Asymmetry Parameter in the Polarized $\Xi_b \rightarrow \Xi_c \ell \nu$ Decay

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We predict an asymmetry parameter $\varepsilon(\hat{e})$ involved in the polarized $\Xi_b$ baryon semileptonic decay $\Xi_b \rightarrow \Xi_c \ell \nu$. Employing the heavy baryon transition form factors calculated in our previous work based on the perturbative QCD factorization theorem, we derive $\varepsilon(\hat{e}) = (N_+ + N_-) / (N_+ - N_-) = (0.31 \pm 0.6) P \cdot \hat{e}$ where $P$ is the $\Xi_b$ baryon polarization, $\hat{e}$ a unit vector perpendicular to the $\Xi_b$ baryon momentum, and $N_+ (N_-)$ the number of leptons with positive (negative) components of momenta along $\hat{e}$. This result is useful for extracting the transverse polarization of a $\Xi_b$ baryon from the data of exclusive semileptonic decays.

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I. Introduction

Recently, we have developed a perturbative QCD (PQCD) factorization theorem for exclusive heavy baryon decays [1, 2]. In this approach transition form factors are expressed as convolutions of hard heavy quark decay amplitudes with heavy baryon wave functions. The former are calculable in perturbation theory in the kinematic region where a large momentum transfer is involved. The latter, involving nonperturbative dynamics of the decay processes, must be obtained by means outside the PQCD regime. Large logarithmic corrections are organized by the Sudakov resummation and renormalization-group equations to improve the perturbative expansions. Since the wave functions are universal, they can be determined once for all, and then employed to make predictions for other modes containing the same heavy baryons. With this prescription for nonperturbative wave functions, the PQCD factorization theorem possesses predictive power.

In this paper we shall apply the above PQCD results to polarized $\Xi_b$ baryon decays. It is well known that hyperons produced in hadronic machines by the strong interaction are usually polarized. However, the mechanism responsible for such a polarization is still unclear. Hence, it is interesting to investigate if similar phenomena happen for heavy baryons produced in proton-proton and proton-anti-proton collisions at various energies. Since $b$ baryon events in hadronic productions are often selected by the presence of high energy leptons, we shall focus on studying the $\Xi_b$ baryon polarization through the semileptonic decays. The longitudinal polarization of a $\Xi_b$ baryon produced from $Z^0$ decays [3, 4] could be determined by measuring the ratio of the average electron (muon) energy to the average neutrino energy in the inclusive semileptonic decays [5,
This ratio, obtained in the laboratory frame, is then related to the ratio of the corresponding energies in the rest frame of the $\Xi_b$ baryon. When heavy baryons are produced via the strong interaction, they could possess transverse polarization perpendicular to the baryon momenta. In the present work we shall concentrate on the determination of the transverse polarization of a $\Xi_b$ baryon from the angular distribution of leptons produced in the $\Xi_b \rightarrow \Xi_c \ell^+ \ell^-$ decay. For semileptonic decays, it is usually not possible to reconstruct fully the momentum of the parent baryon. In order to determine the heavy baryon polarization through semileptonic events, we propose to measure the asymmetry parameter 

$$\mathcal{A} = \frac{(N_+ - N_-)}{(N_+ + N_-)}$$

where $N_+$ ($N_-$) is the lepton number emitted above (below) a plane that contains the $\Xi_b$ baryon momentum. This quantity is invariant under a longitudinal boost and can be simply evaluated in the rest frame of the $\Xi_b$ baryon. Using the $\Xi_b \rightarrow \Xi_c$ transition form factors obtained from the PQCD factorization theorem [2], we derive the relation between $\mathcal{A}$ and the transverse polarization $P \mathbf{e}$ of the $\Xi_b$ baryon, $\mathcal{A} = 0.31 P \mathbf{e}$ where $\mathbf{e}$ is a unit vector perpendicular to the $\Xi_b$ baryon momentum. Comparing this relation with the experimental data for $\mathcal{A}$, one may extract the $\Xi_b$ baryon transverse polarization.

It has been shown that the PQCD analyses for the $\Xi_b \rightarrow \Xi_c$ transition form factors are reliable at the maximal recoil of the $\Xi_c$ baryon [2]. The percentages of the full contribution to the form factors, that arise from the short-distance region with $\mathcal{R}_2 = \frac{1}{4} - 0.2 - 0.5$, are listed in Table I. It is observed that the contribution from the region with $\mathcal{R}_2 = \frac{1}{4} < 0.3$ becomes dominant gradually when $\mathcal{R}_2$ increases. The PQCD analysis can be regarded as being self-consistent at the maximum $\mathcal{R}_2 \approx 1.4$, for which the perturbative contribution amounts to about 60% of the full contribution.

