On the Stopping Power for Low Energy Positrons

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Data on the energy losses for particles incident on various target materials are needed in medical applications and other areas. The existing studies on the energy losses of positrons with low energy are very limited. In this paper, the stopping power calculations are presented for positrons in the energy range of 10 eV–10 keV. Target materials considered in this study are water, brain, and lung. Parameterizations of the energy dependencies of the stopping powers of positrons in water and hydrogen up to 100 eV and from 100 eV to 1 keV are obtained. Also, the maximum stopping powers versus the atomic numbers of various elements in water, brain, and lung are examined. The results are compared with other studies.

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I. INTRODUCTION

Experimental studies in the field of stopping power have accelerated following the development of advanced systems such as accelerators and detectors while theoretical studies have gathered pace through computers. Since the energy range of the studies could now be extended up to the TeV level from the eV level, the application area is much broader.

Despite the studies on collisions for many years, there is no theoretical scheme that is in perfect agreement with the data over a broad energy range. Further theoretical studies are certainly important in the medical diagnostic and therapy areas. That is to say, the energy transfer of charged particles to biological targets should be accurately known. Accurate energy loss values are crucial for tumor treatment with radiation.

The energy loss properties of electrons and positrons are similar for particle energies greater than 1 keV, but for lower energies there are very apparent differences [1, 2]. Although theoretical calculations of the positron stopping power at energies above 10 keV [3, 4] are in agreement with the experimental data, they are much less certain at energies below 10 keV.

The equation suggested by Batra (1987) gives acceptable results in the energy range of 1–500 keV [5]. Meiring et al. (1991) have developed a theory by taking multiple scattering into consideration [2]. However, they have ignored the differences in the interactions of electrons and positrons with the target and assumed $S_p \approx S_e$. When compared to the studies on the passing electrons in the absorber, studies on positrons and High-LET(Linear...
Energy Transfer) ions are relatively limited.

The usage of positron emission tomography (PET) in the medical area has provided a good understanding of the physical and chemical effects on the biological targets from the projectile positrons. In addition, the investigation and analysis of radiation damage in biological systems using positrons is important. The region damaged by radiation is determined by using the data on the positron range.

The study by Tsoulfanidis [6] is a modified version of the other studies [7–9]. Gümüş has calculated the stopping powers of electrons in various targets using a new stopping power formula [10]. In this study the stopping powers of water, brain, and lung were evaluated for positrons in the energy range 10 eV–1 keV employing the two different equations suggested by Tsoulfanidis [6] and Gümüş [10]. The maximum stopping powers for these targets were also evaluated. Parameterizations of the energy dependence of the stopping powers of positrons in water and hydrogen up to 100 eV and in the 100 eV to 1 keV energy range are presented. Also, the maximum stopping powers versus the atomic numbers of elements in targets used in this study were examined. The results are compared with other studies.

II. METHODOLOGY

Stopping power (SP) is defined by the International Commission on Radiation Units and Measurements (ICRU) [4] as the average energy dissipated by the charged particle as it penetrates into a medium. The stopping power is a property of the material in which a charged particle propagates. Although it is not possible to predict how each charged particle will interact with the atoms of the target medium, the average energy loss can be predicted from the Coulomb interaction between the charged particle and atoms.

In the literature, usually, mass stopping power, $S/\rho$, is used because it is independent of the density of the target. Electronic stopping power is related to the energy transferred by charged particles to target electrons. Tsoulfanidis has defined the stopping power formula of charged particles due to Coulombic interactions (i.e., ionization and electron orbital excitation) [6]. The stopping power developed by Tsoulfanidis (1995) for positrons in units of MeV/m is the following [6]:

$$\frac{dE}{dx} = \frac{4\pi r_0^2 mc^2}{\beta^2} N Z \times \left[ \ln \left( \frac{\beta\sqrt{\gamma - 1} mc^2}{I} \right) - \frac{\beta^2}{24} \left[ 23 + \frac{14}{\gamma + 1} + \frac{10}{(\gamma + 1)^2} + \frac{4}{(\gamma + 1)^3} \right] + \frac{\ln 2}{2} \right], \quad (1)$$

where $dE/dx$ is the stopping power for positrons, $r_0$ is the classical electron radius=2.818 x $10^{-15}$ m, $mc^2$ is the rest mass of the electron=0.511 MeV, $N$ is the number of atoms per m$^3$ in the absorber medium ($N = \rho N_0/A$) where $\rho$ is the absorber density, $N_0$ is the Avogadro number, $A$ and $Z$ are the average atomic weight and effective atomic number, respectively, of the compound target material. $\gamma = (T + Mc^2)/Mc^2 = 1/(1 - \beta^2)^{1/2}$, $T$ is positron kinetic energy in MeV, $M$ is the positron rest mass= 0.511 MeV/c$^2$, and $I$ is the mean excitation potential of the absorber [11].
One of the important parameters for calculating the stopping power is the mean excitation energy, \( I \), which characterizes the stopping properties of the absorbed medium. It can be calculated theoretically employing the quantum mechanical approximation. The excitation energies used in this study were received from the NIST program [3]. The contents of target mediums and their properties were also taken from the NIST program [3].

