$ transitions, which in no domain are well approximated by a single exponential law and do not coincide at all, except for $\Delta t = 0$, as it has to be, see Sect.III-1-3.

3. Interlude: “Praise the Gods Twice for EPR Correlations”

The BABAR and BELLE analyses are based on a glorious application of quantum mechanics and in particular EPR correlations [26]. The CP asymmetry in $B_d \rightarrow \psi K_S$ had been predicted to exhibit a peculiar dependence on the time of decay, since it involves $B_d - \bar{B}_d$ oscillations in an essential way:

$$\text{rate}(B_d(t)[\bar{B}_d(t)] \rightarrow \psi K_S) \propto e^{-t/\tau_B} (1 - [+]A\sin\Delta m_B t) .$$

(99)
At first it would seem that an asymmetry of the form given in Eq. (99) could not be measured for practical reasons. For in the reaction
\[
e^{+}e^{-} \rightarrow \Upsilon(4S) \rightarrow B_{d}\bar{B}_{d}
\]
the point where the \(B\) meson pair is produced is ill determined due to the finite size of the electron and positron beam spots: the latter amounts to about 1 mm in the longitudinal direction, while a \(B\) meson typically travels only about a quarter of that distance before it decays. It would then seem that the length of the flight path of the \(B\) mesons is poorly known and that averaging over this ignorance would greatly dilute or even eliminate the signal.

It is here where the existence of an EPR correlation comes to the rescue. While the two \(B\) mesons in the reaction of Eq. (100) oscillate back and forth between a \(B_{d}\) and \(\bar{B}_{d}\), they change their flavour identity in a completely correlated way. For the \(B\bar{B}\) pair forms a \(C\) odd state; Bose statistics then tells us that there cannot be two identical flavour hadrons in the final state:
\[
e^{+}e^{-} \rightarrow \Upsilon(4S) \rightarrow B_{d}\bar{B}_{d} \neq B_{d}B_{d}, \bar{B}_{d}\bar{B}_{d}.
\]
Once one of the \(B\) mesons decays through a flavour specific mode, say \(B_{d} \rightarrow l^{+}\nu X [\bar{B}_{d} \rightarrow l^{-}\bar{\nu}X]\), then we know unequivocally that the other \(B\) meson was a \(\bar{B}_{d}\) [\(B_{d}\)] at that time. The time evolution of \(\bar{B}_{d}(t)[B_{d}(t)] \rightarrow \psi K_{S}\) as described by Eq. (99) starts at that time as well; i.e., the relevant time parameter is the interval between the two times of decay, not

FIG. 5: The observed decay time distributions for \(K^{0}\) vs. \(\bar{K}^{0}\) from CPLEAR on the left and for \(B_{d}\) vs. \(\bar{B}_{d}\) from BABAR on the right.
those times themselves. That time interval is related to — and thus can be inferred from — the distance between the two decay vertices, which is well defined and can be measured.

The great value of the EPR correlation is instrumental for another consideration as well, namely how to see directly from the data that $\text{CP}$ violation is matched by $\text{T}$ violation. Fig. 6 shows two distributions, one for the interval $\Delta t$ between the times of decays $B_d \rightarrow l^+X$ and $B_d \rightarrow \psi K_S$ and the other one for the $\text{CP}$ conjugate process $\bar{B}_d \rightarrow l^-X$ and $\bar{B}_d \rightarrow \psi K_S$. They are clearly different proving that $\text{CP}$ is broken. Yet they show more: the shape of the two distributions is actually the same (within experimental uncertainties) the only difference being that the average of $\Delta t$ is positive for $(l^-X)_{\bar{B}}(\psi K_S)$ and negative for $(l^+X)_B(\psi K_S)$ events; i.e., there is a (slight) preference for $B_d \rightarrow \psi K_S [\bar{B}_d \rightarrow \psi K_S]$ to occur after [before] and thus more [less] slowly (rather than just more rarely) than $\bar{B} \rightarrow l^-X [B \rightarrow l^+X]$. Invoking $\text{CPT}$ invariance merely for semileptonic $B$ decays — yet not for nonleptonic transitions — synchronizes the starting point of the $B$ and $\bar{B}$ decay ‘clocks’, and the EPR correlation keeps them synchronized. We thus see that $\text{CP}$ and $\text{T}$ violation are ‘just’ different sides of the same coin. As explained above, EPR correlations are essential for this argument!

The reader can be forgiven for feeling that this argument is of academic interest only, since $\text{CPT}$ invariance of all processes is based on very general arguments. Yet the main point to be noted is that EPR correlations, which represent some of quantum mechanics’ most puzzling features, serve as an essential precision tool, which is routinely used in these measurements. I feel it is thus inappropriate to refer to EPR correlations as a paradox.
4. Act II: \(B_d(t) \to 2\pi\) and \(\phi_2\) (a.k.a. \(\phi\))

The situation is theoretically more complex than for \(B_d(t) \to \psi K_S\) due to two reasons:

- While both final states \(\pi\pi\) and \(\psi K_S\) are CP eigenstates, the former unlike the latter is not reached through an isoscalar transition. The two pions can form an \(I = 0\) or \(I = 2\) configuration (similar to \(K \to 2\pi\)), which in general will be affected differently by the strong interactions.

- For all practical purposes \(B_d \to \psi K_S\) is described by two tree diagrams representing the two effective operators \((\bar{c}_L\gamma_\mu b_L)(s_L\gamma^\mu c_L)\) and \((\bar{c}_L\gamma_\nu \lambda_i b_L)(s_L\nu^\mu \lambda_i c_L)\) with the \(\lambda_i\) representing the \(SU(3)_C\) matrices. Yet for \(B \to \pi\pi\) we have effective operators \((\bar{d}_L\gamma_\mu \lambda_i b_L)(\bar{q}_i\nu^\mu \lambda_j q)\) generated by the Cabibbo suppressed Penguin loop diagrams in addition to the two tree operators \((\bar{u}_L\gamma_\mu b_L)(d_L\nu^\mu c_L)\) and \((\bar{u}_L\gamma_\nu \lambda_i b_L)(d_L\nu^\mu \lambda_i u_L)\).

This greater complexity manifests itself already in the phenomenological description of the time dependent CP asymmetry:

\[
\frac{R_+(\Delta t) - R_-(\Delta t)}{R_+(\Delta t) + R_-(\Delta t)} = S\sin(\Delta M_d\Delta t) - C\cos(\Delta M_d\Delta t), \tag{102}
\]

where \(R_{+|-}(\Delta t)\) denotes the rate for \(B^{\text{tag}}(t) \bar{B}_d(t + \Delta)[\bar{B}^{\text{tag}}(t)B_d(t + \Delta)]\) and

\[
S = \frac{2\text{Im}^2 \frac{\tilde{\rho}_{\pi^+\pi^-}}{1 + \left| \frac{\tilde{\rho}_{\pi^+\pi^-}}{\bar{\rho}_{\pi^+\pi^-}} \right|^2}, \quad C = \frac{1 - \left| \frac{\tilde{\rho}_{\pi^+\pi^-}}{\bar{\rho}_{\pi^+\pi^-}} \right|^2}{1 + \left| \frac{\tilde{\rho}_{\pi^+\pi^-}}{\bar{\rho}_{\pi^+\pi^-}} \right|^2}, \quad S^2 + C^2 \leq 1. \tag{103}
\]

As before, due to the EPR correlation between the two neutral \(B\) mesons, it is the relative time interval \(\Delta t\) between the two \(B\) decays that matters, not their lifetime. The new feature is that one has also a cosine dependence on \(\Delta t\).

BABAR and BELLE find

\[
S = \begin{cases} 
-0.53 \pm 0.14 \pm 0.02 & \text{BABAR '06} \\
-0.61 \pm 0.10 \pm 0.04 & \text{BELLE '06} \\
-0.59 \pm 0.09 & \text{HFAG} 
\end{cases} \tag{104}
\]

\[
C = \begin{cases} 
-0.16 \pm 0.11 \pm 0.03 & \text{BABAR '06} \\
-0.55 \pm 0.08 \pm 0.05 & \text{BELLE '06} \\
-0.39 \pm 0.07 & \text{HFAG} 
\end{cases} \tag{105}
\]

While BABAR and BELLE agree nicely on \(S\) making the HFAG average straightforward, their findings on \(C\) indicate different messages making the HFAG average more iffy.

\(S \neq 0\) has been established and thus CP violation also in this channel. While BELLE finds \(C \neq 0\) as well, BABAR’s number is still consistent with \(C = 0\). \(C \neq 0\) obviously represents direct CP violation. Yet it is often overlooked that also \(S\) can reveal
such CP violation. For if one studies $B_d$ decays into two CP eigenstates $f_a$ and $f_b$ and finds

$$S(f_a) \neq \eta (f_a) \eta (f_b) S(f_b),$$

(106)

with $\eta_i$ denoting the CP parities of $f_i$, then one has established direct CP violation. For the case under study that means even if $C(\pi\pi) = 0$, yet $S(\pi^+\pi^-) \neq -S(\psi K_S)$, one has observed unequivocally direct CP violation. For the latter requires, as explained in Sect. I-4, that two different amplitudes contribute coherently to $B_d \to f_b$ with non-zero relative weak as well as strong phases. $S(f_a) \neq \eta (f_a) \eta (f_b) S(f_b)$ on the other hand only requires that the two overall amplitudes for $B_d \to f_a$ and $B_d \to f_b$ possess a relative phase. This can be illustrated with a familiar example from CKM dynamics: If there were no Penguin operators for $B_d \to \pi^+\pi^-$ (or they could be ignored quantitatively), one would have $C(\pi^+\pi^-) = 0$, yet at the same time $S(\psi K_S) = \sin(2\phi_1)$ together with $S(\pi^+\pi^-) = \sin(2\phi_2) \neq -\sin(2\phi_1)$; i.e., without direct CP violation one would have to find $C = 0$ and $S = -\sin2\phi_1$ [27]. Yet since the measured value of $S$ is within one sigma of $-\sin2\phi_1$ this distinction is mainly of academic interest at the moment.

Once the categorical issue of whether there is direct CP violation has been settled, one can take up the challenge of extracting a value for $\phi_2$ from the data.\footnote{The complications due to the presence of the Penguin contribution are all too often referred to as ‘Penguin pollution’. Personally I find it quite unfair to blame our lack of theoretical control on water fowls rather than on the guilty party, namely us.} This can be done in a model independent way by analyzing the $B_d(t) \to \pi^+\pi^-$, $\pi^0\pi^0$, and $B^\pm \to \pi^\pm\pi^0$ transitions and performing an isospin decomposition, for the Penguin contribution cannot affect $B_d(t) \to [\pi\pi]_{I=2}$ modes. Unfortunately there is a serious experimental bottle neck, namely to study $B_d(t) \to \pi^0\pi^0$ with sufficient accuracy. Therefore alternative decays have been suggested, in particular $B \to \rho\pi$ and $\rho\rho$. While those provide various advantages on the experimental side, they introduce also theoretical uncertainties, since they have to be extracted from $3\pi$ and $4\pi$ final states that contain also other resonances and configurations like the $\sigma$ state. I am most hesitant to average over the values obtained at present for $\phi_2$ from $B \to 2\pi$, $\rho\pi$, and $\rho\rho$.

5. Act III, 1st Version: $B_d \to K^+\pi^-$

It was pointed out in a seminal paper [13] that (rare) transitions like $B_d \to K^- + \pi^+$’s have the ingredients for sizable direct CP asymmetries:

- Two different amplitudes can contribute coherently, namely the highly CKM suppressed tree diagram with $b \to u\bar{u}s$ and the Penguin diagram with $b \to s\bar{q}q$.

