Transient Effects of Photon Switching by Quantum Interference

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We discuss the transient effects of the four-state system proposed by Harris and Yamamoto [Phys. Rev. Lett. 81, 3611 (1998)]. In this system, a weak probe laser and a coupling laser form a three-state \( \Lambda \)-type configuration of electromagnetically induced transparency; a switching laser drives a transition between the fourth state and the ground state of the coupling field. The presence of the switching field causes absorption of the probe. We show that the rise time of the probe absorption can not be shorter than \( 2/\Gamma \) and is equal to \( 2/\Gamma (\Omega_c^2 + \Omega_s^2) \) in the low-intensity limit, where \( \Gamma \) is the spontaneous decay rate of the excited states, and \( \Omega_c \) and \( \Omega_s \) are the Rabi frequencies of the coupling and switching lasers. A simple picture based on dark and non-dark states is provided to explain these results.

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Photon switching by quantum interference is an intriguing idea first proposed by Harris and Yamamoto [1]. Figure 1 describes the four-state system for photon switching. \(|1\rangle \) and \(|2\rangle \) are ground states and \(|3\rangle \) and \(|4\rangle \) are the excited states. A weak probe field and a strong coupling field drive the \(|1\rangle \rightarrow |3\rangle \) and the \(|2\rangle \rightarrow |3\rangle \) transitions, respectively. The two fields form a \( \Lambda \)-type configuration of electromagnetically induced transparency (EIT): absorption of the probe field is suppressed due to the EIT effect [2–4]. A switching field drives the \(|2\rangle \rightarrow |4\rangle \) transition. Its presence enables absorption of the probe and induces a three-photon transition from the ground state \(|1\rangle \) to the excited state \(|4\rangle \). A similar four-state system, that exhibits the phenomenon of interacting dark resonances (IDR) and the effect of photon switching, has been proposed by Lukin et al. [5]. In the IDR system, state \(|4\rangle \) becomes a ground state and the switching field is a microwave. The presence of the microwave also enables absorption of the probe and induces a three-photon transition. The photon switching first proposed by Harris and Yamamoto, and the IDR phenomenon proposed by Lukin et al., have been observed recently in laser-cooled \(^{87}\)Rb atoms [6, 7].

With the four-state system in Fig. 1, Harris and Yamamoto reached a universal result: that one probe field switching event costs one switching photon, under ideal conditions. This result leads to the very interesting idea of the three-fold entangled state depicted in Fig. 3 of Ref. [1]. The three-fold entangled state has potential applications in the field of quantum information [8–10]. To achieve the result, an energy cost of one photon per switching event in the four-state system, the pulse length of the switching field is set to the delay time of the probe propagation in the medium in the calculation of Ref. [1]. It is desirable to know whether this delay time is longer than the probe absorption rise time. This has prompted our study of the transient effects of photon switching by quantum interference.

We are interested in the situation wherein the probe and coupling fields satisfy the
resonance of the two-photon transition from $|1\rangle$ to $|2\rangle$, and the two fields together with the switching field satisfy the resonance of the three-photon transition from $|1\rangle$ to $|4\rangle$. The two-photon resonance inhibits probe absorption in the absence of the switching field, and the three-photon resonance enables probe absorption in the presence of the switching field. To ensure the situation and simplify the calculation, we set the frequency of each field resonant to the individual transition. The density-matrix operator, $\hat{\rho}$, of the system obeys the following equation:

$$\frac{d\hat{\rho}}{dt} = i \left[ -\frac{H_0}{\hbar} + \left( \frac{\Omega_c}{2} e^{i\omega_c t} |2\rangle \langle 3| + \frac{\Omega_s}{2} e^{i\omega_s t} |2\rangle \langle 4| + \frac{\Omega_p}{2} e^{i\omega_p t} |1\rangle \langle 3| + \text{h.c.} \right), \hat{\rho} \right] + \{d\hat{\rho}/dt\}. \tag{1}$$

$H_0$ is the atom Hamiltonian. $\Omega_c$, $\Omega_s$, and $\Omega_p$ are the Rabi frequencies of the coupling, switching, and probe fields, respectively. $\omega_c$, $\omega_s$, and $\omega_p$ are the frequencies of the three fields and also the resonance frequencies of the transitions. The $\{d\hat{\rho}/dt\}$ term describes the relaxation of $\hat{\rho}$. Only the relaxation process of spontaneous decay is considered in the calculation.

