Some Urgent Problems in Medium and High-Energy Nuclear Physics

W-Y. Pauchy Hwang
Department of Physics, National Taiwan University,
Taipei, Taiwan 106, R.O.C.
(Received October 10, 1994)

In this article, I wish to elucidate some problems which I believe are urgent in the area of medium and high energy nuclear physics. First, the adoption of the parton-model description of hadrons in high energy physics experiments has, over the last two decades, generated a conceptual gap from the meson-baryon picture which is still used as the standard language in medium or high energy nuclear physics. We suggest that the key to bridge the gap is to try to understand, or to derive, the various parton distributions at low or moderate $Q^2$ (say, up to a few GeV$^2$) using the meson-baryon picture. As the second urgent problem in nuclear physics, I wish to echo the standard wisdom that strong interaction physics, as based on quantum chromodynamics (QCD), must be treated in a quantitative manner for low- or medium-energy processes involving hadrons. Here we have chosen to follow the route of using QCD sum rules to offer some solutions, as an alternative to the commonly accepted approach based upon lattice simulations. Finally, I believe that it is highly essential, and of urgently important, to employ what we have learned in nuclear and particle physics to formulate, or to reformulate, certain problems in related areas such as astrophysics and cosmology.

PACS. 24.85.+p - Quarks, gluons, and QCD in nuclei and nuclear processes.
PACS. 24.80.-x - Fundamental interactions in nuclei.
PACS. 97.60.Sm - Other objects believed to be disintegrating or collapsing.

I. Prelude: a Few Words About Dr. Ta-You Wu

It is indeed my great pleasure, as well as an honor, to give an invited talk at the Symposium in honor of Dr. Ta-You Wu on his honorable retirement from the presidency of Academia Sinica. Before jumping into the subject of the present article which deals primarily with a personal review concerning important problems in the area of medium and high energy nuclear physics, I wish to say a few words about Dr. Wu.

Unlike some of the people in the physics community here in Taiwan, I in fact did not have the chance to meet Dr. Wu until in 1987 when I returned to Taiwan on a permanent basis. Soon after I returned, Dr. Edward Yen at TsingHua took me to see Dr. Wu at
his residence. Edward told me how easy it would be to get along with Dr. Wu should I refrain myself to have conversations only on physics-related topics. After all these years, I still follow Edward's advice and really enjoy and value the various encounters with Dr. Wu—quite often on a daily basis.

Indeed, I was so struck by the vivid memory of Dr. Wu when he spent almost an afternoon trying to explain his lecture notes on relativistic quantum mechanics. He went through the 200-page note in great detail and many times he took out from the book shelf other books in order to elucidate the subtle differences between what is in his note and what the other people had said—he is often quite blunt in pointing out what has been said by the others is in fact flawed. This was when I agreed to Dr. Wu to prepare the second half of the book on \textit{Relativistic Quantum Mechanics and Quantum Fields} (World Scientific, Singapore, 1991). It took me almost three years in finishing up the book project and if I try to do it again I would still think that I could do a better job. Nevertheless, these years also taught me how to treasure my friendship with Dr. Wu. As I said, Dr. Wu does not hesitate to point out the mistake which the other people make, in physics as well as in politics. One important reason for it, if I interpret it correctly, has to do with Dr. Wu's unusually deep appreciation of the knowledge and of the life. It is always a great pleasure to chat with Dr. Wu on physics-related topics because of his deep appreciation of the subject.

It is therefore quite natural that Dr. Wu is always so serious towards what he is going to teach in a class. He does not compromise on what he believes is correct. Over the last five years, I was the one who helps to arrange to have Dr. Wu to lecture Taida's students. I had witnessed again and again how Dr. Wu spent an unusual amount of time preparing his lecture notes, sometimes months before the class began and often all the way till the end of the semester. Unfortunately, a genuine scientist and such an insightful teacher no longer receives due respect (and appreciation) from the present-day chaotic society. Nevertheless, the fact that we have video-taped Dr. Wu's lectures during the last two years and will make it available for teaching and learning in the future could be regarded as a minimal gesture for paying our tribute—we should never let the real good thing lost in nowhere in human history.