- The tree diagram contains a large weak phase from $V(ub)$. 
• The Penguin diagram with an internal charm quark loop exhibits an imaginary part, which can be viewed — at least qualitatively — as a strong phase generated by the production and subsequent annihilation of a $c\bar{c}$ pair (the diagram with an internal $u$ quark loop acts merely as a subtraction point allowing a proper definition of the operator).

• While the Penguin diagram with an internal top quark loop is actually not essential, the corresponding effective operator can be calculated quite reliably, since integrating out first the top quarks and then the $W$ boson leads to a truly local operator. Determining its matrix elements is however another matter.

To translate these features into accurate numbers represents a formidable task, which we have not mastered yet. In Ref. [28] an early and detailed effort was made to treat $\bar{B}_d \to K^-\pi^+$ theoretically with the following results:

$$\text{BR}(\bar{B}_d \to K^-\pi^+) \sim 10^{-5}, \quad A_{\text{CP}} \sim -0.10.$$

(107)

Those numbers turn out to be rather prescient, since they are in gratifying agreement with the data

$$\text{BR}(\bar{B}_d \to K^-\pi^+) = (1.85 \pm 0.11) \cdot 10^{-5},$$

$$A_{\text{CP}} = \begin{cases} -0.133 \pm 0.030 \pm 0.009 & \text{BABAR} \\ -0.113 \pm 0.021 & \text{BELLE} \end{cases}.$$  

(108)

Cynics might point out that the authors in [28] did not give a specific estimate of the theoretical uncertainties in Eq. (107). More recent authors have been more ambitious — with somewhat mixed success [29, 30]. While the observed asymmetry in $B_d \to K\pi$ agrees with CKM expectations, we do not have an accurate prediction.

6. Act III, 2nd Version: $\phi_3$ from $B^+ \to D_{\text{neut}}K^+$ vs. $B^- \to D_{\text{neut}}K^-$

As first mentioned in 1980 [11], then explained in more detail in 1985 [31] and further developed in [32], the modes $B^\pm \to D_{\text{neut}}K^\pm$ should exhibit direct CP violation driven by the angle $\phi_3$, if the neutral $D$ mesons decay to final states that are common to $D^0$ and $\bar{D}^0$. Based on simplicity the original idea was to rely on two-body modes like $K_S\pi^0$, $K^+K^-$, $\pi^+\pi^-$, $K^\pm\pi^\mp$. Drawbacks of that method are the small branching ratios and low efficiencies.

A new method was pioneered by BELLE and then implemented also by BABAR, namely to employ $D_{\text{neut}} \to K_S\pi^+\pi^-$ and perform a full Dalitz plot analysis. This requires a very considerable analysis effort — yet once this initial investment has been made, it will pay handsome profit in the long run. For obtaining at least a decent description of
the full Dalitz plot population provides considerable cross checks concerning systematic uncertainties and thus a high degree of confidence in the results. BELLE and BABAR find:

$$\phi_3 = \begin{cases} 53^\circ \pm 18^\circ (\text{stat}) \pm 3^\circ (\text{syst}) \pm 9^\circ (\text{model}) & \text{BELLE}, \\ 92^\circ \pm 41^\circ (\text{stat}) \pm 11^\circ (\text{syst}) \pm 12^\circ (\text{model}) & \text{BABAR}. \end{cases}$$ (109)

At present these studies are severely statistics limited; one should also note that with more statistics one will be able to reduce in particular the model dependence. I view this method as the best one to extract a reliable value for $\phi_3$, where the error estimate can be defended due to the many constraints inherent in a Dalitz plot analysis. It exemplifies how the complexities of hadronization can be harnessed to establish confidence in the accuracy of our results.

7. Act IV: $\phi_1$ from CP Violation in $B_d \to 3$ Kaons: Snatching Victory from the Jaws of Defeat — or Defeat from the Jaws of Victory?

Analysing CP violation in $B_d \to \phi K_S$ decays is a most promising way to search for New Physics. For the underlying quark-level transition $b \to s\bar{s}s$ represents a pure loop-effect in the SM, it is described by a single $\Delta B = 1 & \Delta I = 0$ operator (a ‘Penguin’), a reliable SM prediction exists for it [33] — $\sin^2 \phi_1(B_d \to \psi K_S) \approx \sin^2 \phi_1(B_d \to \phi K_S)$ — and the $\phi$ meson represents a narrow resonance.

Great excitement was created when BELLE reported a large discrepancy between the predicted and observed CP asymmetry in $B_d \to \phi K_S$ in the summer of 2003:

$$\sin^2 \phi_1(B_d \to \phi K_S) = \begin{cases} -0.96 \pm 0.5 \pm 0.10 & \text{BELLE(’03)} \\ 0.45 \pm 0.43 \pm 0.07 & \text{BABAR(’03)} \end{cases}. \quad (110)$$

Based on more data taken, this discrepancy has shrunk considerably: the BABAR/BELLE average for 2005 yields [17]

$$\sin^2 \phi_1(B_d \to \psi K_S) = 0.685 \pm 0.032 \quad (111)$$

versus

$$\sin^2 \phi_1(B_d \to \phi K_S) = \begin{cases} 0.44 \pm 0.27 \pm 0.05 & \text{BELLE(’05)} \\ 0.50 \pm 0.25^{+0.07}_{-0.04} & \text{BABAR(’05)} \end{cases}; \quad (112)$$

while the 2006 values read as follows:

$$\sin^2 \phi_1(B_d \to \psi K_S) = 0.675 \pm 0.026 \quad (113)$$

compared to

$$\sin^2 \phi_1(B_d \to \phi K_S) = \begin{cases} 0.50 \pm 0.21 \pm 0.06 & \text{BELLE(’06)} \\ 0.12 \pm 0.31 \pm 0.10 & \text{BABAR(’06)} \\ 0.39 \pm 0.18 & \text{HFAG(’06)} \end{cases}. \quad (114)$$

I summarize the situation as follows:
• Performing dedicated CP studies in channels driven mainly or even predominantly by $b \to sq\bar{q}$ to search for New Physics signatures makes eminent sense since the SM contribution, in particular from the one-loop Penguin operator, is greatly suppressed.

• The experimental situation is far from settled, as can be seen also from how the central value have moved over the years. It is tantalizing to see that the $S$ contribution for the modes in this category — $B_d \to \pi^0 K_S, \rho^0 K_S, \omega K_S, f_0 K_S$ — are all low compared to the SM expectation Eq. (113). Yet none of them is significantly lower, and for none of these modes a non-zero CP asymmetry has been established except for

$$\sin^2 \phi_1(B_d \to \eta' K_S) = \begin{cases} 0.64 \pm 0.10 \pm 0.04 & \text{BELLE('06)} \\ 0.58 \pm 0.10 \pm 0.03 & \text{BABAR('06)} \\ 0.61 \pm 0.07 & \text{HFAG('06)} \end{cases}.$$


• Obviously there is considerable space still for significant deviations from SM predictions. It is ironic that such a smaller deviation, although not significant, is actually more believable as signaling an incompleteness of the SM than the large one originally reported by BELLE. While it is tempting to average over all these hadronic transitions, I would firmly resist this temptation for the time being, till several modes exhibit a significant asymmetry.

• One complication has to be studied, though, in particular if the observed value of $\sin^2 \phi_1(B_d \to \phi K_S)$ falls below the predicted one by a moderate amount only. For one is actually observing $B_d \to K^+ K^- K_S$. If there is a single weak phase like in the SM one finds

$$\sin^2 \phi_1(B_d \to \phi K_S) = -\sin^2 \phi_1(B_d \to f_0(980) K_S),$$

where $f_0(980)$ denotes any scalar $K^+ K^-$ configuration with a mass close to that of the $\phi$, be it a resonance or not. A smallish pollution by such a $f_0(980)$ by, say, 10% in amplitude — can thus reduce the asymmetry assigned to $B_d \to \phi K_S$ significantly — by 20% in this example.

• In the end it is therefore mandatory to perform a full time dependent Dalitz plot analysis for $B_d \to K^+ K^- K_S$ and compare it with that for $B_d \to 3K_S$ and $B^+ \to K^+ K^- K^+$, $K^+ K_S K_S$ and also with $D \to 3K$. BABAR has presented a preliminary such study. This is a very challenging task, but in my view essential. There is no ‘royal’ way to fundamental insights.\(^8\)

• An important intermediate step in this direction is given by one application of Bianco’s Razor [34], namely to analyze the CP asymmetry in $B_d \to [K^+ K^-] M K_S$ as a function of the cut $M$ on the $K^+ K^-$ mass.

---

\(^8\) The ruler of a Greek city in southern Italy once approached the resident sage with the request to be educated in mathematics, but in a ‘royal way’, since he was very busy with many obligations. Whereupon the sage replied with admirable candor: “There is no royal way to mathematics.”
All of this might well lead to another triumph of the SM, when its predictions agree with accurate data in the future, even for these rare transition rates dominated by loop-contributions, i.e. pure quantum effects. It is equally possible — personally I think it actually more likely — that future precision data will expose New Physics contributions. In that sense the SM might snatch victory from the jaws of defeat — or defeat from the jaws of victory. For us that are seeking indirect manifestations of New Physics the roles of victory and defeat are switched, of course.

In any case the issue has to be pursued with vigour, since these reactions provide such a natural portal to New Physics on one hand and possess such an accurate yardstick from $B_d \to \psi K_S$.

8. The Beginning of Act V — CP Violation in Charged B Decays

So far CP violation has not been established yet in the decays of charged mesons, which is not surprising since meson-antimeson oscillations cannot occur there, and it has to be purely direct CP violation. Now BELLE [35] has found strong evidence for a large CP asymmetry in charged B decays with a 3.9 sigma significance, namely in $B^\pm \to K^\pm \rho^0$ observed in $B^\pm \to K^\pm \pi^\pm \pi^\mp$:

$$A_{CP} \left( B^\pm \to K^\pm \rho^0 \right) = (30 \pm 11 \pm 2.0_{-4}^{+11})\%.$$ (117)

I find it a most intriguing signal since a more detailed inspection of the mass peak region shows a pattern as expected for a genuine effect. Furthermore a similar signal is seen in BABAR’s data, and it would make sense to undertake making a careful average over the two data sets.

I view BELLE’s and BABAR’s analyses of the Dalitz plot for $B^\pm \to K^\pm \pi^\pm \pi^\mp$ as important pilot studies, from which one can infer important lessons about the strengths and pitfalls of such studies in general.

