Some Magnetic Properties of the Amorphous Transverse Spin-$\frac{1}{2}$ Ising System

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The amorphous spin-$\frac{1}{2}$ Ising system with a transverse field is investigated by the use of the effective field theory based on the probability distribution technique, which accounts for the self spin correlation functions. We use the stochastic lattice model to describe the structural disorder. Results for the hysteresis loops, the transverse and longitudinal magnetizations, and susceptibilities are presented for a simple cubic lattice. Due to the fluctuations of the exchange interactions and the applied transverse field, a number of interesting phenomena are found in these quantities.

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I. INTRODUCTION

The magnetism of structurally disordered alloys has become a subject of both experimental and theoretical interest in solid state physics [1–8]. In an amorphous ferromagnet the spins are oriented in the same direction but the topological arrangement of spins is not regular. Some typical examples of amorphous ferromagnetic materials are alloys of the transition metals (Fe, Co, Ni) with metalloid elements (B, C, Si, Ge, P) containing approximately 20% of the latter, such as Fe$_{80}$B$_{20}$ and (Fe$_x$Ni$_{1-x}$)$_{80}$B$_{10}$P$_{10}$. Below the critical (Curie) temperature $T_c$, all the spins are, on the average, oriented parallel to one another, giving rise to a large spontaneous magnetization of the sample in some arbitrary direction if the system is isotropic. In real amorphous materials, there is always some anisotropy, although it may be weak, and the bulk magnetic moment $M(T)$ is oriented along one of the easy magnetization axes. The spontaneous magnetization decreases continuously as the temperature rises, and, in the absence of the external magnetic field, disappears at and above the critical temperature.

A number of experimental and theoretical investigations have led to the result that long-range magnetic order may exist in amorphous systems. At the same time, because of the disordered structure, many interesting physical properties not observed in the corre-
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sponding crystalline magnets are observed [5]. Theoretically, because of the difficulties of description of such complicated magnetics systems, the lattice model of amorphous magnets has often been applied, in which the structural disorder is replaced by the random distribution of the exchange integral. This model has been adopted by several authors in the treatment of various Ising systems within the framework of an effective field theory. These works include studies on pure, dilute [6, 7], and transverse Ising models [8], as well as on semi-infinite systems [9] and thin films [10]. Using these techniques, Kaneyoshi [8, 11] examined both the magnetization and the longitudinal and transverse susceptibilities of the spin-$\frac{1}{2}$ Ising model in a transverse field. The same model on a semi-infinite slab has been studied [12] and the surface phase diagram obtained.

Magnetic hysteresis is a classical example of metastability at experimentally easily accessible time scales. Its ready appearance under simple conditions makes it not only technologically important, but also a theoretical challenge. In addition to macroscopic approaches, relying on multiple-valued equations of state, there has been considerable work done over the past decade by Monte Carlo simulations of lattice systems. The high reproducibility of hysteresis curves, and the fact that among their many observed shapes most are fairly simple, imply that hysteresis delay involves processes similar to ordinary thermalization, which can be modeled by a simple random walk. The Ising model, in the widest sense, has evolved as an important test bed for this kind of question. Its spins are naturally associated with magnetic domains, and can be different physically motivate interactions. It turns out that random fields with cluster updates give Barkhausen noise [13], local spin inversion symmetry is needed for memory effects [14], random couplings produce multicyles at low temperatures [15], while long range and anisotropic interactions give domain patterns very similar to those in some real thin films [16].

In this work, we examine the effect of the transverse field on both the transverse and longitudinal magnetizations, the susceptibilities, the hysteresis magnetic loops, and the pyromagnetic coefficient of the amorphous magnetic simple cubic lattice. We use the effective field theory with a probability distribution technique [17] to clarify how the relevant thermodynamic quantities are subject to the influence of the transverse field. This technique is believed to give more exact results than those of the standard mean-field approximation.

The approximate method is basically a mean field approach which takes into consideration the fluctuations of the effective field, and is based on a probability distribution of random variables which correctly accounts for all the single site kinematical relations. This method is much less sophisticaied than the real space renormalization group; it applies to a wide class of disordered systems. In particular site-dilution, which is physically more relevant, is much easier to treat within effective field theory. A favorable point of this method lies in the simplicity of the calculation. It has been shown that the simplest approximation gives satisfactory results when compared with other techniques [17].
FIG. 1: (a) The temperature dependence of $m_z$ (dashed line), $\chi_{\perp}$ (doted line), and $m_x$ (solid line) for the amorphous transverse spin-1/2 Ising model on a simple cubic lattice for $\Omega/J = 0.1$. (b) The temperature dependence of $m_z$ (dashed line), $\chi_{\perp}$ (doted line), and $m_x$ (solid line) for the amorphous transverse spin-1/2 Ising model on a simple cubic lattice for $\Omega/J = 1.0$.