The average atomic weight and effective atom number were obtained by employing the expression of \( A_{\text{mean}} = \sum_{i=1}^{n} \left( \frac{w_i}{A_i} \right) z_i^2 \), and the equation below described by Tsoulfanidis [6]:

\[
Z_{\text{eff}} = \frac{\sum_{i=1}^{n} \left( \frac{w_i}{A_i} \right) z_i}{\sum_{i=1}^{n} \left( \frac{w_i}{A_i} \right) z_i},
\]

where \( n \) is the number of elements in the absorber, \( w_i \) is the weight fraction \( (w_i = N_i A_i / M) \) of the \( i \)th element, \( N_i \) is the number of atoms of \( i \)th element, and \( A_i \) is the atomic weight of \( i \)th element [6].

In this study, firstly we have calculated the mass stopping power of liquid water for positrons in the energy range of 40 eV–1 keV using the above Equation (1) by Tsoulfanidis (1995) [6]. The SP values calculated using Equation (1) were named as SP-1.

Gümüş [10] has presented a new stopping power formula based on Rohrlich and Carlson [9] modified by Sugiyama for electrons [12, 13]. The Rohrlich and Carlson formula for positron energies below 10 keV, taking into account the effective charge of the positron, the effective number of electrons, and the effective mean excitation energies of the targets, is given as [9, 14],

\[
S = -\frac{dE}{\rho dx} = \frac{4\pi e^4}{m\nu^2} A z^* N_0 Z^* \left\{ \ln \frac{T}{I^*} + \frac{1}{2} \ln \left( 1 + \frac{T}{2} \right) + \frac{f(t)^+}{2} \right\},
\]

\[
F^*(\tau) = 2 \ln 2 - \frac{\beta}{12} \left[ 23 + \frac{14}{\tau + 2} + \frac{10}{(\tau + 2)^2} + \frac{4}{(\tau + 2)^3} \right],
\]

where \( m \) is the electron mass, \( \nu \) is the incident positron velocity, \( T \) is the kinetic energy of incoming positron, * denotes an effective quantity (\( z^* \) is the effective charge of positrons and \( Z^* \) is the effective number of target electrons), \( A \) is the atomic weight of the target element, \( \beta \) is the ratio of \( \nu/c \) with \( c \) being the velocity of light and \( I^* \) is the effective mean excitation energy of the target.

The factor in front of the parenthesis can be written as \( \kappa = 4\pi e^4 N_0/mc^2 = 0.307075 \) MeV.cm², \( \tau = T/mc^2 \) [10, 13]. The analytical expressions for the effective quantities were obtained from Ref. [10]. They are \( z^* = 1 - \exp \left( -2200 \beta^{1.78} \right) \) and \( Z^* = Z \frac{\beta^2 (3x + b)}{(x + b)^2} \) [12, 13]. \( b \) is the normalization constant and is chosen as \( b = \frac{8^{2/3}}{\pi} \) [15],

\[
x = -2 \left( \frac{b}{3} \right) + \frac{\left( \frac{b}{3} \right)^2}{\left[ \frac{a}{2} + \left( \frac{b}{3} \right) + \sqrt{\left( \frac{a}{2} \right) + a \left( \frac{b}{3} \right)^3} \right]^{1/3}} + \left[ \frac{a}{2} + \left( \frac{b}{3} \right) + \sqrt{\left( \frac{a}{2} \right) + a \left( \frac{b}{3} \right)^3} \right]^{1/3},
\]
where \( a = \frac{k_0^2}{0.660647} \frac{\mu_0^2}{\mu^3} Z^{4/3} \) and \( \mu_0 = 2.42 \times 10^6 \) m/s. The effective mean excitation energy of the target can be calculated \([10, 16]\) as

\[
I^* = 213.6 \gamma Z C_0^{-3/2} \exp(\alpha),
\]

\[
C_0 = 0.6064741718,
\]

and

\[
\alpha = \frac{Z}{2Z^*} \left[ x^2(x + 3b) \ln x + x(x + b) + x(\ln 6 - 2)b^2 + b^3(\ln 6 - 10/3) + (3x + b)b^2 \ln \frac{b^2}{(x + b)^4} - (x + b)^3 \ln(x + b) \right] / (x - b)^3.
\]

The \( \gamma \) value for the target atoms was obtained from the \( I \) values listed in NIST \([3]\). SP values calculated using Equation (2) are named as SP-2.