9. Stop the Press: CP Violation in $B_d \to D^+ D^-$?

Most recently BELLE has found evidence for large indirect as well as direct CP violation in the Cabibbo suppressed channel $B_d \to D^+ D^-$ [36] with 4.1 $\sigma$ and 3.2 $\sigma$ significance, respectively, the latter through $C \neq 0$:

$$S(D^+ D^-) = -1.13 \pm 0.37 \pm 0.09, \quad C(D^+ D^-) = -0.91 \pm 0.23 \pm 0.06.$$ BELLE’07 (118)

While the central values do not satisfy the general constraint $S^2 + C^2 \leq 1$, see Sect. III-1-4, this can be attributed to a likely upward statistical fluctuation. The $S$ term, i.e. the coefficient of the sin$\Delta M_{B_d} t$ term in the asymmetry, is just one sigma high compared to its SM prediction of $-\sin 2\phi_1 = -0.675 \pm 0.026$. Yet the $C$ term, which unambiguously represents direct CP violation and is expected to be very close to zero in the SM, appears to be considerably larger. If true, it would establish the presence of New Physics. One should
note though that BABAR’s data indicate a somewhat different message [37] in particular with respect to $C$:

$$S(D^+D^-) = -0.29 \pm 0.63 \pm 0.06 \, . \, C(D^+D^-) = 0.11 \pm 0.35 \pm 0.06 \, . \, \text{BABAR}'06 \, .$$ (119)

In Ref. [36] it is suggested that New Physics could enhance the Cabibbo suppressed Penguin operator $b \to d \bar{q} q$, $q = c$ considerably so as to generate the required weak phase in the $\Delta B = 1$ amplitude. It is quite conceivable that the $b \to s \bar{q} q$ transition operator is much less affected by New Physics. However one needs a very peculiar scenario. For in general such an operator should also for $q = u, d, s$ and presumably with a considerably higher weight. Then it would provide the dominant contribution to the $B_d \to \pi^+\pi^-, 2\pi^0$ and $K_S^0K_S^0$ channels with a branching ratio on the about $10^{-4}$ level or probably more — in clear conflict with the data. That is one has to postulate that the contributions $b \to d \bar{u}u$, $d \bar{d}d$ and $d \bar{s}s$ due to New Physics had to be suppressed greatly relative to $b \to d \bar{c}c$.

### III-2. Summary

While CKM forces are generated by the exchange of gauge bosons, its couplings as explained in Sect. I involve elements of the CKM matrix, the latter being derived from the up- and down-type quark mass matrices. Thus the CKM parameters are intrinsically connected with one of the central mysteries of the SM, namely the generation of fermion masses and family replication. Furthermore the hierarchy in the quark masses and the likewise hierarchical pattern of the CKM matrix elements strongly hints at some deeper level of dynamics about which we are quite ignorant. Nevertheless CKM theory with its mysterious origins has proved itself to be highly successful in describing even quantitatively a host of phenomena occurring over a wide array of scales. It led to the ‘Paradigm of Large $\text{CP}$ Violation in $B$ Decays’ as a prediction in the old-fashioned sense; i.e., predictions were made well before data of the required sensitivity existed. From the observation of a tiny and shy phenomenon — $\text{CP}$ violation in $K_L$ decays on the $O(10^{-3})$ level — it predicted without ‘plausible deniability’ almost ubiquitous manifestations of $\text{CP}$ violation about two orders of magnitude larger in $B$ decays. This big picture has been confirmed now in qualitative as well as impressively quantitative agreement with SM predictions:

- Two $\text{CP}$ insensitive observables, namely $|V(ub)/V(cb)|$ and $\Delta M_{B_d}/\Delta M_{B_s}$, imply that $\text{CP}$ violation has to exist and in a way that at present is fully consistent with the measurements of $\epsilon$ and $\sin 2\phi_1$.

- Time dependent $\text{CP}$ asymmetries in the range of several $\times 10 \%$ have been established in $B_d \to \psi K_S, \pi^+\pi^-$, and $\eta'K_S$ with several others on the brink of being found.

- Direct $\text{CP}$ violations of about 10% or even larger have been discovered in $B_d \to \pi^+\pi^-$, and $K^-\pi^+$. 

- The first significant sign of $\text{CP}$ violation in a charged meson has surfaced in $B^{\pm} \to K^{\pm}\rho^0$. 
• The optimists among us might discern the first signs of tension between the data and
the predictions of CKM theory in $|V(ub)/V(cb)|$ & $\Delta M_{B_d}/\Delta M_{B_s}$ vs. $\sin 2\phi_1$ and in
the CP asymmetries in $b \to sq\bar{q}$ vs. $b \to c\bar{c}s$ driven transitions.

For all these successes it is quite inappropriate to refer any more to CKM theory as an
‘ansatz’ with the latter’s patronizing flavour. Instead I would characterize these develop-
ments as “the expected triumph of a peculiar theory”. I will indulge myself in three more
‘cultural’ conclusions:

• The aforementioned “CKM Paradigm of Large CP Violation in $B$ Decays” is due to
the confluence of several favourable, yet a priori less than likely factors that must be
seen as gifts from nature: she had (i) arranged for a huge top mass, (ii) a “long” $B$
lifetime, (iii) the $\Upsilon(4S)$ resonance being above the $BB$, yet below the $BB^*$ thresholds,
and (iv) regaled us previously with charm hadrons, which prompted the development
of detectors with an effective resolution that is needed to track $B$ decays.

• ‘Quantum mysteries’ like EPR correlations with their intrinsic non-local features were
essential for observing CP violation involving $B_d - \bar{B}_d$ oscillations in $\Upsilon(4S) \to B_d\bar{B}_d$
and to establish that indeed there is T violation commensurate with CP violation.

• While hadronization is not easily brought under quantitative theoretical control, it
enhances greatly observable CP asymmetries and can provide most valuable cross
checks for our interpretation of data.

IV. PROBING THE FLAVOUR PARADIGM OF THE EMERGING NEW STANDARD
MODEL

IV-1. On the Incompleteness of the SM

As described above the SM has scored novel — i.e., qualitatively new — successes in
the last few years in the realm of flavour dynamics. Due to the very peculiar structure of
the latter they have to be viewed as amazing. Yet even so the situation can be characterized
with a slightly modified quote from Einstein: “We know a lot — yet understand so little.”
That is these successes do not invalidate the general arguments in favour of the SM being
incomplete — the search for New Physics is as mandatory as ever.

You have heard about the need to search for New Physics before and what the outcome
has been of such efforts so far. And it reminds you of a quote by Samuel Beckett: “Ever
tried? Ever failed? No matter. Try again. Fail again. Fail better.” Only an Irishman can
express profound skepticism concerning the world in such a poetic way. Beckett actually
spent most of his life in Paris, since Parisians like to listen to someone expressing such a
world view, even while they do not share it. Being in the service of Notre Dame du Lac,
the home of the ‘Fighting Irish’, I cannot just ignore such advice.

9 The German ‘ansatz’ refers to an educated guess.
Yet there are — in my judgement compelling — *theoretical* arguments pointing to the existence of New Physics.

- While electric charge quantization $Q_e = 3Q_d = -\frac{3}{2}Q_u$ is an essential ingredient of the SM — it allows violating the Adler-Bell-Jackiw anomaly — it does not offer any understanding. It would naturally be explained through Grand Unification at very high energy scales implemented through, e.g., $SO(10)$ gauge dynamics. I call this the ‘guaranteed New Physics’.

- We infer from the observed width of $Z^0$ decays that there are three (light) neutrino species. The hierarchical pattern of CKM parameters as revealed by the data is so peculiar as to suggest that some other dynamical layer has to underlie it. I refer to it as ‘strongly suspected New Physics’ or ssNP. We are quite in the dark about its relevant scales. Saying we pin our hopes for explaining the family replication on Super-String or M theory is a scholarly way of saying we have hardly a clue what that ssNP is.

- What are the dynamics driving the electroweak symmetry breaking of $SU(2)_L \times U(1) \rightarrow U(1)_{QED}$? How can we tame the instability of Higgs dynamics with its quadratic mass divergence? I find the arguments compelling that point to New Physics at the $\sim 1$ TeV scale — like low-energy SUSY; therefore I call it the ‘confidently predicted’ New Physics or cpNP.

- Last and possibly least the ‘Strong CP Problem’ of QCD has not been resolved. Similar to the other shortcomings it is a purely theoretical problem in the sense that the offending coefficient for the $P$ and CP odd operator $\tilde{G} \cdot G$ can be fine-tuned to zero, see Sect. I-1-1, — yet in my eyes it is an intriguing problem.

Even better, there is strong experimental evidence for New Physics:

- The occurrence of at least two classes of neutrino oscillations has been established.

- *Dark Matter*: Analysis of the rotation curves of stars and galaxies reveal that there is a lot more ‘stuff’ — i.e. gravitating agents — out there than meets the eye. About a quarter of the gravitating agents in the Universe are such dark matter, and they have to be mostly nonbaryonic. The SM has no candidate for it.

- *The Baryon Number of the Universe*: one finds only about one baryon per $10^9$ photons with the latter being mostly in the cosmic background radiation; there is no evidence for primary antimatter. We know standard CKM dynamics is irrelevant for the Universe’s baryon number. Therefore New Physics has to exist. The success of CKM dynamics tells us that CP violating phases can be large — an insight that should be helpful for any attempt at explaining baryogenesis.
Observation Lesson learned

$\tau - \theta$ Puzzle $P$ violation

production rate $\gg$ decay rate concept of families

suppression of flavour changing neutral currents GIM mechanism & existence of charm

$K_L \rightarrow \pi\pi$ $CP$ violation & existence of top

<table>
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<th>TABLE I: On the History of $\Delta S \neq 0$ Studies.</th>
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<td>$\tau - \theta$ Puzzle</td>
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<td>Suppression of flavour changing neutral currents</td>
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- **Dark Energy.** Type Ia supernovae are considered as ‘standard candles’: considering their real light output as known allows one to infer their distance from their apparent brightness. When in 1998 two teams of researchers studied them at distance scales of about five billion light years, they found them to be fainter as a function of their redshift than what the conventional picture of the Universe’s decelerating expansion would yield. Unless gravitational forces are modified over cosmological distances, one has to conclude the Universe is filled with an hitherto completely unknown agent accelerating the expansion. A tiny, yet non-zero cosmological constant would apparently ‘do the trick’ — yet it would raise more fundamental puzzles.

These heavenly signals are unequivocal in pointing to New Physics, yet leave wide open the nature of this New Physics.

Thus we can be assured that New Physics exists ‘somehow’ ‘somewhere’, and quite likely even ‘nearby’, namely around the TeV scale; above I have called the latter $cpNP$. The LHC program and the Linear Collider project are justified — correctly — to conduct campaigns for $cpNP$. That is unlikely to shed light on the $ssNP$, though it might. Likewise I would not count on a comprehensive and detailed program of heavy flavour studies to shed light on the $ssNP$ behind the flavour puzzle of the SM. Yet the argument is reasonably turned around: such a program will be essential to elucidate salient features of the $cpNP$ by probing the latter’s flavour structure and having sensitivity to scales of order 10 TeV or even higher. One should keep in mind the following: one very popular example of $cpNP$ is supersymmetry; yet it represents an organizing principle much more than even a class of theories. I find it unlikely we can infer all required lessons by studying only flavour diagonal transitions. Heavy flavour decays provide a powerful and complementary probe of $cpNP$. Their potential to reveal something about the $ssNP$ is a welcome extra not required for justifying efforts in that direction.

Accordingly I see a dedicated heavy flavour program as an essential complement to the studies pursued at the high energy frontier at the TEVATRON, LHC, and hopefully ILC. I will illustrate this assertion in the remainder of this review.

**IV-2. $\Delta S \neq 0$ — the ‘Established Hero’**

The chapter on $\Delta S \neq 0$ transitions is a most glorious one in the history of particle physics, as sketched in Table I. We should note that all these features, which now are pillars of the SM, were New Physics at that time. Yet the discovery potential in strange decays
might not have been exhausted.

1. The ‘Dark Horse’

The T odd moment (see Sect. I-2)

\[
\text{Pol}_\perp (\mu) = \frac{\langle \vec{s}(\mu) \cdot (\vec{p}(\mu) \times \vec{p}(\pi)) \rangle}{|\vec{p}(\mu) \times \vec{p}(\pi)|}
\]  

measured in \( K^+ \to \mu^+ \nu \pi^0 \) would

- represent genuine T violation (as long as it exceeded the order \( 10^{-6} \) level) and
- constitute prima facie evidence for CP violation in scalar dynamics.

