Investigation of Fuel Consumption and Pollution Emissions in Cellular Automata

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This paper studies the fuel consumption and emissions in cellular automata (CA), and presents an approach for estimating the amount of fuel consumption and emissions in CA models. The three models of Nagel-Schreckenberg (NS), finite deceleration (FD), and adaptive cruise control (ACC) are applied to the simulation. Among the three models, the ACC model is the most fuel-efficient one in the lower density region, and the FD model is the most fuel-efficient one in the higher density region. Finally, the results of the simulation are analyzed in detail.

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I. INTRODUCTION

Vehicle fuel consumption and engine emissions are two critical aspects that are considered in the transportation planning process of highway facilities. In recent years, the problems about fuel and pollution are also more and more serious. On the one hand, the problems caused by a lack of petroleum come forth more frequently, for example, a rising cost of living and international conflict. On the other hand, motor vehicles are the largest source of man-made polluting emissions [1, 2]. For example, in the USA, more than 75 percent of the national CO emissions and about 35 percent of the emissions of HC and NOx are emitted by motor vehicles [2]. In some urban areas, the motor vehicle contribution to CO emissions can exceed 90 percent [3].

Furthermore, estimation of fuel consumption and pollutant emissions is useful for the design, operations, and planning purposes in urban traffic control (UTC), especially for evaluating intersection and mid-block traffic conditions. For example, the well-known traffic control system split cycle offset optimization technique (SCOOT) [4], required estimation of operating costs caused by vehicles delay. Consequently estimation of fuel consumption and emissions needs to be considered.

In the early 1990s, the rapid development of computer capacity allowed cellular automata (CA) to display its high practical importance. Cellular automata are conceptually

simpler, and can be easily implemented on computers for numerical investigations compared with other dynamical approaches. In CA models, space (road), time, and the velocities of vehicles are assumed to take discrete values [5, 6]. When applied to traffic research, CA uses a cellular state to describe the position and velocity of each car, updates every cell state with rules deduced from practical traffic experience, and obtains the dynamical evolution and final steady result of the whole system.

II. CA MODELS

Cellular automata models applied to traffic flow have attracted a large amount of interest since the 1990s. One of the prototype CA models is the so-called Nagel-Schreckenberg (NS) model [7], which describes a single-lane traffic flow. It consists of \( N \) cars moving in a one-dimensional lattice of \( M \) cells with periodic boundary conditions (the number of vehicles is conserved). Each cell is either occupied by a vehicle, or is empty. The velocity of each vehicle is equivalent to the number of sites that a vehicle advances in one update, and it is an integer between zero and \( V_{\text{max}} \). We take \( X(i,t) \) and \( V(i,t) \) to denote the position and the velocity of the \( i \)th car at time \( t \), respectively. The number of empty cells in front of the \( i \)th car is denoted by \( \text{gap}(i,t) = X(i+1,t) - X(i,t) - L \) and is referred to hereafter as the gap. \( L \) is the length of a car. As mentioned above, space and time are discrete. At each discrete time step \( t \rightarrow t + 1 \) the system update is performed in parallel for all cars according to the following subrules:

1. Acceleration: \( V(i,t+1) \leftarrow \min(V(i,t) + 1, V_{\text{max}}) \),
2. Deceleration to avoid accidents: \( V(i,t+1) \leftarrow \min(V(i,t+1), \text{gap}(i,t)) \),
3. Randomization with a certain probability \( p \), i.e., \( V(i,t+1) \leftarrow \max(V(i,t) - 1, 0) \),
4. Movement: \( X(i,t+1) \leftarrow X(i,t) + V(i,t+1) \).

Briefly, if we do not consider the randomization deceleration, the velocity update rule reads

\[
V(i,t+1) = \min \left( V_{\text{max}}, V(i,t) + 1, \text{gap}(i,t) \right).
\]

In the NS model, the update rules for the \( i \)th car velocity depend on only the headway \( \text{gap}(i,t) \), which is determined by the positions of the two successive cars, \( i \) and \( (i+1) \). The \((i+1)\)th car is regarded as stock-still, whereas actually it may move ahead at the same time.

