Influences of Temporal Dispersion and Nonlinear Absorption on Discrete Interface Solitons in Hetero-Structure Waveguide Arrays

Hongcheng Wang,* Xiaolan Peng, Shaoqiang Zhang, and Dongxiong Ling

School of Electronic Engineering, Dongguan University of Technology, Dongguan 523808, China

(Received March 16, 2009)

A theory is presented to investigate the formation of unstaggered and staggered (or Tamm) discrete interface solitons via the Kerr effect in hetero-structure waveguide arrays. The influences of temporal dispersion and nonlinear absorption are discussed in detail for different initial pulse power levels. It is shown that an unstaggered soliton with low power is destroyed by normal dispersion while a high power unstaggered soliton can propagate, keeping the intensity profiles, although an energy loss is evident. For a staggered soliton with moderate or high input power, the nonlinear absorption causes substantial energy loss, while the normal dispersion breaks the input pulse into a train of short and high peak pulses, localized within the central waveguide. We also investigate the influences of anomalous dispersion on staggered solitons, and find that it would be better if the wavelength of the light is set at the anomalous dispersion region to excite these staggered solitons.

PACS numbers: 42.65.Tg, 42.65.Jx, 42.82.Et

I. INTRODUCTION

In recent years, optical spatial solitons in periodic optical media, such as waveguide arrays (WAs) and photonic lattices (PLs), have been the focus of many investigations [1–5]. Many intriguing phenomena, e.g., discrete (or lattice) solitons and discrete gap solitons, were found to be fundamentally different from those observed in homogeneous media. In 1988, Christodoulides and Joseph obtained the analytical soliton solutions of the discrete nonlinear Schrödinger (DNLS) equation and predicted that light can trap itself in nonlinear WAs [6]. Ten years later, Eisenberg et al. made this idea a reality in aluminum gallium arsenide (AlGaAs) WAs via the Kerr effect [1]. This experiment has triggered intensive investigations on discrete solitons, and AlGaAs WAs have become one of the most popular and convenient experimental configurations for observing these solitons. Very recently, discrete surface solitons (DSSs) at the interface between a WA and a homogenous substrate and discrete interface solitons (DISs) at the hetero-interface of two different WAs have also been found to be of particular interest [7–10]. DSSs at the edge of semi-infinite WAs have been predicted theoretically and demonstrated experimentally [7,11]. In addition, DISs (including some novel modes, such as hybrid, twisted, and flat-top modes) at the hetero-interface of two different semi-infinite WAs have also been predicted theoretically [12]. In all these theories the continuous-wave (CW) model has been used successfully. It can be expected further that if a discrete system exhibits strong dispersion, the propagation of an
ultra-short optical pulse will be significantly influenced by the temporal dispersion effect, and thus the phenomena are evidently different from those of a continuous-wave (CW). Furthermore, nonlinear absorption of this discrete system will become evident because of the high-peak intensity of the ultra-short optical pulse, which means the multi-photon absorption process will become noticeable. Therefore, it is necessary to investigate the influences of temporal dispersion and nonlinear absorption on the propagation of an ultra-short optical pulse in hetero-structure WAs. In this paper, we investigate the existence of unstaggered and staggered DISs hetero-structure WAs, and give a detailed discussion on the influences of temporal dispersion and nonlinear absorption on these solitons to find some novel phenomena.

II. INFLUENCES OF DISPERSION AND NONLINEAR ABSORPTION ON UNSTAGGERED DIS

As shown in Figure 1, we consider two different weakly coupled single-mode semi-infinite periodic AlGaAs slab WAs with normal dispersion around 1.55 µm. These two arrays have the same waveguide site width but different separations between two neighborhood sites. According to the coupled-mode theory [13], the evolution of the modal field amplitude ($\tilde{E}_n = \sqrt{p}E_n$, $p$ is an arbitrary value of power) in the nth waveguide site obeys the following equation:

$$i \frac{\partial E_n}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 E_n}{\partial t^2} + (C_n E_{n+1} + C_{n-1} E_{n-1}) + \frac{k_0 n_2}{A_{\text{eff}}} p |E_n|^2 E_n + i \alpha p^2 |E_n|^4 E_n = 0,$$  