Extrapolating the PQCD results to small $\mathcal{R}_2$ under the requirement of heavy quark symmetry (HQS) [7], we have determined the behavior of the form factors in the whole range of the velocity transfer. This extrapolation leads to the reasonable branching ratio $\mathcal{B}(\Xi_b \rightarrow \Xi_c \ell^+ \ell^-) = 2 \%$, which is consistent with the experimental upper bound of the branching ratio from the data $\mathcal{B}(\Xi_b \rightarrow \Xi_c \ell^+ \ell^- + X) = (8.27 \% \pm 3.38\%)$ [8]. The $\Xi_b \rightarrow \Xi_c$ transition form factors have been evaluated by means of overlap integrals of infinite-momentum frame wave functions [9, 10], relativistic quark models [11], Bethe-Salpeter equations [12] and QCD sum rules [13-15]. Most of the analyses led to the branching ratios about or below 6%. Our result is close to the value (3.4 $\% \pm 0.6\%)$ derived in [13].

<table>
<thead>
<tr>
<th>$\mathcal{R}_2$</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
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</thead>
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<td>18%</td>
<td>49%</td>
<td>66%</td>
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<tr>
<td>1.3</td>
<td>22%</td>
<td>54%</td>
<td>71%</td>
<td>79%</td>
</tr>
<tr>
<td>1.4</td>
<td>25%</td>
<td>58%</td>
<td>74%</td>
<td>82%</td>
</tr>
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</table>
II. Asymmetry parameter

We first define the $\pi_b$ $\pi_c$ transition form factors; for their explicit factorization formulas refer to [2]. The amplitude for the semileptonic decay $\pi_b \rightarrow \pi_c \ell \bar{\nu}_\ell$ is written as

$$M = \frac{G_F}{2} V_{cb} f_{\pi_1}(1_{i} \cdot \sigma_5)_{(0)1} \pi_c(p') j^{\dagger}_a \pi_b(p) i;$$

(1)

where $G_F$ is the Fermi coupling constant, $V_{cb}$ the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, $p$ and $p'$ the $\pi_b$ and $\pi_c$ baryon momenta, respectively. QCD dynamics is contained in the hadronic matrix element

$$H^i \pi_c(p') j_a \pi_b(p);$$

where $f_{\pi}(p)$ and $g_{\pi}(p)$ are the $\pi_b$ and $\pi_c$ baryon spinors, respectively, and the variable $q$ denotes $q = p - p'$. In the case of massless leptons with $qI(1_{i} \cdot \sigma_5)_{(0)1} = 0$, $f_3$ and $g_3$ do not contribute. The contributions from $f_2$ and $g_2$, related to the spin flip of the heavy quark, are suppressed by a power of $1$ $m_b$, $m_b$ being the $b$ quark mass. Therefore, we shall consider only $f_1$ and $g_1$ below.

The polarization density matrix for a $\pi_b$ baryon with polarization $P$ is written as

$$\frac{1}{2} = P^0(\frac{3}{2})_{K K}; \quad \frac{3}{0} = 1;$$

(3)

For the purpose of normalization, we usually choose $P^0 = 1$. The hadronic matrix element, with the helicity of the initial- and final-state baryons specified, is expressed as

$$H^i(\pi_b K) = \frac{3}{2} \left( f_1(\frac{3}{2})_{a b c} + g_1(\frac{3}{2})_{a b c} \pi_b(p) K;\right)$$

(4)

which leads to the covariant density matrix

$$H^{i a b c}(\pi_b K) :$$

(5)

The differential decay rate is written as

$$d_1 = \frac{1}{2} M_{\pi_b} |p_j|^2 \left( \frac{2}{3} \frac{4}{3} E \right) (p_i p_0 i \pi_i p_0) \left( \frac{2}{3} \frac{4}{3} E \right) (p_i p_0 i \pi_i p_0)$$

(6)

$$+ \left( \pi_i p_0 i \pi_i p_0 \right) \left( \frac{2}{3} \frac{4}{3} E \right) (p_i p_0 i \pi_i p_0)$$

$$d_1 = \frac{G_F^2}{2} V_{cb} j^{\dagger} H^i L^\dagger;$$

(7)

with the leptonic covariant tensor,

$$L^\dagger = 8 \pi_i p_0 i \pi_i p_0 i \pi_i p_0 i \pi_i p.$$
Define the velocity transfer \( \frac{1}{2} \) which is related to \( q^2 \) via

\[
\gamma = \frac{M_{\pi_b}^2 + M_{\pi_c}^2}{2M_{\pi_b}M_{\pi_c}} q^2;
\]