The four-state system is initially in the EIT condition and all of the population is in the ground state $|1\rangle$. When the switching field is turned on, we want to know the probe absorption rise time. Treating the weak probe field as a perturbation, we can carry out the
calculation to the first order and obtain the equations,

\[
\frac{d\rho_{31}}{dt} = -\frac{\Gamma}{2}\rho_{31} + i\Omega_c + \frac{i\Omega_p}{2}, \tag{2}
\]

\[
\frac{d\rho_{21}}{dt} = \frac{\Omega_c}{2}\rho_{31} + \frac{i\Omega_s}{2}, \tag{3}
\]

\[
\frac{d\rho_{41}}{dt} = -\frac{\Gamma}{2}\rho_{41} + \frac{i\Omega_s}{2}, \tag{4}
\]

where \( \Gamma \) is the spontaneous decay rate of both excited states, \( \rho_{31} = \hat{\rho}_{31}e^{i\omega_p t} \), \( \rho_{21} = \hat{\rho}_{21}e^{i(\omega_p-\omega_c)t} \), and \( \rho_{41} = \hat{\rho}_{41}e^{i(\omega_p-\omega_c+\omega_s)t} \). The imaginary part of the optical coherence of the probe transition determines the absorption cross section of the probe field, which is given by \( (3\lambda^2/2\pi)(\Gamma/\Omega_p)\text{Im}[\rho_{31}] \), where \( \lambda \) is the wavelength. We combine the above three equations into

\[
\frac{d^3\rho_{31}}{dt^3} = -\left(\Gamma - \frac{1}{\Omega_s} \frac{d\Omega_s}{dt}\right)\frac{d^2\rho_{31}}{dt^2} - \left(\frac{\Gamma^2 + \Omega_c^2 + \Omega_s^2}{4} - \frac{\Gamma}{2\Omega_s} \frac{d\Omega_s}{dt}\right)\frac{d\rho_{31}}{dt} - \left(\frac{\Gamma\Omega_c^2}{8} + \frac{\Gamma\Omega_s^2}{8} - \frac{\Omega_c^2}{4\Omega_s} \frac{d\Omega_s}{dt}\right)\rho_{31} + \frac{i\Omega_p\Omega_s^2}{8} \tag{5}
\]

It is reasonable to assume that the switching field is turned on as a step function to determine the probe absorption rise time. We can then drop all terms containing \( d\Omega_s/dt \) in the above equation. The various ways and the rates of turning on the switching field only contribute different initial conditions to this truncated Eq. (5) (i.e. without the terms containing \( d\Omega_s/dt \)). To find the probe absorption as a function of time, we carry out a numerical calculation with the truncated Eq. (5). After a solution for the probe absorption is obtained, it is fitted with an exponential function. The probe absorption rise time is defined as the time constant of the exponential function with the best fit. Examples of the probe absorption as a function of time and their best fits are shown in Fig. 2. In each plot, the solid line represents the solution of the truncated Eq. (5) and the dashed line is the best fit. For large \( \Omega_c \) or \( \Omega_s \), the transient probe absorption has some oscillation besides exponential increase. The oscillation becomes more rapid at higher Rabi frequencies. In the case where both \( \Omega_c \) and \( \Omega_s \) are small, the probe absorption is simply an exponential function. Figure 3 shows the reciprocal rise time of the probe absorption versus the different \( \Omega_c \)'s and \( \Omega_s \)'s. The rise time asymptotically approaches \( 2/\Gamma \) at large Rabi frequencies and is equal to \( 2\Gamma/(\Omega_c^2 + \Omega_s^2) \) at small Rabi frequencies [11].

We use the dark and non-dark states as a basis in order to clarify the physical picture of the rise time. The concept of dark and non-dark states comes from the phenomenon of coherent population trapping (CPT) [12, 13]. In the CPT case, the dark and non-dark states are superpositions of the two ground states in a three-level system. Here, we transform the excited states from the basis of \( |3\rangle \) and \( |4\rangle \) to that of dark and non-dark states. In the
FIG. 2: Absorption cross section of the probe field versus time at different Rabi frequencies. The solid lines indicate solutions calculated numerically from the truncated Eq. (5). Each dashed line indicates an exponential function with a time constant, $\tau$, and the best fit of the solid line in the same plot. $3\lambda^2/2\pi$ is the absorption cross section of the probe field appearing alone in the system. $\Omega_c$ and $\Omega_s$ described below are in units of the spontaneous decay rate $\Gamma$, and $\tau$ is in units of the excited-state lifetime $1/\Gamma$. (a) $\Omega_c = 0.1$, $\Omega_s = 0.1$, and $\tau = 100$. (b) $\Omega_c = 1$, $\Omega_s = 0.1$, and $\tau = 4.3$. (c) $\Omega_c = 0.1$, $\Omega_s = 1$, and $\tau = 4.3$. (d) $\Omega_c = 1$, $\Omega_s = 1$, and $\tau = 3.2$. (e) $\Omega_c = 10$, $\Omega_s = 1$, and $\tau = 2.0$. (f) $\Omega_c = 1$, $\Omega_s = 10$, and $\tau = 2.0$.