2. The ‘Second Trojan War’: \( K \to \pi \bar{\nu} \nu \)

According to Greek Mythology the Trojan War described in Homer’s Iliad was actually the second war over Troy. In a similar vein I view the heroic campaign over \( K^0 - \bar{K}^0 \) oscillations — \( \Delta M_K, \epsilon_K, \) and \( \epsilon' \) — as a first one to be followed by a likewise epic struggle over the two ultra-rare modes \( K^+ \to \pi^+ \nu \bar{\nu} \) and \( K_L \to \pi^0 \nu \bar{\nu} \). This campaign has already been opened through the observation of the first through three events very roughly as expected within the SM. The second one, which requires CP violation for its mere existence, so far remains unobserved at a level well above SM predictions. These reactions are like ‘standard candles’ for the SM: their rates are functions of \( V(td) \) with a theoretical uncertainty of about 5\% and 2\% respectively, which is mainly due to the uncertainty in the charm quark mass.

While their rates could be enhanced by New Physics greatly over their SM expectation, I personally find that somewhat unlikely for various reasons. Therefore I suggest one should aim for collecting ultimately about 1000 events of these modes to extract the value of \( V(td) \) and/or identify likely signals of New Physics.

IV-3. The ‘King Kong’ Scenario for New Physics Searches

This scenario can be formulated as follows: “One is unlikely to encounter King Kong; yet once it happens one will have no doubt that one has come across something quite out of the ordinary!”

What it means can be best illustrated with the historical precedent of \( \Delta S \neq 0 \) studies sketched above: the existence of New Physics can unequivocally be inferred if there is a qualitative conflict between data and expectation; i.e., if a theoretically ‘forbidden’ process is found to proceed nevertheless — like in \( K_L \to \pi \pi \) — or the discrepancy between expected and observed rates amounts to several orders of magnitude — like in \( K_L \to \mu^+ \mu^- \) or \( \Delta M_K \). History might repeat itself in the sense that future measurements might reveal such qualitative conflicts, where the case for the manifestation of New Physics is easily made.
This does not mean that the effects are large or straightforward to discover — only that they are much larger than the truly minute SM effects.

I have already mentioned one potential candidate for revealing such a qualitative conflict, namely the muon transverse polarization in $K_{\mu3}$ decays.

1. Electric Dipole Moments

The energy shift of a system placed inside a weak electric field can be expressed through an expansion in terms of the components of that field $\vec{E}$:

$$\Delta \mathcal{E} = d_i E_i + d_{ij} E_i E_j + \mathcal{O}(E^3).$$  \hphantom{}(121)

The coefficients $d_i$ of the term linear in the electric field form a vector $\vec{d}$, called an electric dipole moment (EDM). For a non-degenerate system — it does not have to be elementary — one infers from symmetry considerations that this vector has to be proportional to that system’s spin:

$$\vec{d} \propto \vec{s}.$$  \hphantom{}(122)

Yet, since under time reversal $T$

$$E_i \xrightarrow{T} E_i, \quad s_i \xrightarrow{T} -s_i,$$  \hphantom{}(123)

a non-vanishing EDM constitutes $T$ violation.

No EDM has been observed yet; the upper bounds of the neutron and electron EDM read as follows [50]:

$$d_N < 5 \cdot 10^{-26} \text{ e cm} \quad [\text{from ultracold neutrons}],$$  \hphantom{}(124)

$$d_e < 1.5 \cdot 10^{-27} \text{ e cm} \quad [\text{from atomic EDM}].$$  \hphantom{}(125)

The experimental sensitivity achieved can be illustrated as follows: (i) A neutron EDM of $5 \cdot 10^{-26} \text{ e cm}$ of an object with a radius $r_N \sim 10^{-13}$ cm scales to a displacement of about 7 micron, i.e. less than the width of human hair, for an object of the size of the earth. (ii) Expressing the uncertainty in the measurement of the electron’s magnetic dipole moment $\delta((g - 2)/2) \sim 10^{-11}$ — in analogy to its EDM, one finds a sensitivity level of $\delta(F_2(0)/2m_e) \sim 2 \cdot 10^{-22} \text{ e cm}$ compared to $d_e < 2 \cdot 10^{-26} \text{ e cm}$.

Despite the tremendous sensitivity reached these numbers are still several orders of magnitude above what is expected in CKM theory:

$$d_N^{\text{CKM}} \leq 10^{-30} \text{ e cm},$$  \hphantom{}(126)

$$d_e^{\text{CKM}} \leq 10^{-36} \text{ e cm},$$  \hphantom{}(127)

where in $d_N^{\text{CKM}}$ I have ignored any contribution from the strong CP problem. These numbers are so tiny for reasons very specific to CKM theory, namely its chirality structure and
the pattern in the quark and lepton masses. Yet New Physics scenarios with right-handed currents, flavour changing neutral currents, a non-minimal Higgs sector, heavy neutrinos etc. are likely to generate considerably larger numbers: $10^{-28} - 10^{-26}$ e cm represents a very possible range there quite irrespective of whether these new forces contribute to $\epsilon_K$ or not. This range appears to be within reach in the foreseeable future. There is a vibrant multiprong program going on at several places. Such experiments while being of the ‘table top’ variety require tremendous efforts, persistence, and ingenuity — yet the insights to be gained by finding a nonzero EDM somewhere are tremendous.

2. On the Brink of Major Discoveries in Charm Transitions

The study of charm dynamics had a great past: It was instrumental in driving the paradigm shift from quarks as mathematical entities to physical objects and in providing essential support for accepting QCD as the theory of the strong interactions. Yet it is often viewed as one without a future. For charm’s electroweak phenomenology is on the ‘dull’ side: CKM parameters are known from other sources, $D^0 - \bar{D}^0$ oscillations slow, CP asymmetries small and loop driven decays extremely rare.

Yet more thoughtful observers have realized that the very ‘dullness’ of the SM phenomenology for charm provides us with a dual opportunity, namely to

- probe our quantitative understanding of QCD’s nonperturbative dynamics thus calibrating our theoretical tools for $B$ decays and

- perform almost ‘zero-background’ searches for New Physics. Charm transitions actually provide a unique and novel portal to flavour dynamics with the experimental situation being a priori favourable (except for the lack of Cabibbo suppression). While New Physics signals can exceed SM predictions on CP asymmetries by orders of magnitude, they might not be large in absolute terms, as specified later [51].

The former goal provides a central motivation for the CLEO-c program at Cornell University. The latter perspective has received a major boost through the strong, albeit not yet conclusive evidence for $D^0 - \bar{D}^0$ oscillations presented by BABAR and BELLE in March 2007. The main three positive findings are:

- Comparing the effective lifetimes for $D^0 \to K^+K^-$ and $D^0 \to K^-\pi^+$ BELLE obtains a $3.2 \sigma$ signal for a difference [53]:

$$y_{CP} = \frac{\tau(D^0 \to K^-\pi^+)}{\tau(D^0 \to K^+K^-)} - 1 = (1.31 \pm 0.32 \pm 0.25) \cdot 10^{-2} \quad (128)$$

In the limit of CP invariance, which provides a good approximation for charm decays as explained later, the two mass eigenstates of the $D^0 - \bar{D}^0$ complex are

---

10 The meaning of almost ‘zero-background’ has of course to be updated in the light of the increasing experimental sensitivity.
CP eigenstates as well. $D^0 \to K^+ K^-$ yields the width for the CP even state and $D^0 \to K^- \pi^+$ the one averaged over the CP even and odd states. In this limit one has

$$y_{CP} = y_D = \frac{\Delta \Gamma_D}{2 \Gamma_D}$$  \hspace{1cm} (129)

- Analyzing the time dependent Dalitz plot for $D^0(t) \to K_S \pi^+ \pi^-$ BELLE finds [54]

$$x_D \equiv \frac{\Delta M_D}{\Gamma_D} = (0.80 \pm 0.29 \pm 0.17) \cdot 10^{-2} ,\ y_D = (0.33 \pm 0.24 \pm 0.15) \cdot 10^{-2} ,$$  \hspace{1cm} (130)

which amounts to a $2.4 \sigma$ signal for $x_D \neq 0$.

- BABAR has studied the decay rate evolution for the doubly Cabibbo suppressed mode $D^0(t) \to K^+ \pi^-$ and found [55]

$$y'_D = (0.97 \pm 0.44 \pm 0.31) \cdot 10^{-2} ,\ (x'_D)^2 = (-2.2 \pm 3.0 \pm 2.1) \cdot 10^{-4}$$  \hspace{1cm} (131)

representing a $3.9 \sigma$ signal for $[y'_D, (x'_D)^2] \neq [0,0]$ due to the correlations between $y'_D$ and $(x'_D)^2$. The observables $x'_D$ and $y'_D$ are related to $x_D$ and $y_D$ through the strong phase shift $\delta$ of the amplitude for $D^0 \to K^+ \pi^-$ to that for $D^0 \to K^- \pi^+$:

$$x'_D = x_D \cos \delta + y_D \sin \delta ,\ y'_D = -x_D \sin \delta + y_D \cos \delta$$  \hspace{1cm} (132)

A ‘preliminary’ average by the heavy Flavour Averaging Group over all relevant data yields

$$x_D = (0.85 \pm 0.32) \cdot 10^{-2} ,\ y_D = (0.71 \pm 0.21) \cdot 10^{-2} ,\ \cos \delta = 0.40^{+0.23}_{-0.31}$$  \hspace{1cm} (133)

with $5 \sigma$ significance for $[x_D, y_D] \neq [0,0]$ – and the caveat that averaging over the existing data sets has to be taken with quite a grain of salt at present due to the complicated likelihood functions [56].

Establishing $D^0 - \bar{D}^0$ oscillations would provide a qualitatively new insight into flavour dynamics. After having discovered oscillations in all three mesons built from down-type quarks – $K^0$, $B_d$ and $B_s$ – it would be the first observation of oscillations with up-type quarks; it would also remain the only one (at least for three-family scenarios): while top quarks do not hadronize [40] thus removing the condition sine qua non for $T^0 - \bar{T}^0$ oscillations, one cannot have $\pi^0 - \pi^0$ oscillations in the $u$ quark sector. This means that analyzing flavour changing neutral currents (FlChNC) in charm transitions provides a rather unique portal to New Physics, where FlChNC could be much less suppressed for up-type than for down-type quarks.