II. FORMALISM

The pseudospin theory based on the transverse Ising model is generally believed to be a good microscopic description of hydrogen-bonded ferroelectric systems of the KH$_2$PO$_4$ type [18, 19]. It has also been successfully applied to several other systems, for example cooperative Jahn-Teller systems like DyVO$_2$ and ferromagnets in transverse and external longitudinal magnetic fields. The Hamiltonian of the system is given by

$$H = - \sum_{\langle ij \rangle} J_{ij} S_i z S_j z - \Omega \sum_i S_i x - h \sum_i S_i z,$$

where $S_i z$ and $S_i x$ denote the $z$ and $x$ components of a quantum spin $\vec{S}_i$ of magnitude $S = \frac{1}{2}$ at site $i$. $J_{ij}$ is the strength of the exchange interaction between the spins at nearest-neighbors sites $i$ and $j$, and the first summation runs over all pairs of nearest neighbors. $\Omega$ represents the transverse field. In two or more dimensions the transverse Ising model has a finite transition temperature, which can be depressed to zero temperature by increasing the transverse field to a critical value $\Omega_c$. An exact solution for the one-dimensional case has been obtained [20], where no phase transition is found at finite temperature, but at $T = 0$ the system is ordered for $\Omega$ less than some critical value $\Omega_c$. Thus, the phase
boundary starts at some $\Omega_c$ for $T = 0$, and it separates the ferromagnetic region from the paramagnetic one by a second-order phase transition.

In order to describe the structural disorder in a sample way, the lattice model of amorphous magnets is used; the nearest-neighbor exchange interactions are given by independent random variables as

$$P(J_{ij}) = \frac{1}{2} \delta [J_{ij} - J (1 + \Delta)] + \frac{1}{2} \delta [J_{ij} - J (1 - \Delta)]$$

with $\Delta^2 = \langle (\Delta J_{ij})^2 \rangle / J^2$, where $\Delta J_{ij}$ is the random fluctuation from the mean exchange interaction $J$ and $\langle \ldots \rangle$ represents the configurational average. It should be noted here that the distribution (2) corresponds to the so-called stochastic lattice model of amorphous magnets [21], in which the average values of $\Delta J_{ij}$ are approximated by $\langle (\Delta J_{ij})^2 \rangle \approx \langle (\Delta J_{ij})^2 \rangle^n$, $\langle (\Delta J_{ij})^{2n+1} \rangle \approx 0$, where $n$ is an integer.

For the determination of the average magnetization per site defined by $m_\alpha = \langle S_{0\alpha} \rangle$, the effective field theory is used. Here the inner and outer angular brackets denote thermal and random configurational averaging, respectively. The latter is to be carried out after the thermal averaging has been taken. In the simplest form, the effective field theory to be adopted is based on a single cluster theory in which attention is focused on a cluster comprising just a single selected spin, labeled 0, and the neighboring spins with which it
FIG. 3: (a) The temperature dependence of the longitudinal magnetic susceptibility at different values of the amorphization $\Delta$ for $\Omega/J = 0.1$. (b) The temperature dependence of the longitudinal magnetic susceptibility at different values of the amorphization $\Delta$ for $\Omega/J = 1.0$.

directly interacts. To this end, the Hamiltonian is split into two parts, $H = H_0 + H'$, where $H_0$ is that part of the Hamiltonian containing spin 0, namely

$$H_0 = -AS_0z - BS_{0z},$$

with

$$A = \sum_{j=1}^{N} J_{0j} S_{jz} + h \quad \text{and} \quad B = \Omega.$$  

This single-site Hamiltonian can readily be diagonalized and its eigenvalues and eigenvectors found. The two eigenvectors corresponding to the eigenvalues

$$E_{\pm} = \pm \frac{1}{2} \sqrt{A^2 + B^2}$$

are

$$|\psi_{\pm}\rangle = \frac{1}{2\sqrt{A^2 + B^2}} \left[ (2E_{\pm} + A) \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} + (2E_{\pm} - A) \begin{pmatrix} -1/2 \\ -1 \end{pmatrix} \right]$$

in a representation in which $S_{0z}$ is diagonal.
For “classical systems” in which $H_0$ and $H'$ commute, the starting point of single-site cluster theory is a set of formal identities of the type

$$\langle S_{0\alpha} \rangle = \frac{\text{Tr} [S_{0\alpha} e^{-\beta H_0}]}{\text{Tr} [e^{-\beta H_0}]},$$