My research career during the year right after I left Indiana University (in 1986) underwent serious self re-organization. When I was sitting at a theorist's office at Indiana University Cyclotron Facility (IUCF), there was little serious thought concerning urgent problems in nuclear physics (except perhaps those closely related to IUCF). During the summer of 1991 when I just finished the book project with Dr. Wu, I had the opportunity to visit Jülich as a Humboldt fellow. I sort of ventured into an unknown territory (certainly a previously unknown place to me). Upon some discussions with J. Speth (my host) and G. E. Brown (a frequent summer visitor there), I set out to try to understand sea quark dis-
tributions in the quark parton model of Feynman. Surprisingly enough, my first journey to Germany set the stage for me to regain my strength as a theorist in the area of intermediate energy physics and I started paying serious attention to avoid the trap of being trapped in some well-capped topics, such as CEBAF physics or RHIC physics. After all, the truth must be universal—a well-known Chinese proverb, but most of us are inescapably the prisoners of such trap of some kind. Although the progresses which we have made over the last several years and which I am going to describe in what follows might still deserve more of due appreciation from the community, I seriously think that we are on the right track in terms of working on the urgent problems in the area of medium and high energy nuclear physics.

II. TOWARDS UNDERSTANDING THE QUARK PARTON MODEL

The basic fact that pions couple strongly to nucleons has dominated various nuclear physics thinkings since the birth of the field more than sixty years ago. The parton model of Feynman, in which the structure of a nucleon (or a hadron) is characterized by a set of parton distributions, was proposed in late 1960’s to treat high energy deep inelastic scattering (DIS). Introduction of the concept of parton distributions signifies the departure of particle physics from nuclear physics, as the various reactions in particle physics are phrased using this new language whenever it applies. Nevertheless, Sullivan [1] observed in 1972 that, in DIS of a nucleon by leptons, the process in which the virtual photon strikes the pion emitted by the nucleon and smashes the pion into debris scales like the original process where the virtual photon strikes and smashes the nucleon itself. In other words, the process will contribute by a finite amount to cross sections in the Bjorken limit, \( Q^2 \rightarrow \infty \) and \( \nu \rightarrow \infty \) with \( x \equiv Q^2/(2m_N \nu) \) held fixed. This in turn implies that a certain part of the sea quark distributions in a nucleon may be due to the meson cloud surrounding the nucleon. It is therefore of interest to note that, through the Sullivan process, the concept of parton distributions is after all not completely independent of nuclear physics aspects.

Thomas [2], in 1983, and, later, Frankfurt et al. [3], and Kumano [4] employed the original Sullivan process to study the difference \( \frac{1}{2}(\bar{u} + \bar{d}) = \bar{s} \) in order to set a limit on the size of a nucleon or the \( \pi N \) form factor. By considering generalized Sullivan processes that involve lowlying mesons and baryons, Hwang, Speth, and Brown showed that the original argument of Thomas no longer gives rise to a limit on the nucleon size, nor on the \( \pi N \) or \( \pi N \Delta \) form factor. They observed that, through generalized Sullivan processes with the \( \pi N \) form factor in the range of about 1 GeV (when parametrized in a monopole form), the meson-baryon picture in fact provides a reasonable account for the observed sea quarks in a nucleon at low and moderate \( Q^2 \). Not only the strengths of the various sea distributions, but also their shapes in the \( x \) region that by far is relevant experimentally.
are reproduced [5] using the meson-baryon picture.

II-1. Violation of the Gottfried Sum Rule

The idea of linking the original Sullivan processes to the apparent violation of the Gottfried sum rule [6], as observed by the New Muon Collaboration (NMC) [7], has been considered independently (and almost simultaneously) by several authors: by Henley and Miller [8], by Kumano [4], by Signal, Schreiber, and Thomas [9], and by us [5]. The Gottfried sum rule (GSR) [6] has to do with the integral:

\[ I_G \equiv \int_0^1 \frac{dz}{z} \left\{ F_2^p(x) - F_2^n(x) \right\}, \tag{1} \]

which is, in the context of the quark parton model,

\[ I_G = \frac{1}{3} \int_0^1 \left\{ u^p(x) - d^p(x) + \bar{u}^p(x) - \bar{d}^p(x) \right\} dx \]

\[ = \frac{1}{3} \int_0^1 \left\{ u^n(x) - d^n(x) \right\} dx + \frac{2}{3} \int_0^1 \left\{ \bar{u}^p(x) - \bar{d}^p(x) \right\} dx, \tag{2} \]

where \( u^p(x) \) is the up quark distribution in the proton and similarly for the other quark distributions \( d^p(x), u^n(x), \) and \( \bar{d}^p(x) \). In obtaining Eq. (2), we have used isospin invariance in relating the distributions in a neutron to those in a proton, i.e., \( u^n(x) = d^p(x), d^n(x) = u^p(x), s^n(x) = s^p(x), \) etc., in order to obtain an expression that involves only parton distributions in a proton. We have also adopted the 'valence' definition \( u^p(x) \equiv u^p(x) - \bar{u}^p(x) \) and a similar definition for \( d^p(x) \).