The observables $x_D$ and $y_D$ are doubly Cabibbo suppressed – $\Delta M_D/\Gamma_D$, $\Delta \Gamma_D/2 \Gamma_D \propto \tan^2 \theta_C$ – and vanish in the limit of $SU(3)_F$ symmetry. The history of SM predictions for them beyond these general statements is a rather checkered one with ‘suggested’ numbers differing by orders of magnitude. Some of this variation is due to authors following the SM description for $\Delta S = 2$ or $\Delta B = 2$ dynamics literally by inferring $\mathcal{L}(\Delta C = 2)$ from
quark box diagrams treated as short-distance dynamics. This is, however, not justified. It is widely understood now that the usual quark box diagram is irrelevant due to its untypically severe GIM suppression \((m_s/m_c)^4\). Two complementary approaches have been employed for estimating the size of \(x_D\) and \(y_D\). (i) A systematic analysis based on an OPE treatment has been given in Ref.[43]. The novel feature is that the numerically largest contributions do not come from terms of leading order in \(1/m_c\), but from higher-dimensional operators. For those possess a much softer GIM reduction of \((m_s/\mu_{had})^2\) due to nonperturbative ‘condensate’ terms characterized by a mass scale \(\mu_{had}\) (even \(m_s/\mu_{had}\) terms could arise in the presence of right-handed currents). The resulting expansion is expressed through powers of \(1/m_c\), \(m_s\) and \(\mu_{had}\) yielding

\[
x_D(SM)_{OPE}, \ y_D(SM)_{OPE} \sim \mathcal{O}(10^{-3}).
\] (134)

(i) The authors of Refs.[44, 45] find similar numbers, albeit in a quite different approach: estimating \(SU(3)_{FL}\) breaking for \(\Delta \Gamma_D\) from phase space differences for two-, three- and four-body \(D\) modes they obtain \(y_D(SM) \sim 0.01\) and inferring \(x_D\) from \(y_D\) via a dispersion relation they arrive at \(0.001 \leq |x_D(SM)| \leq 0.01\) with \(x_D\) and \(y_D\) being of opposite sign.

While one predicts similar numbers for \(x_D\) and \(y_D\), one should keep in mind that they arise in very different dynamical environments: \(\Delta M_D\) is generated from off-shell intermediate states and thus is sensitive to New Physics, which could affect it considerably. \(\Delta \Gamma_D\) on the other hand is shaped by on-shell intermediate states; while it is hardly sensitive to New Physics (for a dissenting opinion, see Ref.[57]), it involves less averaging or ‘smearing’ than \(\Delta M_D\) making it thus more vulnerable to violations of quark-hadron duality.

In summary: to the best of our present knowledge even values for \(x_D\) and \(y_D\) as ‘high’ as 0.01 could be due entirely to SM dynamics of otherwise little interest. It is likewise possible that a large or even dominant part of \(x_D \sim 0.01\) in particular is due to New Physics. While one should never rule out a theoretical breakthrough, I am less than confident that even the usual panacea, namely lattice QCD, can provide a sufficiently fine instrument in the foreseeable future.

Yet despite this lack of an unequivocal statement from theory one wants to probe these oscillations as accurately as possible even in the absence of the aforementioned breakthrough, since they represent an intriguing quantum mechanical phenomenon and – on the more practical side – constitute an important ingredient for CP asymmetries arising in \(D^0\) decays due to New Physics as explained next.

**(i) CP Violation without Oscillations – Partial Widths, Moments etc.**

As explained before the observed baryon number implies the existence of New Physics in \(CP\) violating dynamics. It would therefore be unwise not to undertake dedicated searches for \(CP\) asymmetries in charm decays, in particular, since those offer several pragmatic advantages. (i) While we do not know how to reliably compute the strong phase shifts required for direct \(CP\) violation to emerge in partial widths, we can expect them to be in general large, since charm decays proceed in a resonance domain. (ii) The branching ratios into relevant modes are relatively large. (iii) \(CP\) asymmetries can be linear in New Physics amplitudes thus enhancing sensitivity to the latter. (iv) The ‘background’ from known physics is small: within the SM the effective weak phase is highly diluted,
namely $\sim \mathcal{O}(\lambda^4)$. Without oscillations only direct $\text{CP}$ violation can occur, and it can arise only in singly Cabibbo suppressed transitions, where one expects them to reach no better than the 0.1 % level; significantly larger values would signal New Physics. *Almost any* asymmetry in Cabibbo allowed or doubly suppressed channels requires the intervention of New Physics, since – in the absence of oscillations – there is only one weak amplitude. The exception are channels containing a $K_S$ (or $K_L$) in the final state like $D \rightarrow K_S \pi$. There are two sources for a $\text{CP}$ asymmetry from known dynamics: (i) There are actually two transition amplitudes involved, namely a Cabibbo favoured and a doubly suppressed one, $D \rightarrow \bar{K}^0 \pi$ and $D \rightarrow K^0 \pi$, respectively. Their relative weak CKM phase is given by $\eta A^2 \lambda^6 \sim \text{few} \cdot 10^{-5}$, which seems to be well beyond observability. (ii) While one has $|T(D \rightarrow \bar{K}^0 \pi)| = |T(\bar{D} \rightarrow K^0 \pi)|$, the well-known $\text{CP}$ impurity $|p_K| \neq |q_K|$ in the $\bar{K}_S$ wave function introduces a difference between $D^{0,+} \rightarrow K_S \pi^{0,+}$ and $\bar{D}^{0,-} K_S \pi^{0,-}$ of $|q_K|^2 - |p_K|^2 = (3.32 \pm 0.06) \cdot 10^{-3}$ [41].

Decays to final states of more than two pseudoscalar or one pseudoscalar and one vector meson contain more dynamical information than given by their widths; their distributions as described by Dalitz plots or $\text{T-odd}$ moments can exhibit $\text{CP}$ asymmetries that can be considerably larger than those for the width. Final state interactions while not necessary for the emergence of such effects, can fake a signal; yet that can be disentangled by comparing $\text{T-odd}$ moments for $\text{CP}$ conjugate modes, as explained below.

All $\text{CP}$ asymmetries observed so far in $K_L$ and $B_d$ decays except one concern partial widths, i.e. $\Gamma(P \rightarrow f)$ vs. $\Gamma(\bar{P} \rightarrow \bar{f})$. The one notable exception can teach us important lessons for future searches both in charm and $B$ decays, namely the $\text{T odd}$ moment found in $K_L \rightarrow \pi^+ \pi^- e^+ e^-$, as discussed around Eq.(22): a large asymmetry $A \simeq 14\%$ driven by the tiny impurity parameter $|\epsilon_K| \sim 0.22\%$ was found in the rare mode with a branching ratio of about $3 \cdot 10^{-7}$! I.e., one can trade the size of the branching ratio for that of a $\text{CP}$ asymmetry.

$D$ decays can be treated in an analogous way. Consider the Cabibbo suppressed channel

$$D \rightarrow \bar{K} K \pi^+ \pi^- \quad (135)$$

and define by $\phi$ now the angle between the $\bar{K} K$ and $\pi^+ \pi^-$ planes. Then one has

$$\frac{d\Gamma}{d\phi}(D \rightarrow \bar{K} K \pi^+ \pi^-) = \Gamma_1 \cos^2 \phi + \Gamma_2 \sin^2 \phi + \Gamma_3 \cos \phi \sin \phi \quad (136)$$

$$\frac{d\Gamma}{d\phi}(\bar{D} \rightarrow \bar{K} K \pi^+ \pi^-) = \bar{\Gamma}_1 \cos^2 \phi + \bar{\Gamma}_2 \sin^2 \phi + \bar{\Gamma}_3 \cos \phi \sin \phi \quad (137)$$

The partial width for $D[\bar{D}] \rightarrow \bar{K} K \pi^+ \pi^-$ is given by $\Gamma_{1,2}[\bar{\Gamma}_{1,2}]$: $\Gamma_1 \neq \bar{\Gamma}_1$ or $\Gamma_2 \neq \bar{\Gamma}_2$ represents direct $\text{CP}$ violation in the partial width. $\Gamma_3$&$\bar{\Gamma}_3$ constitute $\text{T odd}$ correlations.

\[11\] This mode can exhibit direct $\text{CP}$ violation even within the SM.
By themselves they do not necessarily indicate CP violation, since they can be induced by strong final state interactions. However

\[ \Gamma_3 \neq \bar{\Gamma}_3 \implies \text{CP violation!} \] (138)

It is quite possible or even likely that a difference in \( \Gamma_3 \) vs. \( \bar{\Gamma}_3 \) is significantly larger than in \( \Gamma_1 \) vs. \( \bar{\Gamma}_1 \) or \( \Gamma_2 \) vs. \( \bar{\Gamma}_2 \). Furthermore one can expect that differences in detection efficiencies can be handled by comparing \( \Gamma_3 \) with \( \Gamma_{1,2} \) and \( \bar{\Gamma}_3 \) with \( \bar{\Gamma}_{1,2} \). A pioneering search for such an effect has been undertaken by FOCUS [58].

(ii) Oscillations – the New Portal to CP Violation

With oscillations on an observable level – and it seems \( x_D, y_D \sim 0.005 - 0.01 \) satisfy this requirement – the possibilities for CP asymmetries proliferate.

At the very least – i.e. even if \( \mathcal{L}(\Delta C = 2) \) is generated by SM dynamics alone and thus does not contain any appreciable CP violation – oscillations provide another stage for a CP asymmetry to surface in Cabibbo favoured channels like \( D^0 \to K_S \rho^0 \) (or \( D^0 \to K_S \pi^0, K_S \phi \)): in addition to the direct asymmetries mentioned just above – of about \( 10^{-4} \) due to the interference between \( D^0 \to \bar{K}^0 \rho^0 \) and \( D^0 \to K^0 \rho^0 \) and of \( (3.32 \pm 0.06) \cdot 10^{-3} \) due to \( |p_K| \neq |q_K| \) – one obtains a time dependent asymmetry in qualitative analogy to \( B_d \to \psi K_S \) given by

\[ x_D \frac{t}{\tau_D} \cdot \text{Im}\left\{ \frac{q T(\bar{D}^0 \to K_S \rho^0)}{p T(D^0 \to K_S \rho^0)} \right\} \approx x_D \frac{t}{\tau_D} \cdot \frac{V^*(u) V(d)}{V(c) V^*(u)} \cdot \eta(A\lambda^2)^2 \sim x_D \frac{t}{\tau_D} \cdot 10^{-3} \] (139)

\[ \text{Im}\left\{ \frac{q T(\bar{D}^0 \to K_S \phi)}{p T(D^0 \to K_S \phi)} \right\} \approx 2\eta(A\lambda^2)^2 \] is an accurate SM prediction without a hadronic uncertainty. Alas with \( x_D \sim 0.01 \) it amounts to a \( 10^{-5} \) effect and is presumably too small to be observed.

The more intriguing scenario arises, when New Physics contributes significantly to \( \mathcal{L}(\Delta C = 2) \), which is still quite possible. I will begin by drawing on analogies with two other cases, namely the retrospective one of \( K_L \) and the very topical one of \( B_s \). (i) Let us assume that – contrary to history – at the time of the discovery of \( K^0 - \bar{K}^0 \) oscillations the community had already established the SM with two families, been aware of the possibility of CP violation and the need for three families to implement the latter. They would then have argued that \( \Delta M_K \) could be generated by long-distance dynamics through off-shell \( K^0 \to \pi^0, \eta, \eta', 2 \pi^+ \to \bar{K}^0 \) etc. Indeed roughly half the observed size of \( \Delta M_K \) can be produced that way. Yet they would have realized that long-distance dynamics cannot induce CP violation in \( K^0 \to \bar{K}^0 \), i.e. \( \epsilon_K \neq 0 \). The latter observable is thus controlled by short-distance dynamics. Finding a time dependent CP asymmetry would then show the presence of physics beyond the SM then, namely the third family. (ii) \( \Delta M(B_s) \) has been observed to be consistent with the SM prediction within mainly theoretical uncertainties; yet since those are still sizable, we cannot rule out that New Physics impacts \( B_s - \bar{B_s} \) oscillations significantly. This issue, which is unlikely to be resolved theoretically, can be decided experimentally by searching for a time dependent CP violation in \( B_s(t) \to \psi \phi \). For within the SM one predicts [12] a very small asymmetry not exceeding 4% in this
transition since on the leading CKM level quarks of only the second and third family contribute; this will be discussed in detail in Sect.IV IV-51. Yet in general one can expect New Physics contributions to $B_s - \bar{B}_s$ oscillations to exhibit a weak phase that is not particularly suppressed. Even if New Physics affects $\Delta M(B_s)$ only moderately, it could greatly enhance $\sin 2\phi(B_s \rightarrow \psi \phi)$, possibly even by an order of magnitude!