where $\beta_2$ is the group velocity dispersion (GVD), which is about $+1.3$ ps$^2$/m at $\lambda_0 = 1.55$ µm; $C_n$ is the linear coupling coefficient between the $n$th and $(n+1)$th waveguide; $k_0 = 2\pi/\lambda_0$; the factor $n_2 (= 2.47 \times 10^{-13}$ cm$^2$/W ) is the self-focusing Kerr coefficient [14]; $A_{\text{eff}}$ is the effective cross-sectional area of each waveguide element in the array, which is set by 15 µm$^2$; $\alpha$ is the three-photon absorption coefficient, about $10^{-4}$ m$^{-1}$W$^{-2}$ [14, 15]. For convenience, we introduce $\gamma (= k_0 n_2/A_{\text{eff}})$ as the effective nonlinear coefficient, which is about $6.6$ m$^{-1}$W$^{-1}$. The linear coupling coefficient depends on the separation between two neighborhood sites $D_i$ ($i = 1, 2$) and the refractive index of the WA. In our
In simulations, we choose $D_1 = 4 \, \mu \text{m}$, $D_2 = 7 \, \mu \text{m}$, and the refractive index is about 3.34 at 1.55 $\mu \text{m}$. Therefore, the linear coupling coefficients are calculated to be 728 $\text{m}^{-1}$ and 520 $\text{m}^{-1}$ for different semi-infinite periodic WAs, respectively, as shown in Figure 1. In addition, two-photon nonlinear absorption is ignored since the wavelength 1.55 $\mu \text{m}$ is below the half-band-gap of the AlGaAs material. Without losing generality, we choose $p = 1000$ W in our simulations.

To obtain the DIS solution, we ignore the dispersion and three-photon absorption terms of Eq. (1) and obtain

$$\frac{\partial E_n}{\partial z} + (C_n E_{n+1} + C_{n-1} E_{n-1}) + \gamma p |E_n|^2 E_n = 0.$$  

(2)

A stationary DIS solution is obtained by adopting $E_n(z) = U_n \exp(i\Gamma z)$, where $\Gamma$ is the propagation constant and $U_n$ is the amplitude of the DIS determined by $\Gamma$. Substituting this form into Eq. (2), we obtain

$$-\Gamma U_n + (C_n U_{n+1} + C_{n-1} U_{n-1}) + \gamma p |U_n|^2 U_n = 0.$$  

(3)

Obviously Eq. (3) has no analytic solutions, and thus it should be solved by a numerical method. We employ the Newton fixed-point iteration method to find the numerical solution of Eq. (3) for a fixed $\Gamma$. Figure 2 shows some examples of unstaggered DISs centered $n = 0$ with $\Gamma = 2200 \, \text{m}^{-1}$, 5000 $\text{m}^{-1}$, and 9000 $\text{m}^{-1}$. From Figure 2, one can see that the intensity profiles of this kind of DISs present an asymmetric distribution, which is because the structure of the hetero-structure WAs is asymmetric. As $\Gamma$ increases, this asymmetry becomes inconspicuous. Furthermore, the peak power becomes high and the soliton FWHM becomes narrow for large $\Gamma$.

![FIG. 2: Unstaggered discrete interface solitons centered at the site $n = 0$ for several values of $\Gamma$.](image)

In the following, we will investigate the influence of dispersion effects on the propagation of ultra-short optical pulses that are used to excite DISs centered at $n = 0$. In
our simulations, the array considered is 1 cm long and involves 41 waveguides with the hetero-interface located in the central waveguide (site \( n = 0 \)), as shown in Figure 1. The array is initially excited at \( z = 0 \) with some time domain hyperbolic secant (sech) pulses \( E_n(t, z = 0) = U_n \cdot \text{sech}(t) \), with an intensity FWHM 1.76 ps, where \( U_n \) have the same optical amplitudes as those of the DISs shown in Figure 2. Ignoring absorption terms, Eq. (1) is solved numerically by the split-step Fourier scheme, with the nonlinear part treated on the basis of the fourth order Runge-Kutta method. The accuracy of the numerical simulation is checked against the total “power” \( \sum_n \int_{-\infty}^{+\infty} |E_n|^2 \, dt \), an invariant of this equation during propagation.