where $\alpha = z, x$ and $\beta = 1/k_B T$, with $k_B$ being the Boltzmann constant and $T$ is the temperature. While these relations are no longer exact for an Ising system in a transverse field, they have nevertheless, since the original work of Sà Barreto et al. [22], been accepted as a reasonable starting point in many studies of that system. To proceed, one has to effect the inner traces in Eq. (7) over the states of the spin 0, and this is most easily performed using the eigenstates of Eq. (6) as the basic states. In this way it is found that

$$\langle S_{0z} \rangle = \langle f_z ((S_{jz}, J_{0j}, h); \Omega) \rangle = \left\langle \frac{1}{2} \frac{A}{\sqrt{A^2 + B^2}} \tanh \left( \frac{1}{2} \beta \sqrt{A^2 + B^2} \right) \right\rangle,$$  

and

$$\langle S_{0x} \rangle = \langle f_x ((S_{jz}, J_{0j}, h); \Omega) \rangle = \langle f_z (\Omega; (S_{jz}, J_{0j}, h)) \rangle,$$

where $A$ and $B$ are defined in Eq. (4).
The sum in Eq. (4) is over the nearest neighbors of the site 0, \( N \) being the nearest neighbor coordination number of the lattice. In a mean field approximation one would simply replace these spin operators by their thermal values. However, it is at this point that a substantial improvement to the theory is made by noting that the spin operators have a finite set of basis states so that the average over the function \( f \) can be expressed as averages over a finite polynomial of spin operators belonging to the neighboring spins. This procedure can be effected by the combinatorial method and correctly accounts for the single site kinematic relations. Up to this point the theory is exact, but the right hand side of Eqs. (8) and (9) will contain multiple spin correlation functions. Usually, at this stage a Zernike type decoupling of the multiple spin correlation functions is made that neglects the correlations between quantities pertaining to different sites. The above thermal averages were for a fixed spatial configuration. In the next step, we have to carry out the spatial configurational averaging. To make progress, the simplest approximation of neglecting the correlations between different sites will be made.

To perform thermal averaging on the right hand side of Eqs. (8) and (9), we follow the general approach described in Ref. [17]. First of all, in the spirit of the effective field theory, multispin-correlation functions are approximated by products of single spin averages. We then take advantage of the integral representation of the Dirac delta distribution, in order to write Eqs. (8) and (9) in the following form:

\[
m_\alpha = \langle S_{0\alpha} \rangle = \sum_{S_{0z} = -\frac{1}{2}}^{\frac{1}{2}} \int dJ_{0j} P(J_{0j}) P(S_{0z}) f_\alpha ((S_{0z}, J_{0j}, h); \Omega),
\]

\[
m_\alpha = \int d\omega dJ_{0j} P(J_{0j}) f_\alpha ((\omega, J_{0j}, h); \Omega) \frac{1}{2\pi} \int [dt \exp(i\omega t) \exp(-iht) \prod_j \langle \exp(itJ_{0j}S_{0z}) \rangle].
\]

We now introduce the probability distribution of the spin variables (for details see Ref. [17])

\[
P(S_{0z}) = \frac{1}{2} [(1 - 2m_z)\delta(S_{0z} + \frac{1}{2}) + (1 + 2m_z)\delta(S_{0z} - \frac{1}{2})].
\]

Next we average Eq. (10) over the interactions according to Eq. (2). This leads to the following expression of the longitudinal magnetization of the amorphous spin-\( \frac{1}{2} \) Ising model:

\[
m_\alpha = 2^{-2N} \sum_{i=0}^{N} \sum_{j=0}^{N-i} \sum_{k=0}^{N-j-i} C_i^N C_j^N C_k^N (1 - 2m_z)^j (1 + 2m_z)^{j+k} F(X, h, \Omega),
\]

with

\[
F(X, h, \Omega) = \frac{1}{2} \frac{X + h}{\sqrt{(X + h)^2 + \Omega^2}} \tanh \left( \frac{1}{2} \beta \sqrt{(X + h)^2 + \Omega^2} \right)
\]
cubic lattice which is considered here, one has
values of the amorphization (∆ = 0 of the longitudinal and transverse magnetizations and transverse susceptibility for different of an external transverse field. Indeed, in Figs. (1a) and (1b) we examine the behaviors susceptibilities, the magnetic hysteresis, and the pyromagnetic coefficient in the presence magnetic properties of the system, such as the longitudinal and transverse magnetizations, the

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In this equation, \( N \) denotes the number of nearest neighbors and for the case of a simple cubic lattice which is considered here, one has \( N = 6 \) and \( \mathcal{C}_k^l \) are the binomial coefficients, \( \mathcal{C}_k^l = \frac{n!}{l!(n-l)!} \).