interaction picture, the transformation is given by

$$|D\rangle = \frac{\Omega_c}{\sqrt{\Omega_c^2 + \Omega_s^2}} |4\rangle - \frac{\Omega_s}{\sqrt{\Omega_c^2 + \Omega_s^2}} |3\rangle \quad (6)$$

$$|N\rangle = \frac{\Omega_c}{\sqrt{\Omega_c^2 + \Omega_s^2}} |4\rangle + \frac{\Omega_s}{\sqrt{\Omega_c^2 + \Omega_s^2}} |3\rangle \quad (7)$$

$|D\rangle$ and $|N\rangle$ are the dark and non-dark states. The coupling and switching fields do not interact with the dark state, i.e. $\langle 2 | (\Omega_c |2\rangle \langle 3 | + \Omega_s |2\rangle \langle 4 |) |D\rangle = 0$. For this basis, the transition diagram is shown in Fig. 4(a). The probe field drives both the $|1\rangle \rightarrow |D\rangle$ and the $|1\rangle \rightarrow |N\rangle$ transitions, with Rabi frequencies $\Omega_D$ and $\Omega_N$, respectively. The coupling and switching fields cooperatively drive the $|2\rangle \rightarrow |N\rangle$ transition with Rabi frequency $\Omega_{sc}$. 
The Rabi frequencies are
\[ \Omega_{sc} = \sqrt{\Omega_c^2 + \Omega_s^2}, \]
\[ \Omega_D = -\Omega_p \Omega_s/\Omega_{sc}, \]
\[ \Omega_N = \Omega_p \Omega_s/\Omega_{sc}. \]

The absorption cross section of the probe field, in the dark and non-dark states basis, is given by

\[
\frac{3\lambda^2}{2\pi} \left\{ \left( \frac{\Omega_s}{\Omega_{sc}} \right)^2 \Gamma \text{Im}[\rho_{D1}] + \left( \frac{\Omega_c}{\Omega_{sc}} \right)^2 \Gamma \text{Im}[\rho_{N1}] \right\},
\]

where \( \rho_{D1} \) and \( \rho_{N1} \) are the optical coherences of the \( |1\rangle \rightarrow |D\rangle \) and \( |1\rangle \rightarrow |N\rangle \) transitions. \( (\Omega_s/\Omega_{sc})^2 \Gamma \) is the spontaneous decay rate of \( |D\rangle \) to \( |1\rangle \) and \( (\Omega_c/\Omega_{sc})^2 \Gamma \) is that of \( |N\rangle \) to \( |1\rangle \).

With the weak-probe conditions, we are able to further decompose the four-state system into the two subsystems shown in Fig. 4(b). One subsystem has two states, \( |1\rangle \) and \( |D\rangle \), which are only coupled by the probe field. The other subsystem has three states in the A-type EIT configuration, in which the coupling and switching fields drive the \( |2\rangle \rightarrow |N\rangle \)
transition, and the probe field drives the \( |1\rangle \rightarrow |N\rangle \) transition. In each subsystem, the population is all in \( |1\rangle \) and is not affected by the probe field according to the perturbation nature. Define \( \sigma_2 = (\Omega_s/\Omega_{sc})^2(\Gamma/\Omega_D)\text{Im}[\rho_{D1}] \) in the two-state subsystem; it obeys a simple equation:

\[
\frac{d\sigma_2}{dt} = -\frac{\Gamma}{2}\sigma_2 + \frac{\Gamma}{2} \left( \frac{\Omega_s}{\Omega_{sc}} \right)^2.
\]  

(11)