Yang et al. presented a CA model considering the finite deceleration and braking distance [8] (represented by FD model hereafter). The velocity update rule is as follows:

\[
V(i,t+1) = \min \left( V(i,t) + \text{acc}, V_{\text{max}}, V(d), V_{\text{safe}} \right),
\]

where \( \text{acc} = 0 \) if \( V(i,t-1) = 0 \) and \( V(i,t) = 1 \), or \( \text{acc} = 1 \) otherwise; \( d = \text{gap}(i,t) \) if the braking light of the preceding vehicle is on, or \( d = \text{gap}(i,t) + V(i+1,t) - 1 \) otherwise. \( V(d) \) and \( V_{\text{safe}} \) are used for restricting the velocity of the vehicle and ensuring comfortable driving and safety, respectively.
In 2006, Jiang et al. presented a cellular automata model of adaptive cruise control (ACC) vehicles in which the constant time headway (CTH) policy was fulfilled [9]. Compared with the NS model, the acceleration or deceleration rule and randomization was modified. The velocity update rule is

$$V(i,t) \leftarrow \min(V_{\text{max}}, \lceil \text{gap}(i,t)/T \rceil)$$

(3)

and the randomization probability is

$$p = \begin{cases} 
\lceil \text{gap}(i,t)/T \rceil, & \text{for } \text{gap}(i,t)/T < V_{\text{max}}, \\
0, & \text{for } \text{gap}(i,t)/T > V_{\text{max}},
\end{cases}$$

(4)

where $T$ is the constant time headway and $\lceil x \rceil$ denotes the minimum integer that is not smaller than $x$. The model reduces to the deterministic Fukui-Ishibashi model [10] in the case of $T = 1$.

III. ESTIMATE MODEL OF FUEL CONSUMPTION AND EMISSIONS

In recent years, some fuel consumption and emissions models were presented [11–13]. The model adopted in this paper is a nonlinear regression model [12]. The following function is used to estimate the value of the fuel consumed (mL) or emission produced (g), $\Delta F$, during a simulation interval (duration $\Delta t$ seconds):

$$\Delta F = \begin{cases} 
\alpha + \beta_1 R_T v + (\beta_2 M_v a^2/1000)_{a > 0} \Delta t, & \text{for } R_T > 0, \\
\alpha \Delta t, & \text{for } R_T \leq 0,
\end{cases}$$

(5)

where

- $M_v$ = vehicle mass (kg) including occupants and any other load;
- $v$ = instantaneous speed (m/s);
- $a$ = instantaneous acceleration rate (m/s$^2$), negative for deceleration;
- $\alpha$ = constant idle fuel rate (mL/s) or emission rate (g/s), which applies during all modes of driving (as an estimate of fuel used to maintain engine operation);
- $\beta_1$ = the efficiency parameter which relates fuel consumed or pollutant emitted to the energy provided by the engine, i.e., fuel consumption or emission per unit of energy (mL/kJ or g/kJ);
- $\beta_2$ = the efficiency parameter which relates fuel consumed or pollutant emitted during positive acceleration to the product of inertia energy and acceleration, i.e., fuel consumption or emission per unit of energy-acceleration (mL/(kJ·m/s$^2$)) or (g/(kJ·m/s$^2$));
- $R_T$ = total traction force (kN) required to drive the vehicle, whose value is equal to the sum of rolling resistance, air drag force, cornering resistance, inertia force and grade force.

In this paper, we don’t consider the vehicles turning and the road gradient, and the total traction force can therefore be computed as follows:

$$R_T = F_f + F_w + F_j = M_v g_f + \frac{1}{2} C_D \rho u_r^2 + \delta M_v a,$$

(6)
herein \( f \), \( C_D \), and \( \delta \) are the rolling friction coefficient, air resistance coefficient, and rotating mass inertia factor, respectively. \( A \) is the vehicle frontal area, \( \rho \) is the air density which could be generally regarded as a fixed constant. \( u_r \) is the relative speed of the vehicle and air; \( u_r \approx v \) is adopted in this paper.