![Figure 3: Propagation of unstaggered optical pulses corresponding to \( U_n \) of (a, d, g) \( \Gamma =2200 \text{ m}^{-1} \), (b, e, h) 5000 \text{ m}^{-1} \), and (c, f, i) 9000 \text{ m}^{-1} \); the figures show the intensity profiles at (a)–(c) \( z = 0 \); (d)–(f) \( z = 1 \text{ cm} \) when they undergo normal dispersion effects only; and (g)–(i) \( z = 1 \text{ cm} \) when they undergo both normal dispersion and nonlinear absorption effects.

Figures 3a–3c are the intensity profiles of input pulses at \( z = 0 \) corresponding to \( U_n \) with \( \Gamma =2200 \text{ m}^{-1} \), 5000 \text{ m}^{-1} \), and 9000 \text{ m}^{-1} \) shown in Figure 2, and the peak power of these pulses are 510 W, 855 W, and 1160 W, respectively. The evolutions of these pulses are simulated, and their intensity profiles at \( z = 1 \text{ cm} \) are shown in Figures 3d–3f. As can be seen in Figures 3d and 3e, when the power of the input pulse is not high enough (\( \Gamma =2200 \text{ m}^{-1} \) or 5000 \text{ m}^{-1} \), the dispersion effect will destroy the profile of the pulse, and much of the energy of the pulse within the central waveguide will couple into its neighboring waveguide.
While for a high input power ($\Gamma = 9000 \text{ m}^{-1}$), the major part of the initial pulse is localized within the central waveguide with a rectangular-like intensity profile at the output face, as shown in Figure 3f, and the pulse edges become narrower than those in Figures 3d, 3e. In this case, an interface localized wave is formed due to temporal dispersion, discrete diffraction, and nonlinearity effects.

Up to this point, we have discussed the influence of dispersion on the propagation of ultra-short optical pulses used to excite DISs. The nonlinear absorption, however, will become important at high power levels. In the following, we consider further the influence of the three-photon nonlinear absorption effect while ignoring the two-photon one, based on the consideration that the energy of 1.55 $\mu$m light is below the half-band-gap of the AlGaAs material. We directly simulate Eq. (1) by the split-step Fourier method and the fourth order Runge-Kutta method. Three examples are shown in Figure 3g–3i. Comparing Figures 3d–3f with Figures 3g–3i respectively, one can see that, for a low input power ($\Gamma = 2200 \text{ m}^{-1}$), the nonlinear absorption is very small (about 1.68%), and the dispersion is the main factor influencing the propagation of the light pulse [Figure 3g]. So it is difficult to excite this DIS with a small $\Gamma$. For a moderate input power ($\Gamma = 5000 \text{ m}^{-1}$), the nonlinear absorption not only causes a distinct loss of pulse energy but also results in an evident coupling of pulse energy into a neighboring waveguide [Figure 3h]. For a high input power ($\Gamma = 9000 \text{ m}^{-1}$), the loss of the pulse energy is substantial (about 39.7%), but the major part of remaining energy still localizes at the interface. So we can still regard this localized wave as a DIS.

III. INFLUENCES OF DISPERSION AND NONLINEAR ABSORPTION ON STAGGERED DISSS

To investigate the influences of dispersion and nonlinear absorption on staggered DISs, we first study the existence and their properties. This type of DIS in hetero-structure WAs, staggered modes known as Tamn states, was first found like localized electronic states at the edge of a truncated periodic potential [16]. In nonlinear optics, these solitons have been theoretically predicted to occur in Dirac-comb lattices [17], at the surface of semi-infinite WAs [8], and the interface of hetero-structure PLs and WAs [9, 12]. However, no experimental observations of these DISs were reported. In the following, we will discuss the conditions for experimental observations of these solitons, and discuss in detail the influences of dispersion and nonlinear absorption on them.

To obtain the staggered DISs, we numerically simulate Eq. (3) with $\gamma < 0$ (for computation convenience, we choose $\gamma = -6.6$), which means the media of hetero-structure WAs is self-defocusing. We still solve Equation (3) by numerical methods to get the soliton solutions. Staggered DISs seated at the site $n = 0$ with $\Gamma = -2200 \text{ m}^{-1}$, -5000 $\text{ m}^{-1}$, and $-9000 \text{ m}^{-1}$ are shown in Figure 4. Similarly to the unstaggered DISs, the intensity profiles of staggered DISs also present an asymmetric distribution.