### III. RESULTS AND DISCUSSION

The collision stopping powers (SP) of water (liquid) for positrons were determined using Equation (1) and Equation (2) in the 10 eV–1 keV energy range (Equation (1) cannot calculate the SP of water for positrons below 40 eV energy). \( Z_{\text{eff}} \) (effective atomic number of target) and \( A_{\text{m}} \) (mean atomic mass of target) were used in Equation (1). Equation (2) suggested by Rohrlieh and Carlson \([9]\) and modified by Gümüş \(\text{et al.} [14]\) was used to calculate the SP by taking into account the algorithm presented for electrons in the study by Gümüş \([10]\).

Fig. 1 shows the comparison of the stopping powers of water between the results evaluated by Refs. \([14, 17, 18]\) and this study. In this study the maximum stopping power value of water for positrons has been found at \( \approx 100 \) eV positron energy. This is in good agreement with the results of the other studies, but the values of SP are different. Various studies in the literature \([4, 14, 17, 18]\) show the comparison of the stopping power of water. It can be seen that there are important disagreements between them.

Figure 2 shows the collision stopping power curves of brain as a function of the positron kinetic energy in the range of 10 eV–1 keV. The stopping power reaches the maximum value at about 100 eV for brain. However, the SP-1 values are considerably smaller than that of the SP-2. The difference between the SP-1 and SP-2 values at 100 eV is 14.2%.

Although the energy loss mechanism of positrons is similar to that of electrons, the track of a positron through the absorber is significantly different. The difference between their stopping powers is more important at low energy \([17]\). Equation (2) with the algorithm used in this study has provided us the SP values at low positron energies down to 1 eV.

Figure 3 shows the stopping powers of lung for positrons. It can be seen that SP-2 values are higher than that of the SP-1. The difference between SP-1 (372.95 MeV.cm^2/g)
and SP-2 (416.79 MeV.cm²/g) values at the 100 eV is about 12%. The maximum SP values are at 100 eV as expected.

The energy loss with bremsstrahlung radiation can be ignored, so all plots were drawn for mass collision energy loss.

The stopping powers values calculated using SP-2 of water and hydrogen below and above 100 eV are fitted to the polynomial function and exponential function respectively (Fig. 4 and Fig. 5). It can be seen that the energy dependence follows ∼ $E^{-0.77}$ for hydrogen and ∼ $E^{-0.693}$ for water above 100 eV. This is similar to the ∼ $E^{-3/4}$ dependence at high energy [17].

Fig. 6 shows the change of the maximum stopping powers versus atomic number of elements contained in water, brain, and lung. The SP of hydrogen which has the smallest atomic number among the others is considerably larger than that of the other elements. The atomic number dependence of the mass stopping power was found as ∼ $Z^{-0.64}$.

IV. CONCLUSIONS

As known, the positron emitters have continuous energy spectrum (0 → $E_{\text{max}}$) and the maximum SP is found at low particle energy. In this study, the properties of stopping power for incident positrons have been examined in the 10 eV–10 keV energy range. Equation (2) can be used to calculate the positron stopping power especially for low energies using the algorithm by Güümüş [10]. The stopping powers obtained using the formalism described by Equation (2) are considerably higher than others.
FIG. 2: The mass stopping power of brain for the positron energy range of 10 eV–1 keV. The maximum stopping power was found at \( \approx 100 \) eV.

FIG. 3: The mass stopping power of lung for the positron energy range of 10 eV–10 keV. The maximum stopping power was found at 100 eV.
FIG. 4: The mass stopping powers (SP-2) versus positron energy up to 100 eV. The important differences between the SP values of water and hydrogen (it is 69 % at 100 eV) are displayed.

Maximum stopping power value for a given medium is required for studies of radiation damage for biological structures and other applications. Therefore, the stopping power for positrons has been evaluated both theoretically and experimentally. The results from this study of stopping powers of biological structures for especially low energy positrons should be useful in PET, diagnostics, and therapy with nuclear radiation.

References

FIG. 5: The mass stopping powers (SP-2) versus positron energy above 100 eV.

FIG. 6: The maximum stopping power-2 was drawn according to the atomic numbers of elements consisting of water and brain. As seen, the maximum SP-2 decreases as \( Z \) increases.

<table>
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<th>Target</th>
<th>( Z )</th>
<th>( A_{\text{avr}} )</th>
<th>( I(\text{eV})^a )</th>
<th>( \gamma )</th>
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\( ^a \) ESTAR(2003) [19], \( ^* Z_{\text{eff}} \)