When calculating the fuel consumption or emissions, we naturally think of doing one cumulative count using formula (5) for every time step of the CA, i.e., \( \Delta t \) in (5) equals to the length of time step in the CA model. However, the length of the time step is small (often 1s) in the CA models, so that the velocity of the car may change very frequently and the absolute value of the acceleration may be too large. For example, in the FI model, the vehicle speed could achieve maximum immediately from zero, which means that the acceleration achieves an unrealistic \( 75 m/s^2 \) if the value of acceleration is regarded as fixed constant during one time step. The estimate of fuel consumption and emissions may be larger than reality. Therefore, in this paper, we must use other approaches.

We adopt a longer simulation interval to ensure a reasonable velocity and acceleration. For every interval, the value of the acceleration is regarded as an invariant constant, and the vehicle is regarded as in uniform motion in the previous interval. The following formulas are utilized to calculate the values of the mean velocity and acceleration in every interval.

\[
V_{i,N} = \frac{S_{i,N}}{\Delta t}, \quad (7)
\]

\[
a_{i,N} = \frac{2(S_{i,N} - S_{i,N-1})}{\Delta t^2}. \quad (8)
\]

Here the subscripts \( i \) and \( N \) are the vehicle and simulation interval indices, respectively. \( S_{i,N} \) denotes the driving distance of vehicle \( i \) in the \( N \)th interval time. \( V_{i,N} \) and \( a_{i,N} \) replace \( v \) and \( a \) in (5).

### IV. SIMULATION RESULTS AND ANALYSIS

In the simulation, the fuel consumptions and emissions of the three models (NS, FD, and ACC) are compared and analyzed. A closed periodic road is adopted, which is 15km in length. For simplicity, all cars are assumed to be identical compact passenger cars (termed light cars in the following). The randomization probability is assumed to be 0.3 in the NS model. The length of the simulation interval \( \Delta t \) is 7s. Initially, \( N \) cars are located randomly on the \( M \) cells and the velocity of each car is designated by an integer randomly chosen from zero to the maximum \( V_{\text{max}} \). The cell division, the initial condition, and other parameters of the three models are the same as in the articles [7], [8], and [9] (the constant time headway \( T \) is assumed to be 1s in the ACC model, i.e., the deterministic Fukui-Ishibashi model is adopted), respectively (see TABLE I). The data are collected after the time evolution reaches the 10000th second.

Figs. 1–4 are the simulation results. Fig. 1 is the average fuel consumption of per vehicle (L/100 km) vs. traffic density and Figs. 2–4 are the average emission rates (g/s) per car vs. traffic density.
TABLE I: The parameters selected in the simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NS model</th>
<th>ACC model</th>
<th>FD model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of each cell (m)</td>
<td>7.5</td>
<td>7.5</td>
<td>3.75</td>
</tr>
<tr>
<td>Length of each car (cell)</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Total number of cells</td>
<td>2000</td>
<td>2000</td>
<td>4000</td>
</tr>
<tr>
<td>Maximum velocity (cell)</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Constant time headway (s)</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Randomization probability</td>
<td>0.3</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Abrupt deceleration threshold (cell)</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Maximum deceleration (cell)</td>
<td>-</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>Initial state of braking light</td>
<td>-</td>
<td>-</td>
<td>on</td>
</tr>
</tbody>
</table>

*“-” denotes that the appointed parameter does not appear in the indicated model.*

From Fig. 1, we can see that the trend of the curve of the ACC model is different from the others. ACC is the most fuel-efficient model among the three models in the lower density region, and the FD model is the most fuel-efficient model in the higher density region. It is surprising that ACC is the worst fuel-efficient model when the traffic density exceeds a critical value. The phenomena could be analyzed in detail as follows.