It is interesting to study the behavior of the longitudinal susceptibility with amorphization change, which is defined by

\[
\chi_\parallel = \left( \frac{\partial m_\parallel}{\partial h} \right)_{h \to 0}
\]

In the Ising model with a transverse field, at high temperatures the \( S_z \) components are disordered, and at temperatures below a critical value \( T_c \) an ordered phase sets in with \( m_z \neq 0 \), although in all temperature intervals there exists an ordering with \( m_x \neq 0 \). In this way it is interesting to investigate the behavior of the transverse susceptibility which is defined by

\[
\chi_\perp = \left( \frac{\partial m_\perp}{\partial \Omega} \right)_{h \to 0}
\]

By differentiating the expressions of the longitudinal and transverse magnetizations with respect to \( h \) and \( \Omega \), we can easily obtain the expressions of the susceptibilities as follows:

\[
\chi_\parallel = \frac{2^{-2N} \sum_{i=0}^{N} \sum_{j=0}^{N-i} \mathcal{C}_i^N \mathcal{C}_j^N \mathcal{C}_k^N \mathcal{C}_l^N (1-2m_x)^{j+k} (1+2m_x)^{N-j-k} (\partial F(X,h,\Omega))/\partial h)_{h \to 0}}{1 - 2^{-2N} \sum_{i=0}^{N} \sum_{j=0}^{N-i} \sum_{k=0}^{N-j-k} \sum_{\nu=0}^{\mu=0} (-1)^\nu 2^{\nu+\nu} \mathcal{C}_i^N \mathcal{C}_j^N \mathcal{C}_k^N \mathcal{C}_l^N \mathcal{C}_m^N \mathcal{C}_n^N \mathcal{C}_o^N \mathcal{C}_p^N \mathcal{C}_q^N (\mu + \nu) m_x^{\nu+\nu+\nu+\nu} F(X,h,\Omega)_{h \to 0}}
\]

and

\[
\chi_\perp = \frac{2^{-2N} \sum_{i=0}^{N} \sum_{j=0}^{N-i} \sum_{k=0}^{N-j-k} \sum_{\nu=0}^{\mu=0} (-1)^\nu 2^{\nu+\nu} \mathcal{C}_i^N \mathcal{C}_j^N \mathcal{C}_k^N \mathcal{C}_l^N \mathcal{C}_m^N \mathcal{C}_n^N \mathcal{C}_o^N \mathcal{C}_p^N \mathcal{C}_q^N \mathcal{C}_r^N \mathcal{C}_s^N (\mu + \nu) m_x^{\nu+\nu+\nu+\nu} F(X,h,\Omega)_{h \to 0}}{1 - 2^{-2N} \sum_{i=0}^{N} \sum_{j=0}^{N-i} \sum_{k=0}^{N-j-k} \sum_{\nu=0}^{\mu=0} (-1)^\nu 2^{\nu+\nu} \mathcal{C}_i^N \mathcal{C}_j^N \mathcal{C}_k^N \mathcal{C}_l^N \mathcal{C}_m^N \mathcal{C}_n^N \mathcal{C}_o^N \mathcal{C}_p^N \mathcal{C}_q^N \mathcal{C}_r^N \mathcal{C}_s^N (\mu + \nu) m_x^{\nu+\nu+\nu+\nu} F(X,h,\Omega)_{h \to 0}}
\]

III. RESULTS AND DISCUSSION

In this section we present and discuss the effects of the amorphization on some magnetic properties of the system, such as the longitudinal and transverse magnetizations, the susceptibilities, the magnetic hysteresis, and the pyromagnetic coefficient in the presence of an external transverse field. Indeed, in Figs. (1a) and (1b) we examine the behaviors of the longitudinal and transverse magnetizations and transverse susceptibility for different values of the amorphization (\( \Delta = 0.0, 0.5, 1.0, \) and \( 1.25 \)) in the cases of weak and high
applied transverse field. Firstly, it will be worthwhile to examine the pure Ising ferromagnetic simple cubic lattice with a transverse field (curves with $\Delta = 0.0$). The results show that the larger the transverse field the smaller is the longitudinal magnetization $m_z$. The role of the transverse field is essentially to inhibit the ordering of the $S_z$-component. In relation to the result, the values of the transverse magnetization $m_x$ increase on increasing the value of $\Omega$.