(9)

with \( M_{\pi_b} \) and \( M_{\pi_c} \) being the \( \pi_b \) and \( \pi_c \) baryon masses, respectively. Performing the phase space integrations straightforwardly, Eq. (6) reduces to

\[
\frac{d^4}{dE_1^0 dE_1} = \frac{1}{256\pi^2 M_{\pi_b}} \frac{1}{\mu_k \sin \mu \sin \theta} jM_{p_j^2}^2;
\]

(10)

where \( E_1^0 = \frac{1}{2} M_{\pi_c} E_1 \) is the \( \pi_c \) baryon (lepton) energy, \( \mu_k (\mu) \) the polar angle of the \( \pi_c \) baryon momentum \( p_0^0 \) (the lepton momentum \( p_l^0 \)) with respect to the \( \pi_b \) baryon polarization, \( \hat{\theta} \) the azimuthal angle of \( p_l \) and \( d \cdot d_1 = \cos \mu \cos \hat{\theta} \) the solid angle. The azimuthal angle \( \hat{\theta} \) and the angle \( \mu \) between \( p_0 \) and \( p_l \) satisfy the relations

\[
\cos \hat{\theta} = \frac{1}{\sin \mu \sin \mu} [\cos \mu_l \cos \mu_k \cos \mu];
\]

(11)

\[
\cos \mu_l = \frac{M_{\pi_b}^2 + M_{\pi_c}^2}{2M_{\pi_b}} \frac{2M_{\pi_b}E_0^0 \mu_k + 2M_{\pi_b}E_1 + 2E_0 E_1}{2 E_0^2 \mu_k + M_{\pi_c} E_1};
\]

(12)

The kinematic ranges of \( E_1^0 \) and \( E_1 \) are

\[
\frac{1}{2} M_{\pi_b} \mu_k E_0^0 \mu_k \mu_k \mu_k \mu_k M_{\pi_c} E_1 \cdot \frac{1}{2} \mu_k E_0^0 \mu_k + \mu_k E_0^0 \mu_k + \mu_k M_{\pi_c} E_1 ;
\]

(13)

Requiring \( \cos^2 \hat{\theta} \cdot 1 \), it is easy to derive the ranges of \( \cos \mu_k \) and \( \cos \mu_l \) from Eq. (11),

\[
j \cos \mu_j \cdot 1;
\]

(14)

The projection operator associated with a spin-1/2 particle of mass \( m \) is written as [16]

\[
\psi(p, K) \psi(p, K') = \frac{1}{2} (m + \not{p}) [(l)_K \not{K} + \not{K}_l (3/4)_K \not{K}];
\]

(15)

where \( 1 \) is the 2 \( \times \) 2 unit matrix, \( K \) and \( K' \) represent the helicities \( \frac{1}{2} \), and \( \frac{3}{2} \) are the Pauli matrices. In the rest frame of a \( \pi_b \) baryon with \( p^0 = \text{const} \), we have the boost vectors \( n_i^1 = \pm \), \( i, j = 1, 2, 3 \). Using Eqs. (3), (5), and (15), we obtain the expression of \( jM_{p_j^2} \),

\[
jM_{p_j^2} = 8M_{\pi_b} G_F^2 F_{\pi_b}(E_0^0, E_1) + F_1(E_0^0, E_1) P_0^0 \not{p} + F_2(E_0^0, E_1) P_1 \not{p} ;
\]

(16)
with the functions
\[
F_0 = i (M_{\pi^0} M_{\rho^0} E_0 E_1 [M_{\pi^0}^2 + M_{\rho^0}^2 i 2(E_0 + E_1) M_{\pi^0}]) (f_{1\frac{1}{2}} - g_{1\frac{1}{2}}) \\
+ E_1 (M_{\pi^0}^2 M_{\rho^0} E_1) f_{2\frac{1}{2}} + 2 M_{\pi^0} E_0 (f_{1\frac{1}{2}} - g_{1\frac{1}{2}}) + 2 M_{\rho^0} E_0 (f_{1\frac{1}{2}} + g_{1\frac{1}{2}}) + 2 M_{\pi^0} E_1 (f_{2\frac{1}{2}} - g_{2\frac{1}{2}}); \\
F_1 = [M_{\pi^0}^2 + M_{\rho^0}^2 i 2(E^0 + E_1) M_{\pi^0}^2] (f_{1\frac{1}{2}} g_{1\frac{1}{2}}) + 2 M_{\rho^0} E_0 (f_{2\frac{1}{2}} + g_{2\frac{1}{2}}); \\
F_2 = [M_{\pi^0}^2 + M_{\rho^0}^2 i 2(E^0 + E_1) M_{\rho^0}^2] (f_{1\frac{1}{2}} g_{1\frac{1}{2}}) + 2 M_{\rho^0} E_1 (f_{2\frac{1}{2}} + g_{2\frac{1}{2}})
\]

and the scalar products
\[
p^0 \cdot p = E^i \cdot M_{\pi^0}^2 \cdot j \cdot p \cos \mu; \quad p_1 \cdot p = E_1 \cdot j \cdot p \cos \mu.
\]