These examples can be seen as ‘qualitative’ analogies only, not quantitatives one with $D^0-\bar{D}^0$ oscillations being (at best) quite slow. Since $y_D, x_D \ll 1$, it suffices to give the decay rate evolution to first order in those quantities only (the general expressions can be found in Ref.[41]):

$$\Gamma(D^0(t) \rightarrow K^+K^-) \propto e^{-\tau_D t}\left|\Gamma(D^0 \rightarrow K^+K^-)\right|^2 \times$$

$$\left[1 + y_D \frac{t}{\tau_D} \left(1 - \frac{\text{Re}\frac{q}{p} \rho_{K+K^-}}{1}\right) - x_D \frac{t}{\tau_D} \text{Im}\frac{q}{p} \rho_{K+K^-}\right]$$

$$\Gamma(\bar{D}^0(t) \rightarrow K^+K^-) \propto e^{-\tau_D t}\left|\Gamma(D^0 \rightarrow K^+K^-)\right|^2 \times$$

$$\left[1 + y_D \frac{t}{\tau_D} \left(1 - \frac{\text{Re}\frac{p}{q} \rho_{K+K^-}}{1}\right) - x_D \frac{t}{\tau_D} \text{Im}\frac{p}{q} \rho_{K+K^-}\right]$$

Some comments might elucidate Eqs.(140):

- **CP** invariance implies (in addition to $|T(D^0 \rightarrow K^+K^-)| = |T(\bar{D}^0 \rightarrow K^+K^-)|$) $\frac{q}{p} \rho_{K+K^-} = 1$ (and $|q| = |p|$). The transitions $D^0(t) \rightarrow K^+K^-$ and $\bar{D}^0(t) \rightarrow K^+K^-$ are then described by the same single lifetime. That is a consequence of the Theorem given by Eq.(?7), since $K^+K^-$ is a CP eigenstate.

- The usual three types of **CP** violation can arise, namely the direct and indirect types $|\rho_{K+K^-}| \neq 0$ and $|q| \neq |p|$, respectively – as well as the one involving the interference between the oscillation and direct decay amplitudes – $\text{Im}\frac{q}{p} \rho_{K+K^-} \neq 0$ leading also to $\text{Re}\frac{q}{p} \rho_{K+K^-} \neq 1$.

- Assuming for simplicity $|T(D^0 \rightarrow K^+K^-)| = |T(\bar{D}^0 \rightarrow K^+K^-)|$ (CKM dynamics is expected to induce an asymmetry not exceeding 0.1%) and $|q/p| = 1 - \epsilon_D$ one has $(q/p) \rho_{K+K^-} = (1 - \epsilon_D)e^{i\phi_{KK}}$ and thus

$$A_\Gamma = \frac{\Gamma(D^0(t) \rightarrow K^+K^-) - \Gamma(D^0(t) \rightarrow K^-K^+)}{\Gamma(D^0(t) \rightarrow K^+K^-) + \Gamma(D^0(t) \rightarrow K^-K^+)} \simeq x_D \frac{t}{\tau_D} \sin \phi_{KK} - y_D \frac{t}{\tau_D} \epsilon_D \cos \phi_{KK} \cdot$$

where I have assumed $|\epsilon_D| \ll 1$. BELLE has found [53]

$$A_\Gamma = (0.01 \pm 0.30 \pm 0.15)\%$$

(142)

While there is no evidence for **CP** violation in the transition, one should also note that the asymmetry is bounded by $x_D$. For $x_D, y_D \leq 0.01$, as indicated by the data, $A_\Gamma$ could hardly exceed the 1% range. I.e., there is no real bound on $\phi_D$ or $\epsilon_D$ yet.
The good news is that if $x_D$ and/or $y_D$ indeed fall into the 0.5 - 1 % range, then any improvement in the experimental sensitivity for a CP asymmetry in $D^0(t) \to K^+K^-$ constrains New Physics scenarios – or could reveal them [59]!

Another promising channel for probing CP symmetry is $D^0(t) \to K^+\pi^-$: since it is doubly Cabibbo suppressed, it should a priori exhibit a higher sensitivity to a New Physics amplitude. Furthermore it cannot exhibit direct CP violation in the SM.

$$\frac{\Gamma(D^0(t) \to K^+\pi^-)}{\Gamma(D^0(t) \to K^-\pi^+)} = \frac{|T(D^0 \to K^+\pi^-)|^2}{|T(D^0 \to K^-\pi^+)|^2} \times \left[ 1 + \left( \frac{t}{\tau_D} \right)^2 \left( \frac{x_D^2 + y_D^2}{4tg\theta_C^2} \right) \frac{p^2}{q} |\hat{\rho}_{K\pi}|^2 + \left( \frac{t}{\tau_D} \right) \frac{q}{p} |\hat{\rho}_{K\pi}| \left( \frac{y_D^D\cos\phi_{K\pi} + x_D^D\sin\phi_{K\pi}}{tg\theta_C^2} \right) \right]$$

(143)

$$\frac{\Gamma(\bar{D}^0(t) \to K^-\pi^+)}{\Gamma(D^0(t) \to K^+\pi^-)} = \frac{|T(D^0 \to K^-\pi^+)|^2}{|T(D^0 \to K^+\pi^-)|^2} \times \left[ 1 + \left( \frac{t}{\tau_D} \right)^2 \left( \frac{x_D^2 + y_D^2}{4tg\theta_C^2} \right) \frac{p^2}{q} |\hat{\rho}_{K\pi}|^2 + \left( \frac{t}{\tau_D} \right) \frac{q}{p} |\hat{\rho}_{K\pi}| \left( \frac{y_D^D\cos\phi_{K\pi} - x_D^D\sin\phi_{K\pi}}{tg\theta_C^2} \right) \right]$$

(144)

with

$$\frac{q}{p} \frac{T(D^0 \to K^+\pi^-)}{T(D^0 \to K^-\pi^+)} = -\frac{1}{tg^2\theta_C} (1 - \epsilon_D)|\hat{\rho}_{K\pi}|e^{-i(\delta - \phi_{K\pi})}$$

$$\frac{q}{p} \frac{T(D^0 \to K^-\pi^+)}{T(D^0 \to K^+\pi^-)} = -\frac{1}{tg^2\theta_C} \frac{1}{1 - \epsilon_D}|\hat{\rho}_{K\pi}|e^{-i(\delta + \phi_{K\pi})}$$

(145)

yielding an asymmetry

$$\frac{\Gamma(\bar{D}^0(t) \to K^-\pi^+)}{\Gamma(D^0(t) \to K^+\pi^-) + \Gamma(D^0(t) \to K^+\pi^-)} \approx \left( \frac{t}{\tau_D} \right) |\hat{\rho}_{K\pi}| \left( \frac{y_D^D\cos\phi_{K\pi} + x_D^D\sin\phi_{K\pi}}{tg\theta_C^2} \right) + \left( \frac{t}{\tau_D} \right)^2 |\hat{\rho}_{K\pi}|^2 \frac{\epsilon_D(x_D^2 + y_D^2)}{2tg\theta_C^2}$$

(146)

where I have again assumed for simplicity $|\epsilon_D| \ll 1$ and no direct CP violation.

BABAR has also searched for a time dependent CP asymmetry in $D^0 \to K^+\pi^-$ vs. $\bar{D}^0(t) \to K^-\pi^+$, yet so far has not found any evidence for it [55] on the about 1 % level. Yet again, with $x_D^D$ and $y_D^D$ capped by about 1%, no nontrivial bound can be placed on the weak phase $\phi_{K\pi}$ that can be induced by New Physics. On the other hand any further increase in experimental sensitivity could reveal a signal.
Oscillations, even in the extreme scenario of \( x_D = 0, y_D \neq 0 \), will induce a time dependence in the T odd moments \( \Gamma_3 \) and \( \bar{\Gamma}_3 \) of \( D \to K^+K^-\pi^+\pi^- \).

In close qualitative analogy to \( B^0 \) decays one can observe \( \text{CP} \) violation in \( D^0 \) decays also through the existence of a transition. The reaction

\[
\epsilon^+\epsilon^- \to \psi(3770) \to D^0\bar{D}^0 \to f_\pm f_\pm' \tag{147}
\]

with \( \text{CP} |f_0(t)\rangle = \pm|f(t)\rangle \) can occur only, if \( \text{CP} \) is violated, since \( \text{CP} |\psi(3770)\rangle = +1 \neq \text{CP} |f_\pm f_\pm'\rangle = (-1)^{L-1} = -1 \). The final states \( f \) and \( f' \) can be different, as long as they possess the same \( \text{CP} \) parity. More explicitly one has for \( x_D \ll 1 \)

\[
\text{BR}(\psi(3770) \to D^0\bar{D}^0 \to f_\pm f_\pm' \simeq \text{BR}(D \to f_\pm)\text{BR}(D \to f_\pm'));
\]

\[
\left[ (2 + x_D^2) \left| \frac{q}{p} \right|^2 \left| \bar{\rho}(f_\pm) - \bar{\rho}(f_\pm') \right|^2 + x_D^2 \left| 1 - \frac{q}{p} \bar{\rho}(f_\pm) \right|^2 \left| \frac{q}{p} \bar{\rho}(f_\pm') \right|^2 \right] \tag{148}
\]

The second contribution in the square brackets can occur only due to oscillations and then also for \( f_\pm' = f_\pm \); yet it is heavily suppressed by \( x_D^2 \lesssim 10^{-4} \) making it practically unobservable. The first term arises even with \( x_D = 0 \), yet requires \( f_\pm' \neq f_\pm \). It is possible that \( \left| \bar{\rho}(f_\pm) - \bar{\rho}(f_\pm') \right|^2 \) provides a larger signal of \( \text{CP} \) violation than either \( \left| 1 - |\rho(f_\pm)|^2 \right| \) or \( \left| 1 - |\rho(f_\pm')|^2 \right| \).

Eq.148 also holds, when the final states are not \( \text{CP} \) eigenstates, yet still modes common to \( D^0 \) and \( \bar{D}^0 \). Consider for example \( \epsilon^+\epsilon^- \to D^0\bar{D}^0 \to f_\pm f_\mp \) with \( f_a = K^+K^- \), \( f_b = K^\pm\pi^\mp \). Measuring those rates will yield unique information on the strong phase shifts.

(iii) \( \text{CP} \) Violation in Semileptonic \( D^0 \) Decays

\( |q/p| \neq 1 \) unambiguously reflects \( \text{CP} \) violation in \( \Delta C = 2 \) dynamics. It can be probed most directly in semileptonic \( D^0 \) decays leading to ‘wrong sign’ leptons:

\[
a_{S,L}(D^0) \equiv \frac{\Gamma(D^0(t) \to l^-X) - \Gamma(D^0(t) \to l^+X)}{\Gamma(D^0(t) \to l^-X) + \Gamma(D^0(t) \to l^+X)} = \frac{|q|^4 - |p|^4}{|q|^4 + |p|^4} \tag{149}
\]

The corresponding observable has been studied in semileptonic decays of neutral \( K \) and \( B \) mesons. With \( a_{S,L} \) being controlled by \( (\Delta\Gamma/\Delta M)\sin\phi_{\text{weak}} \), it is predicted to be small in both cases, albeit for different reasons: (i) While \( (\Delta\Gamma_K/\Delta M_K) \sim 1 \) one has \( \sin^2\phi_{\text{weak}} \ll 1 \) leading to \( a_{S,L} = 6 \zeta \simeq (3.32 \pm 0.06) \cdot 10^{-3} \) as observed. (ii) For \( B^0 \) on the other hand one has \( (\Delta\Gamma_B/\Delta M_B) \ll 1 \) leading to \( a_{S,L}^B < 10^{-3} \) (see Sect.IV IV-5 for details).