The longitudinal magnetization curves with a finite value of $\Delta$ express the depression from that of its corresponding non-random ferromagnet (curve with $\Delta = 0.0$) over the entire temperature range below the transition temperature, which phenomenon has been generally observed in amorphous ferromagnets with zero transverse field. The saturation magnetization of $m_z$ at $T = 0$ decreases, when the disorder of the exchange interaction increases. Comparing Fig. (1a) with Fig. (1b), the decrease is more rapid in the high applied transverse field (Fig. (1b)) than in low applied transverse field (Fig. (1a)).

The critical temperature $T_c$ is defined as the temperature at which the longitudinal magnetization $m_z$ goes to zero. From these figures we can see clearly that the critical temperature decreases when the degree of the amorphization $\Delta$ increases.

For a fixed value of $\Omega$ and $T < T_c$, the transverse magnetization and susceptibility increase with the increase of the amorphization.

Below the transition temperature, the susceptibility $\chi_\perp$ and the magnetization $m_x$ are insensitive to temperature, and $\chi_\perp$ has a gap at the transition depending on the value of $\Omega$. Upon increasing the value of $\Delta$, however, $m_x$ and $\chi_\perp$ become temperature dependent below $T_c$ and the discontinuity of $\chi_\perp$ at $T = T_c$ increases. Especially for the case $\Delta = 1.25$ in Fig. (1b), some characteristic behavior is observed below $T_c$. Thus the results imply that some characteristic behavior of $\chi_\perp$ may be observed in the amorphization of the Ising model with an applied transverse field.

The $(m_z - h)$ hysteresis loops of the amorphous ferromagnets are obtained by changing, cyclically, the value of the external applied magnetic field $h$. In Fig. (2a), we plot the hysteresis loops with different values of the amorphization $\Delta$ ($\Delta = 0.0$, $\Delta = 0.5$, $\Delta = 1.0$, $\Delta = 1.25$, and $\Delta = 1.5$) for a fixed value of the temperature and the transverse field ($T/J = 0.5$, $\Omega/J = 0.5$). We can see that the magnetization curves are symmetric for both positive and negative longitudinal magnetic field. This type of hysteresis loop becomes narrower with increasing amorphization $\Delta$. Then the hysteresis loop disappears when $\Delta$ is greater than a large value of $\Delta = \Delta_c$. When $\Delta$ becomes larger than $\Delta = 1.0$, the exchange interaction can take positive and negative values with equal probability. The so-called frustration may appear in the system, and, if we admit the existence of a spin-glass phase, the transition from the spin-glass phase to the paramagnetic phase passing through the ferromagnetic one is possible. Then the hysteresis loop disappears when $\Delta$ is greater than a large value of $\Delta = \Delta_c$, with $\Delta_c \gg 1$.

The dependence of the hysteresis on the transverse field is shown in Fig. (2b). From the later, we can see that the tendency of the change of the hysteresis loops with the increase of the transverse field is similar to that with the increase of the amorphization. The explanation for the different hysteresis at different $\Omega$ is clear; the higher the transverse field, the more disordered the system.
In Figs (3a) and (3b), the numerical results of the longitudinal susceptibility are depicted for the studied system for various values of the amorphization $\Delta$ in the cases of low and high transverse field. The thermal dependence of the longitudinal susceptibility is presented for $\Delta = 0.0, 0.5, 1.0,$ and $\Delta = 1.25$. We see clearly a sharp cusp near the critical temperature which corresponds to the divergence of $\chi_{\parallel}$. The temperature at which the magnetic susceptibility maximizes shifts to lower temperatures with the increase of the amorphization $\Delta$. These shifts increase at high transverse field.

Finally, we plot in Figs. (4a) and (4b) the temperature dependence of the pyromagnetic coefficient defined as $\frac{\partial m}{\partial T}(h \to 0)$ for different values of the amorphization $\Delta$ at low and high transverse fields. For a fixed value of $\Delta$, the pyromagnetic coefficient presents a peak at the critical temperature $T_c$. The critical temperature decreases when $\Delta$ becomes significant at high transverse field.

IV. SUMMARY

In this paper, we have investigated the effects of the amorphization and the transverse field on hysteresis loops and some physical quantities of the amorphous spin-$\frac{1}{2}$ Ising system, especially for the transverse and longitudinal magnetizations and susceptibilities and the pyromagnetic coefficient. We have used the effective field theory based on the probability distribution technique that accounts for the self spin correlation functions. As results, the critical temperature decreases as the amorphization parameter increases and this shift increases with the transverse field. The hysteresis loops become narrower with increasing amorphization. The numerical results of the hysteresis curves obtained in this work seem to be reasonable, and we think that experimental data can be explained by fitting some parameters in this simple model.

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