The first term \(F_0\) is associated with the unpolarized \(\pi^0\) baryon decay. Integrating out \(\cos \mu\) according to the range in Eq. (14), Eq. (10) becomes
\[
\frac{d^3 \phi}{dE dE d\phi} = \frac{G_F^2}{16 \pi^4} (V_{tb} \cdot j \cdot p_1 \cdot F_0 + (M_{\pi^0} \cdot p \cdot \frac{1}{\sqrt{2}} \cdot \cos \mu \cdot j \cdot F_1 + E_1 F_2) \phi_1 \cdot p);
\]

where \(\phi_1 = p_1 \cdot \epsilon \cdot p\) is the unit vector along the lepton momentum.

To obtain the decay rate, we need the information of the form factors \(f_{1\frac{1}{2}}\) and \(g_{1\frac{1}{2}}\) in the whole range of \(\frac{1}{2}\). The behaviors of \(f_{1\frac{1}{2}}\) and \(g_{1\frac{1}{2}}\) at large \(\frac{1}{2}\) have been evaluated in [2], by adopting the CKM matrix element \(V_{tb} = 0.04\), the masses \(M_{\pi^0} = 5.624\) GeV and \(M_{\rho^0} = 2.285\) GeV. We then extrapolated the PQCD predictions to the small \(\frac{1}{2}\) region under the requirement of HQS [7]. Assuming the parameterization,
\[
f_{1\frac{1}{2}} = \frac{c_f}{\sqrt{2}}; \quad g_{1\frac{1}{2}} = \frac{c_g}{\sqrt{2}};
\]

we have obtained the constants \(c_f = 1.32\) and \(c_g = 1.19\), and the powers \(\alpha_f = 5.18\) and \(\alpha_g = 5.14\), by fitting Eq. (20) to the PQCD results at large \(\frac{1}{2}\). The values of the form factors at zero recoil, \(f_{1\frac{1}{2}}(1) = 1.32\) and \(g_{1\frac{1}{2}}(1) = 1.19\), are close to those derived based on wave function overlap integrals [9]. The approximate equality of \(f_{1\frac{1}{2}}\) and \(g_{1\frac{1}{2}}\) is consistent with the conclusion drawn from heavy quark effective theory (HQET) [17, 18].

We emphasize that PQCD and HQET complement each other in the analyses of heavy hadron decays. A portion of the heavy hadron mass used for the energy release involved in the decay processes is sufficient to guarantee the applicability of PQCD. That is, the decay processes move into the perturbative regime, as the velocity transfer increases from unity. In [2] we have made a quantitative investigation of this issue and found that at the maximal recoil PQCD is applicable to \(\pi^0\) baryon decays. Besides, HQET gives the relations among various transition form factors, while PQCD can be employed to calculate the behavior of these form factors near the high end of the velocity transfer. Combining the PQCD predictions and the HQS requirement for the form factors, we have derived Eq. (20).
To extract the transverse polarization of a $\Xi_b$ baryon, we propose to measure a lepton asymmetry parameter in the laboratory frame. Choose a unit vector $\hat{e}$ perpendicular to the $\Xi_b$ baryon momentum, whose direction is known if the interaction point of the hadron beams as well as the decay vertex of the $\Xi_b$ baryon are observed. The lepton asymmetry parameter $\mathcal{A}(\hat{e})$ with respect to $\hat{e}$ is defined as

$$\mathcal{A}(\hat{e}) = \frac{N_+ - N_1}{N_+ + N_1},$$

where $N_+(N_1)$ is the number of leptons with positive (negative) values of $\hat{p}_l \cdot \hat{e}$. Since $\mathcal{A}(\hat{e})$ is invariant under a boost along the $\Xi_b$ baryon momentum, we can simply calculate $\mathcal{A}(\hat{e})$ in the rest frame of the $\Xi_b$ baryon, and derive its relation to the $\Xi_b$ baryon polarization.