For \( D^0 \) on the other hand both \( \Delta M_D \) and \( \Delta\Gamma_D \) are small, yet \( \Delta\Gamma_D/\Delta M_D \) is not: present data indicate it is about unity or even larger: \( a_{S,L} \) is given by the smaller of \( \Delta\Gamma_D/\Delta M_D \) or its inverse multiplied by \( \sin^2\phi_{\text{weak}} \); which might not be that small: i.e., while the rate for ‘wrong-sign’ leptons is certainly small in semileptonic decays of neutral \( D \) mesons, their \( \text{CP} \) asymmetry might not be at all, if New Physics intervenes to induce \( \phi_{\text{weak}} \).
3. CP Violation in the Lepton Sector

I find the conjecture that baryogenesis is a secondary phenomenon driven by primary leptogenesis a most intriguing one also for philosophical reasons. Yet then it becomes mandatory to search for CP violation in the lepton sector in a dedicated fashion.

In Sect. IV-3-1 I have sketched the importance of measuring electric dipole moments as accurately as possible. The electron’s EDM is a most sensitive probe of CP violation in leptodynamics. Comparing the present experimental and CKM upper bounds, respectively

\[ d_{e}^{\exp} \leq 1.5 \cdot 10^{-27} \text{ e cm vs. } d_{e}^{CKM} \leq 10^{-36} \text{ e cm}, \]

we see there is a wide window of several orders of magnitude where New Physics could surface in an unambiguous way. This observation is reinforced by the realization that New Physics scenarios can naturally generate \( d_{e} > 10^{-28} \text{ e cm} \), while of only secondary significance in \( \epsilon_{K}, \epsilon' \), and \( \sin2\phi_{i} \).

The importance that at least part of the HEP community attributes to finding CP violation in leptodynamics is best demonstrated by the efforts contemplated for observing CP asymmetries in neutrino oscillations. Clearly hadronization will be the least of the concerns, yet one has to disentangle genuine CP violation from matter enhancements, since the neutrino oscillations can be studied only in a matter, not an antimatter environment. Our colleagues involved in such endeavours will rue their previous complaints about hadronization and remember the wisdom of an ancient Greek saying:

"When the gods want to really harm you, they fulfill your wishes."

4. The Decays of \( \tau \) Leptons — the Next ‘Hero Candidate’

Like charm hadrons the \( \tau \) lepton is often viewed as a system with a great past, but hardly a future. Again I think this is a very misguided view, and I will illustrate it with two examples.

Searching for \( \tau^{\pm} \rightarrow \mu^{\pm} \mu^{+}\mu^{-} \) (and its variants) — processes forbidden in the SM — is particularly intriguing, since it involves only ‘down-type’ leptons of the second and third family and is thus the complete analogy of the quark lepton process \( b \rightarrow s\bar{s}s \) driving \( B_{s} \rightarrow \phi K_{S} \), which has recently attracted such strong attention. Following this analogy literally one guestimates \( \text{BR}(\tau \rightarrow 3\mu) \sim 10^{-8} \) to be compared with the present bound from BELLE

\[ \text{BR}(\tau \rightarrow 3\mu) \leq 2 \cdot 10^{-7}. \]

\[ ^{12} \text{For it would complete what is usually called the Copernican Revolution [46]: first our Earth was removed from the center of the Universe, then in due course our Sun, our Milky Way and local cluster; few scientists believe life exists only on our Earth. Realizing that the stuff we are mostly made out of — protons and neutrons — are just a cosmic ‘afterthought’ fits this pattern, which culminates in the dawning realization that even our Universe is just one among innumerable others, albeit a most unusual one.} \]
It would be very interesting to know what the $\tau$ production rate at the hadronic colliders is, and whether they could be competitive or even superior with the $B$ factories in such a search.

In my judgment $\tau$ decays — together with electric dipole moments for leptons and possibly $\nu$ oscillations referred to above — provide the best stage to search for manifestations of CP breaking leptodynamics.

The most promising channels for exhibiting CP asymmetries are $\tau \to \nu K \pi$, since due to the heaviness of the lepton and quark flavours they are most sensitive to nonminimal Higgs dynamics, and they can show asymmetries also in the final state distributions rather than integrated rates [47].

There is also a unique opportunity in $e^+e^- \to \tau^+\tau^-$: since the $\tau$ pair is produced with its spins aligned, the decay of one $\tau$ can ‘tag’ the spin of the other $\tau$; i.e., one can probe spin-dependent CP asymmetries with unpolarized beams. This provides higher sensitivity and more control over systematic uncertainties.

I feel these features are not sufficiently appreciated even by proponents of Super-B factories. It has been recently pointed [48] out that based on known physics one can actually predict a CP asymmetry:

$$\frac{\Gamma(\tau^+ \to K_S\pi^+\nu) - \Gamma(\tau^- \to K_S\pi^-\nu)}{\Gamma(\tau^+ \to K_S\pi^+\nu) + \Gamma(\tau^- \to K_S\pi^-\nu)} = (3.27 \pm 0.12) \times 10^{-3}$$

(152)

due to $K_S$’s preference for antimatter.

IV-4. Future Studies of $B_{u,d}$ Decays

The successes of CKM theory to describe flavour dynamics do not tell us at all that New Physics does not affect $B$ decays; the message is that typically we cannot count on a numerically massive impact there. Shifting an asymmetry by, say, ten percentage points — for example from 40 % to 50 % — might already be on the large side. Thus our aim has to be to aim for uncertainties that do not exceed a few percent.

An integrated luminosity of $1 \text{ab}^{-1}$ at the $B$ factories will fall short of such a goal for $B_d \to \pi\pi$, $B^\pm \to D^{\text{neat}}K^{\pm}$, and in particular also for the modes driven by $b \to sqq$. Even ten times that statistics would not suffice in view of the ‘big picture’, i.e. when one includes other rare transitions. Of course we are in the very fortunate situation that one of the LHC experiments, namely LHCb, is dedicated to undertaking precise measurements of the weak decays of beauty hadrons. Thus we can expect a stream of high quality data to be forthcoming over the next several years. I will briefly address one class of rare decays.

1. $B \to l^+l^-X$

We are just at the beginning of studying $B \to l^+l^-X$, and it has to be pursued in a dedicated and comprehensive manner for the following reasons:

• With the final state being more complex than for $B \to \gamma X$, it is described by a
larger number of observables: rates, spectra of the lepton pair masses and the lepton energies, their forward-backward asymmetries, and CP asymmetries.

- These observables provide independent information, since there is a larger number of effective transition operators than for \( B \to \gamma X \). By the same token there is a much wider window to find New Physics and even diagnose its salient features.

- It will take the statistics of a Super-Flavour factory to mine this wealth of information on New Physics.

- Essential insights can be gained also by analyzing the exclusive channel \( B \to \ell^+\ell^-K^* \) at hadronic colliders like the LHC, in particular the position of the zero in the lepton forward-backward asymmetry. For the latter appears to be fairly insensitive to hadronization effects in this exclusive mode [38]. It will be important to analyze quantitatively down to which level of accuracy this feature persists.

IV-5. \( B_s \) Decays — an Independent Chapter in Nature’s Book

When the program for the \( B \) factories was planned, it was thought that studying \( B_s \) transitions will be required to construct the CKM triangle, namely to determine one of its sides and the angle \( \phi_3 \). As discussed above a powerful method has been developed to extract \( \phi_3 \) from \( B^\pm \to D^{\text{neut}}K^\pm \), and a meaningful value for \( |V(td)/V(ts)| \) has been inferred from the measured value of \( \Delta M_{B_d}/\Delta M_{B_s} \). None of this, however, reduces the importance of a future comprehensive program to study \( B_s \) decays — on the contrary! With the basic CKM parameters fixed or to be fixed in \( B_{u,d} \) decays, \( B_s \) transitions can be harnessed as powerful probes for New Physics and its features.

In this context it is essential to think ‘outside the box’ — pun intended. The point here is that several relations that hold in the SM (as implemented through quark box and other loop diagrams) are unlikely to extend beyond minimal extensions of the SM. In that sense \( B_{u,d} \) and \( B_s \) decays constitute two different and complementary chapters in Nature’s book on fundamental dynamics.

1. CP Violation in Non-Leptonic \( B_s \) Decays

One class of nonleptonic \( B_s \) transitions does not follow the paradigm of large CP violation in \( B \) decays [12]:

\[
A_{CP} (B_s(t) \to [\psi\phi]_{t=0}/\psi\eta) = \sin2\phi(B_s)\sin\Delta M(B_s)t ,
\]

\[
\sin2\phi(B_s) = \text{Im} \left[ \frac{(V^*(tb)\bar{V}(ts))^2 (V(cb)\bar{V}^*(cs))^2}{|V^*(tb)\bar{V}(ts)|^2 (V(cb)\bar{V}^*(cs))^2} \right] \approx 2\lambda^2 \eta \sim 0.03 .
\]

This is easily understood: on the leading CKM level only quarks of the second and third families contribute to \( B_s \) oscillations and \( B_s \to \psi\phi \) or \( \psi\eta \); therefore on that level there can
be no CP violation making the CP asymmetry Cabibbo suppressed. Yet New Physics of various ilks can quite conceivably generate \( \sin 2\phi(B_s) \sim \text{several} \times 10\% \).

Analyzing the decay rate evolution in proper time of

\[ B_s(t) \rightarrow \phi \phi \]  

with its direct as well as indirect CP violation is much more than a repetition of the \( B_d(t) \rightarrow \phi K_S \) saga:

- \( \mathcal{M}_{12}(B_s) \) and \( \mathcal{M}_{12}(B_d) \) — the off-diagonal elements in the mass matrices for \( B_s \) and \( B_d \) mesons, respectively — provide in principle independent pieces of information on \( \Delta B = 2 \) dynamics.

- While the final state \( \phi K_S \) is described by a single partial wave, namely \( l = 1 \), there are three partial waves in \( \phi \phi \), namely \( l = 0, 1, 2 \). Disentangling the three partial rates and their CP asymmetries — or at least separating \( l = \text{even} \) and odd contributions — provides a new diagnostics about the underlying dynamics.

Even in the limit of \( x_s \rightarrow \infty \) CP violation can be searched for through the existence of a transition, namely \( e^+e^- \rightarrow B_s \bar{B}_s|_{C=\bar{C}} \rightarrow f_\pm f'_\pm \), where \( f_\pm \) and \( f'_\pm \) denote CP eigenstates of the same CP parity:

\[
\text{BR}(B_s \bar{B}_s|_{C=\bar{C}} \rightarrow f_\pm f'_\pm) \approx \text{BR}(B_s \rightarrow f_\pm)\text{BR}(B_s \rightarrow f'_\pm);
\]

\[
\left[ |\bar{\rho}(f_\pm) - \rho(f'_\pm)|^2 + \left| 1 - \frac{q}{p} \rho(f_\pm) \frac{q}{p} \rho(f'_\pm) \right|^2 \right],
\]

where the two terms in the square brackets have the coefficients \( 1 + \frac{1}{1 + x_s^2} \) and \( \frac{x_s^2}{1 + x_s^2} \), respectively, both of which go to unity for \( x_s \rightarrow \infty \). The analogous expression describes also \( e^+e^- \rightarrow B_d \bar{B}_d \rightarrow f_\pm f'_\pm \), where the two terms in the square brackets carry coefficients of 1.62 and 0.38, respectively. Available data sets should be large enough to produce candidates.