The influences of dispersion and nonlinear absorption on these DISs are shown in Figure 5, where the array is still initially excited at $z = 0$ with some time domain hyperbolic secant (sech) pulses $E_n(t, z = 0) = U_n \cdot \text{sech}(t)$ with different power levels. In Figures 5a–
the field amplitudes correspond to $U_n$ with $\Gamma = -2200 \text{ m}^{-1}$, -5000 m$^{-1}$, and -9000 m$^{-1}$, and the peak power of these pulses are 510 W, 855 W, and 1160 W, respectively. For the medium with only normal dispersion (e.g., $\beta_2 = +1.3 \text{ ps}^2/\text{m}$ in our simulations), the simulated intensity profiles of the pulses at $z = 0$ and $z = 1 \text{ cm}$ are shown in Figures 5d–5f. If the pulses experience both normal dispersion and nonlinear absorption simultaneously, the simulated intensity profiles of the pulses at $z = 1 \text{ cm}$ are shown in Figures 5g–5i. Comparing Figures 5d–5f with Figures 5g–5i, respectively, one can see that the nonlinear absorption is very small and the dispersion is the main factor destroying the profile of the DISs for a low input power ($\Gamma = -2200 \text{ m}^{-1}$). For a moderate input power ($\Gamma = -5000 \text{ m}^{-1}$) or a high input power ($\Gamma = -9000 \text{ m}^{-1}$), the influence of nonlinear absorption becomes evident, which leads to a substantial loss of the pulse energy [Figures 5h and 5i]. However, the major part of the remaining energy still localizes at the interface strongly, though the normal dispersion effect breaks the pulse into a train of short and high peak pulses. The origin of this phenomenon is the ‘neck’ instability which is found in self-focusing media for an anomalous dispersion region [18].

We finally study the propagation behavior of pulses with different power levels in a medium with anomalous dispersion and nonlinear absorption. For convenience, we choose $\beta_2 = -1.3 \text{ ps}^2/\text{m}$ and take those shown in Figures 5a–5c as initial pulses. The intensity profiles of the output pulses at $z = 1 \text{ cm}$ are calculated and shown in Figure 6. It is clear that Figure 6 is similar to Figures 3g–3i. To leave more energy in the central waveguide, we should use optical pulses with high powers to excite the hetero-structure waveguide array, although they will experience a large energy loss. Furthermore, comparing Figure 6 with Figures 5g–5i, one can see that anomalous dispersion can compensate for the discrete diffraction better than the normal dispersion. Thus, we can conclude that, to excite staggered DISs, it would be better that the working wavelength of the light pulse is set at the anomalous dispersion region.
FIG. 5: Propagation of staggered optical pulses corresponding to $U_n$ of (a, d, g) $\Gamma = -2200 \text{ m}^{-1}$, (b, e, h) $-5000 \text{ m}^{-1}$, and (c, f, i) $-9000 \text{ m}^{-1}$; the figures show the intensity profiles at (a)–(c) $z = 0$; (d)–(f) $z = 1$ cm when they undergo a normal dispersion effect only; and (g)–(i) $z = 1$ cm when they undergo both normal dispersion and nonlinear absorption effects.

FIG. 6: (a)–(c) The intensity profiles of output pulse at $z = 1$ cm corresponding to Figures 5a–6c when the staggered pulse undergoes both anomalous dispersion and nonlinear absorption.

IV. CONCLUSION

In conclusion, taking the practical experimental conditions into consideration, we have investigated the influences of temporal dispersion and nonlinear absorption on DISs in hetero-structure WAs. The normal dispersion effect of hetero-structure WAs has different
influences on unstaggered and staggered DIS. For an unstaggered soliton with low power, the power in the central waveguide spreads into the neighborhood waveguides, and finally normal dispersion destroys its localized configuration; for the high power case, normal dispersion and nonlinear absorption does not destroy the localized configuration of this soliton, although absorption is evident. Furthermore, for a moderate or a high power staggered DIS, besides the substantial power losses due to the nonlinear absorption, the normal dispersion effect results in the breakdown of the input pulse into a train of short and high peak power pulses, localized within the central waveguide. The influences of anomalous dispersion on staggered solitons have also been investigated, and the numerical simulations indicate that it would be better that the working wavelength of the light pulse be set at the anomalous dispersion region to excite these staggered solitons.

Acknowledgements

This work is partially supported by Guangdong Natural Science Foundation (8151170003000010).

References

* Electronic address: hc_wang@126.com