It is obvious that the solution of the above equation is an exponential-growth function with a time constant of \( 2/\Gamma \) and a steady-state value of \( (\Omega_s/\Omega_{sc})^2 \). Define \( \sigma_3 = (\Omega_c/\Omega_{sc})^2(\Gamma/\Omega_N)\text{Im}[\rho_{N1}] \) in the three-state subsystem. The equation for \( \sigma_3 \) is given by

\[
\frac{d^2\sigma_3}{dt^2} = -\frac{\Gamma}{2} \frac{d\sigma_3}{dt} - \left( \frac{\Omega_{sc}}{2} \right)^2 \sigma_3.
\]  

(12)

This equation is the same as the equation of motion for a damped oscillator. Initially, \( [d\sigma_3/dt]_{t=0} \) has a finite value of \(-\Gamma\Omega_s^2/2\Omega_{sc}^2\), which comes from the ground-state coherence, \( \rho_{21} \), in the system. The ground-state coherence has been established by the probe and the coupling fields together before the switching field is turned on. For various \( \Omega_c \)'s and \( \Omega_s \)'s,
the probe absorption calculated analytically from Eqs. (11) and (12) is in good agreement with that numerically calculated from the truncated Eq. (5).

The behavior of the rise time in Fig. 3 can be explained on the basis of the dark and non-dark states. In the two-state subsystem, $\sigma_{\text{2}}$ always exponentially increases from 0 to $(\Omega_s/\Omega_{sc})^2$ with a time constant of $2/\Gamma$. In the strongly underdamped case ($\Omega_{sc} \gg \Gamma/4$) in the three-state system, $\sigma_{\text{3}}$ oscillates at a frequency of $\Omega_{sc}$, and its amplitude damps exponentially with a time constant of $4/\Gamma$. However, the amplitude of $\sigma_{\text{3}}$ is initially $(\Gamma/\Omega_{sc})(\Omega_s/\Omega_{sc})^2$, much smaller than the steady-state magnitude of $\sigma_{\text{2}}$. Such a small $\sigma_{\text{3}}$ is expected as a consequence of a stiff oscillator, i.e. an oscillator with a large spring constant of the $\Omega_{sc}$ in Eq. (12). Since $\sigma_{\text{2}}$ is dominant, the rise time of the probe absorption is just $2/\Gamma$, the $\sigma_{\text{2}}$ time constant. In the strongly overdamped case ($\Omega_{sc} \ll \Gamma/4$) in the three-state system, $\sigma_{\text{3}}$ decreases rapidly to $-\langle\Omega_s/\Omega_{sc}\rangle^2$ with a time constant of $2/\Gamma$, then increases slowly to 0 with a time constant of $2\Gamma/\Omega_{sc}^2$. Both the increase and decrease behaviors are exponential, without any oscillation. Since $\sigma_{\text{2}}$ and $\sigma_{\text{3}}$ are of the same order of magnitude, the rise time of the probe absorption is $2\Gamma/\Omega_{sc}^2$, the much longer time constant of the two.

We can now answer the question as to whether the probe propagation delay time in a medium is longer than the rise time of the probe absorption. The delay time is given by $OD(\Omega_{sc}^2/\Omega_{ec}^2)$, according to Eq. (4) of Ref. [1], where $OD$ is the optical density of the medium. As long as $OD > 2(\Omega_{sc}/\Omega_{ec})^2$ in the low-intensity limit (the situation discussed in Ref. [1]), the delay time is longer than the rise time and an energy cost of one photon per switching event can be achieved.

In conclusion, we have systematically studied the transient effect of photon switching by quantum interference. The probe absorption rise time in a four-state system can not be shorter than half the excited-state lifetime. It is equal to $2\Gamma/(\Omega_{sc}^2 + \Omega_{ec}^2)$ at low Rabi frequencies. A three-fold entangled state can be realized in a medium of large optical density. The realization of the three-fold entangled state may provide a new means for the manipulation of quantum information.

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References

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[11] In the case of two excited states with unequal lifetimes, the rise time asymptotically approaches $\frac{2}{\Gamma_{\text{eff}}}$ at large Rabi frequencies and is equal to $\frac{2}{\Gamma_{3}^{2}/\Gamma_{3} + \Gamma_{4}^{2}/\Gamma_{4}}$ at small Rabi frequencies. $\Gamma_{3}$ and $\Gamma_{4}$ are the spontaneous decay rates for the $|3\rangle$ and $|4\rangle$ states, and $\Gamma_{\text{eff}} = (\Omega_{3}^{2}\Gamma_{4} + \Omega_{4}^{2}\Gamma_{3})/(\Omega_{3}^{2} + \Omega_{4}^{2})$.