2. Semileptonic Modes

Due to the rapid \( B_s \) oscillations those mesons have a practically equal probability to decay into ‘wrong’ and ‘right’ sign leptons. One can then search for an asymmetry in the wrong sign rate for mesons that initially were \( B_s \) and \( \bar{B}_s \):

\[
a_{SL}(B_s) \equiv \frac{\Gamma(\bar{B}_s \rightarrow l^+X) - \Gamma(B_s \rightarrow l^-X)}{\Gamma(B_s \rightarrow l^+X) + \Gamma(B_s \rightarrow l^-X)}. \tag{156}
\]

This observable is necessarily small; among other things it is proportional to \( \frac{\Delta \Gamma_{B_s}}{\Sigma \Gamma_{B_s}} \ll 1 \). The CKM predictions are not very precise, yet certainly tiny [18]:

\[
a_{SL}(B_s) \sim 2 \cdot 10^{-5} \, , \, a_{SL}(B_d) \sim 4 \cdot 10^{-4} ; \tag{157}
\]
a_{SL}(B_s) suffers a suppression quite specific to CKM dynamics; analogous to $B_s \to \psi\phi$ quarks of only the second and third family participate on the leading CKM level, which therefore cannot exhibit CP violation. Yet again, New Physics can enhance $a_{SL}(B_s)$, this time by two orders of magnitude up to the 1% level.

IV-6. Instead of a Summary: On the Future HEP Landscape — a Call to Well-Reasoned Action

The situation of the SM, as it enters the third millennium, can be characterized through several statements:

1. There is a new dimension due to the findings on $B$ decays: one has established the first CP asymmetries outside the $K^0 - \bar{K}^0$ complex in four $B_d$ modes — as predicted qualitatively as well as quantitatively by CKM dynamics: $B_d(t) \to \psi K_S$, $B_d(t) \to \pi^+\pi^-$, $B_d \to K^+\pi^-$, and $B_d(t) \to \eta'K_S$. Taken together with the other established signals — $K^0(t) \to 2\pi$ and $|\eta_{++}| \neq |\eta_{00}|$ — we see that in all these cases except for $B_d \to K^+\pi^-$ the intervention of meson-antimeson oscillations was instrumental in CP violation becoming observable. This is why I write $B_d(K^0)(t) \to f$. For practical reasons this holds even for $|\eta_{++}| \neq |\eta_{00}|$.

For the first time strong evidence has emerged for CP violation in the decays of a charged state, namely in $B^\pm \to K^\pm \rho^0$.

The SM’s success here can be stated more succinctly as follows:

- From a tiny signal of $|\eta_{++}| \simeq 0.0023$ one successfully infers CP asymmetries in $B$ decays two orders of magnitude larger, namely $\sin^2\phi_1 \simeq 0.7$ in $B_d(t) \to \psi K_S$.
- From the measured values of two CP insensitive quantities — $|V(ub)/V(cb)|$ in semileptonic $B$ decays and $|V(td)/V(ts)|$ in $B^0 - \bar{B}^0$ oscillations — one deduces the existence of CP violation in $K_L \to 2\pi$ and $B_d(t) \to \psi K_S$ even in quantitative agreement with the data.

We know now that CKM dynamics provides at least the lion’s share in the observed CP asymmetries. The CKM description thus has become a tested theory. Rather then searching for alternatives to CKM dynamics we hunt for corrections to it. We have already learnt one lesson of a general lesson: CP violation has been ‘demystified; i.e., weak phases expressing CP violation can be as large as 90°.

2. None of these novel successes of the SM invalidate the theoretical arguments for it being incomplete. There is also clean evidence of mostly heavenly origin for New Physics, namely (i) neutrino oscillations, (ii) dark matter, (iii) presumably dark energy, (iv) probably the baryon number of our Universe, and (v) possibly the Strong CP Problem.

3. Flavour dynamics has become even more intriguing due to the emergence of neutrino oscillations. We do not understand the structure of the CKM matrix in any profound
way — and neither the PMNS matrix, its leptonic counterpart. Presumably we do understand why they look different, since only neutrinos can possess Majorana masses, which can give rise to the ‘see-saw’ mechanism.

Sometimes it is thought that the existence of two puzzles makes their resolution harder. I feel the opposite way: having a larger set of observables allows us to direct more questions to Nature, and if we are sufficiently persistent, learn from her answers.\(^{13}\)

4. The next ‘Grand Challenge’ after studying the dynamics behind the electroweak phase transition is to find CP violation in the lepton sector — anywhere.

5. While the quantization of electric charge is an essential ingredient of the SM, it does not offer any understanding of it. It would naturally be explained through Grand Unification at very high energy scales. I refer to it as the ‘guaranteed New Physics’, see Sect. IV-1.

6. The SM’s success in describing flavour transitions is not matched by a deeper understanding of the flavour structure, namely the patterns in the fermion masses and CKM parameters. For those do not appear to be of an accidental nature. I have referred to the dynamics generating the flavour structure as the ‘strongly suggested’ New Physics (ssNP), see Sect. IV-1.

7. Discovering the cpNP that drives the electroweak phase transition has been the justification for the LHC program, which will come online soon. Personally I am very partisan to the idea that the cpNP will be of the SUSY type. Yet SUSY is an organizing principle rather than a class of theories, let alone a theory. We are actually quite ignorant about how to implement the one empirical feature of SUSY that has been established beyond any doubt, namely that it is broken.

8. The LHC is very likely to uncover the cpNP, and I have not given up hope that the TEVATRON will catch the first glimpses of it. Yet the LHC and a forteriori the TEVATRON are primarily discovery machines. The ILC project is motivated as a more surgical probe to map out the salient features of that cpNP.

9. This cpNP is unlikely to shed light on the ssNP behind the flavour puzzle of the SM, although one should not rule out such a most fortunate development. On the other hand New Physics even at the \(\sim 10 \text{ TeV}\) scale could well affect flavour transitions significantly through virtual effects. A comprehensive and dedicated program of heavy flavour studies might actually elucidate salient features of the cpNP.

\(^{13}\) Allow me a historical analogy: in the 1950’s it was once suggested to a French politician that the French government’s lack of enthusiasm for German re-unification showed that the French had not learnt to overcome their dislike of Germany. He replied with aplomb: “On the contrary, Monsieur! We truly love Germany and are therefore overjoyed that there are two Germanies we can love. Why would we change that?”
that could not be probed in any other way. Such a program is thus complementary to the one pursued at the TEVATRON, the LHC, and hopefully at the ILC and — I firmly believe — actually necessary rather than a luxury to identify the \textit{cpNP}.

To put it in more general terms: Heavy flavour studies are of fundamental importance, many of its lessons cannot be obtained any other way and they cannot become obsolete; i.e., no matter what studies of high $p_{\perp}$ physics at the LHC and ILC will or will not show — comprehensive and detailed studies of flavour dynamics will remain crucial in our efforts to reveal Nature’s Grand Design.

10. Yet a note of caution has to be expressed as well. Crucial manifestations of New Physics in flavour dynamics are likely to be subtle. Thus we have to succeed in acquiring data as well as interpreting them with \textit{high precision}. Obviously this represents a stiff challenge — however one that I believe we can meet, if we prepare ourselves properly.

One of three possible scenarios will emerge in the next several years.

1. \textit{The optimal scenario:} New Physics has been observed in “high $p_{\perp}$ physics”, i.e. through the production of new quanta at the TEVATRON and/or LHC. Then it is \textit{imperative} to study the impact of such New Physics on flavour dynamics; even if it should turn out to have none, this is an important piece of information, no matter how frustrating it would be to my experimental colleagues. Knowing the typical mass scale of that New Physics from collider data will be of great help to estimate its impact on heavy flavour transitions.

2. \textit{The intriguing scenario:} Deviations from the SM have been established in heavy flavour decays — like the $B \to \phi K_S$ \textit{CP} asymmetry or an excess in $\Gamma(K \to \pi\nu\bar{\nu})$ — without a clear signal for New Physics in high $p_{\perp}$ physics. A variant of this scenario has already emerged through the observations of neutrino oscillations.

3. \textit{The frustrating scenario:} No deviation from SM predictions have been identified.

I am optimistic it will be the ‘optimal’ scenario, quite possibly with some elements of the ‘intriguing’ one. Of course one cannot rule out the ‘frustrating’ scenario; yet we should not treat it as a case for defeatism: a possible failure to identify New Physics in future experiments at the hadronic colliders (or the $B$ factories) does not — in my judgment — invalidate the persuasiveness of the theoretical arguments and experimental evidence pointing to the incompleteness of the SM. It ‘merely’ means we have to increase the sensitivity of our probes. \textbf{I firmly believe a Super-flavour factory with a luminosity of order $10^{36}\text{ cm}^{-2}\text{ s}^{-1}$ or more for the study of beauty, charm, and $\tau$ decays has to be an integral part of our future efforts towards deciphering Nature’s basic code.}

For a handful of even perfectly measured transitions will not be sufficient for the task at hand — a \textit{comprehensive} body of \textit{accurate} data will be essential. \textbf{Likewise we need a new round of experiments that can measure the rates for $K \to \pi\nu\bar{\nu}$ accurately}
with sample sizes $\sim O(10^3)$ and mount another serious effort to probe the muon transverse polarization in $K_{\mu 3}$ decays.

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References

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[10] The most general time evolution for the $P^0-\bar{P}^0$ complex, including its decays, is given by an infinite-component vector in Hilbert space reading for $P^0 = K^0 |\Psi(t)\rangle = a(t)|K^0\rangle + b(t)|\bar{K}^0\rangle + c(t)|2\pi\rangle + d(t)|3\pi\rangle + e(t)|\pi l \bar{\nu}_l\rangle + ...$, which is the solution of an infinite-component Schrödinger equation. We do not know how to solve the latter, which describes a strong interaction problem. Fortunately we do not have to and can solve Eq.(46) instead, since for our purposes we restrict ourselves to times much longer than typical strong interaction times as appropriate for the Weisskopf-Wigner approximation: V. F. Weisskopf and E. P. Wigner, *Z. Phys.* **63** (1930) 54; *ibid.* **65** (1930) 18.
[17] The Heavy Flavour Averaging Group provides continuously updated experimental numbers on their web site: http://www.slac.stanford.edu/xorg/hfag/


[37] BABAR Collab., B. Aubert et al., hep-ex/0505092.
[42] CDF note 7867.
[46] The usual tale that the Dark Ages of the Middle Ages were overcome by the Copernican Revolution being born like the goddess Athena jumping out of the head of her father Zeus fully developed and in full armor is unfair to the Middle Ages. Yet more importantly it completely overlooks the immeasurable service to Human culture rendered by Arab Science. For the truly committed student I recommend reading: Ahmed Djebbar, Une histoire de la science arabe, Editions du Seuil, 2001.


[53] BELLE Collab., K. Abe et al., BELLE-CONF-0701.