Integrating over the variables $E_0$ (or $\frac{1}{2}$) and $E_1$ in Eq. (19), we arrive at

$$\frac{d\mathcal{A}}{dE_1} = \frac{M_{\Xi_b} G_0^2}{16\pi^2} j V_d j^2 (G_0 + G_1 \hat{p}_l \cdot \hat{e});$$

with the integrals

$$G_0 = \int d\hat{p}_l E_1 F_0(E_0;E_1);$$
$$G_1 = \int d\hat{p}_l [M_{\Xi_b} \hat{p}_l \cdot \frac{1}{2} \cos \mu F_1(E_0;E_1) + E_1 F_2(E_0;E_1)].$$

It is straightforward to show, from Eq. (22), that

$$N_+ = G_0 \frac{1}{2} G_1 \hat{p}_l \cdot \hat{e};$$

$G_0$ being a constant depending on the production rate of $\Xi_b$ baryons. Therefore, we have

$$\mathcal{A}(\hat{e}) = \frac{G_1}{2G_0} \hat{p}_l \cdot \hat{e} = \hat{1}: 0.31 \hat{P} \hat{e};$$

where the numerical results of $G_0$ and $G_1$ from Eq. (23) have been inserted. This implies that the lepton tends to be emitted in the direction opposite to the transverse polarization of the $\Xi_b$ baryon.

We have tested the sensitivity of our predictions to the variation of the heavy baryon wave functions. The asymmetry parameters $\mathcal{A}(\hat{e})$ change by only a few percent for various choices of the baryon wave functions. This observation is expected, since nonperturbative effects from the wave functions cancel in the ratio of $(N_+ - N_1)/(N_+ + N_1)$. For a similar reason, this ratio does not depend on the normalizations of the heavy baryon wave functions. Therefore, Eq. (25) is insensitive to nonperturbative dynamics, and can be regarded as being almost model-independent.

Other potential corrections to the asymmetry parameter are under control and can be included systematically. To estimate their effects, we consider higher-order corrections to the hard amplitudes, which are suppressed by $\mathcal{A} \hat{1}: 4 < 0.3$, and higher-twist contributions, which are of order $\mathcal{A} \hat{1}: 0.1$ with the mass difference $\hat{A} = M_{\Xi_b} - M_b$. Hence, it is reasonable to associate 30% uncertainty from these corrections with the large $\frac{1}{2}$ behavior of the form factors. Similarly, we extrapolate the PQCD results to the zero-recoil region under the requirement of HQS. It is
easy to observe that the above uncertainty in the form factors leads to about a 20% uncertainty in the asymmetry parameter \( @@ \). We then arrive at

\[
@@ = (i 0.31 \pm 0.6) \cdot \text{Pe}^e
\]

The \( \pi_b \) baryon, produced together with the \( \pi_b \) baryon in proton-anti-proton collisions, possesses a polarization opposite to that of the \( \pi_b \) baryon, while the lepton from the \( \pi_b \) decay tends to be produced along the transverse polarization of the \( \pi_b \) baryon. As a result, for colliding experiments at the Tevatron, it is possible to combine events of the \( \pi_b \) and \( \pi_b \) baryons to double the statistics, and measure the lepton asymmetry parameter irrespective of whether the leptons are from \( \pi_b \) or \( \pi_b \) baryon decays. With about 200 \( \pi_b \) baryon events available currently, the asymmetry parameter could be measured to 10% accuracy corresponding to a 30% polarization. For Run II of the Tevatron, it is likely to collect 20 times more events so that a polarization of a few percent could be determined. In addition, the \( \pi_b \) baryon polarization can be measured through the meson asymmetry parameter in nonleptonic decays. The application of the PQCD formalism to this case is under investigation.

III. Conclusion

In this paper we have studied the polarized \( \pi_b \) baryon semileptonic decay \( \pi_b \! \pi \ell^0 \) using the PQCD factorization theorem. We have proposed a boost invariant quantity \( @@ \) related to the transverse polarization \( \text{Pe}^e \) of a \( \pi_b \) baryon, which can be measured with available data from current and future experiments. This quantity is defined as the number asymmetry of the leptons which are produced with positive and negative components of momenta along \( \text{Pe}^e \). With respect to the production plane of \( \pi_b \) baryons at the Tevatron, it is the asymmetry of the lepton numbers above and below this plane. Using the transition form factors obtained in the PQCD approach, we have predicted \( @@ = (i 0.31 \pm 0.6) \cdot \text{Pe}^e \). This indicates that the lepton tends to be emitted in the direction opposite to the \( \pi_b \) baryon transverse polarization. Our result is useful for extracting the transverse polarization of a \( \pi_b \) baryon from its exclusive semileptonic decay.

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References

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