Firstly, we know that the two main direct factors of vehicle fuel consumption are velocity and acceleration from (5). Under the condition of uniform motion, we can find the optimal speed whose fuel-efficiency is the highest. Suppose a car is driving a certain distance with a uniform speed \( s \), and \( \Delta F' \) is the fuel consumption. Then the optimal speed...
can be depicted as follows:

\[ s^* = \arg \min_s \Delta F' = \arg \min_s \left\{ \alpha + \beta_1 R_TS \right\}. \]  \hspace{1cm} (9)

When all the parameters (according to the parameters of light cars) are confirmed, we can find the optimal speed \( s^* \approx V_{\text{max}} \). We can discuss the results in three regions.
In the free flow region, the headways of the vehicles are large enough, and all the vehicles almost drive with maximum speed. The only factor which causes the difference of the fuel consumption is stochastic deceleration. Therefore we can see that the curves are almost horizontal lines in the free flow region. Because there is no stochastic deceleration in the ACC model under this situation, i.e., the vehicles move with uniform speed, the fuel-efficiency is highest. The stochastic probability of the NS model is largest in the free flow region. This means that the average speed is smallest and the fluctuation of the velocity is the most frequent, so the fuel efficiency of the NS model is lowest. Naturally, the widths of the free flow region are different for different models because they assume different stochastic probabilities.

In the seriously jammed region (such as density larger than 0.7), the acceleration and velocities of most vehicles are zero in every time step. The fuel consumption is mainly from idle fuel consumption, i.e., \( \Delta F \approx \alpha \Delta t \). The average fuel consumption increases with an increase of the traffic density. Under the extreme condition, when the traffic density is 1, the average fuel consumption tends to infinity. Here the stochastic deceleration mainly plays a role that reduces the fluctuations of the velocities. The stochastic probability of the FD model is largest under this highly density condition, and therefore the average fuel consumption is least. In contrast, the average fuel consumption in the ACC model is the largest.

Finally, consider the region between the above two situations. In the ACC model, when the traffic density is higher, the cars' average speed is smaller, and the cars' velocities fluctuate more frequently, which is caused by the fact that each car tends to go as fast as it can whenever the headway allows. Thus the average fuel consumption increases with increasing traffic density. (The fluctuation is caused by the discrete character of cellular
automata. If the velocities of the vehicles are permitted to be nonintegral values, all the vehicles could always drive with a uniform speed and the velocities would not fluctuate after the system achieves equilibrium, and the fuel-efficiency would always be higher than for other models.) However, the case of the NS model or the FD model is different and more complicated. It is mainly the effect of the stochastic deceleration. On the one hand the stochastic deceleration may cause unnecessary deceleration, which leads to the speed decreasing and velocities fluctuating. On the other hand, the stochastic deceleration may play another role that delays the start of a stock-still car, decreases the acceleration, and consequently reduces velocity fluctuation. On account of this the average headway and average velocity of the cars would decrease with an increase of the density. When the traffic density is higher, the effect of the former is weaker, and the effect of the latter is stronger. Therefore we can see the two stages of increasing and decreasing of the average fuel consumption with increasing traffic density in the NS model and FD model.

In addition, the vehicles drive more smoothly in the FD model than that in the NS model [8]. Thus the result is reasonable that the fuel-efficiency is higher in the FD model whatever the value of the traffic density.

There are similar results for the engine emissions (Fig. 2-4), and they can be explained using similar approaches. The only difference is that the emission rate is the ratio of the emission produced not to distance, but to time.

V. CONCLUSION

This paper presents an approach to investigating fuel consumption and engine emissions in cellular automata which avoids the problem of unrealistic acceleration. We smoothed out the abrupt changes inherent in the models by taking the average of a quantity across several time steps. For every interval, the value of the acceleration is regarded as invariant constant, and the vehicle is regarded as having uniform motion in the previous interval.

Then the NS model, the FD model, and the ACC model are applied in the simulation. The results of the average fuel consumption and engine emissions are estimated and analyzed in detail. Among the three models (NS, FD, and ACC), the ACC model is the most fuel-efficient one at low density, and the FD model is the most fuel-efficient one in the high density region. In fact, the ACC model is the smoothest driving model under the condition of low traffic density, and the FD model is the smoothest one under the condition of high traffic density. In other words, if you want to reduce the expenditure of fuel and emission pollution, you should try your best to keep your driving smooth.

Naturally, in a continuous space, the vehicles could always keep smooth driving if all the cars are ACC vehicles, whatever the value of traffic density is. Therefore it is beneficial to use the adaptive cruise control in vehicles for saving fuel and reducing pollution.
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References

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