Using identical input parameters adopted earlier [5] (in the dipole scenario with the valence distributions taken from the EHLQ parametrization [10]), we obtained [11]

\[ \int_{0.002}^1 \frac{dz}{z} \left\{ F_2^p(x) - F_2^n(x) \right\} = 0.251. \tag{3} \]

In Fig. 1, we plot the structure function difference \( F_2^p(x) - F_2^n(x) \) as a function of \( x \). We plot separately the two contributions to the structure function difference \( F_2^p(x) - F_2^n(x) \) as a function of \( x \) — the dotted curve from the valence contribution \( \frac{1}{2}(u_v(x) - d_v(x)) \) and the dashed curve from the calculated sea contribution \( \frac{1}{2}(\bar{u}(x) - \bar{d}(x)) \).

We should emphasize that, despite the fact that the integrated value as listed in Eq. (3) may come close to the data, it is nontrivial to also reproduce the experimental data as a function of \( x \). The curves shown in Fig. 1 reflect directly the shape of the proposed valence distribution convoluted according to Sullivan processes. The general agreement may be taken as an additional evidence towards the suggestion that the sea distributions of a hadron, at low and moderate \( Q^2 \) (at least up to a few GeV\(^2\)), may be attributed primarily to generalized Sullivan processes.
The structure function difference $F_2^p(z) - F_2^n(z)$ as a function of $z$ in the case of EHLQ [10] is decomposed into two contributions, the dotted curve from the valence contribution $\frac{1}{2}(u_v(x) - d_v(x))$ and the dashed curve from the calculated sea distribution $\frac{3}{2}(\bar{u}(x) - \bar{d}(x))$.

11.2. Proton Spin Crisis

In 1989, the European Muon Collaboration (EMC) [12] measured the proton spin structure function $g_1(x,Q^2)$ using the scattering of polarized electrons on a polarized proton target. They obtained the first moment of $g_1^p(x,Q^2)$ at $(Q^2) = 10.7$ GeV$^2$,

$$\Gamma_1^p : \int_{0}^{1} g_1^p(x) \, dx = 0.126 \pm 0.010 \pm 0.015,$$  \hspace{1cm} (4)

where we have, in the naive parton model,

$$\Gamma_1^p = \frac{1}{2} \left( \frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right),$$  \hspace{1cm} (5)

with

$$Aq \equiv \int_{0}^{1} \left[ (q \uparrow (x) + \bar{q} \uparrow (x)) - (q \downarrow (x) + \bar{q} \downarrow (x)) \right] \, dx, \quad q = u, d, s.$$  \hspace{1cm} (6)

Note that, recently in 1993, the Spin Muon Collaboration (SMC) [13] measured the spin structure function $g_1^d(x,Q^2)$ for the deuteron.
while the most recent SLAC data [14] on the neutron spin structure function $g_1^n(x, Q^2 = 2 \text{ GeV}^2)$ yield

$$\Gamma_1^n = -0.022 \pm 0.011.$$  \hfill (8)

The results as from the generalized Sullivan processes at $Q^2 = 10.7 \text{ GeV}^2$ are listed in Table I, where we have used the notation of [15] in that $\Gamma_{\text{bare}}^1$ and $\Gamma_{\text{Sullivan}}^1$ represent the bare and full calculations. In Fig. 2, we plot the predicted proton and neutron spin structure functions $zg_1^p(x, Q^2)$ and $zg_1^n(x, Q^2)$ at $Q^2 = 10.7 \text{ GeV}^2$ as a function of $z$, using the parametrization of EHLQ [10] as the input for $u_v(z)$ and $d_v(z)$. Note that the two experiments EMC [12] and SLAC E42 [14], when combined together, are not consistent with the Bjorken sum rule [16]. The level of uncertainty in obtaining our present numerical predictions prevent us from deciding which of the two experiments should be checked further.

11-3. Further Experimental Tests

To offer further tests of the conjecture, we have investigated the extent to which Drell-Yan processes may be employed [17]. The numerical results indicate that the measurement of the p/N (proton-to-nucleon, i.e. normalized proton-to-deuteron) Drell-Yan production cross section ratio is a suitable choice for unraveling the $\bar{d}$ over $\bar{u}$ asymmetry in a proton – such asymmetry is suggested by the observed violation of the Gottfried sum rule.

<table>
<thead>
<tr>
<th>Table I. The calculated first moments of $g_1(x, Q^2)$ at $Q^2 = 10.7 \text{ GeV}^2$ for the proton and neutron, using the parametrization of EHLQ [10] as the input for $u_v(z, Q^2)$ and $d_v(z, Q^2)$. Note that $\Gamma_{\text{bare}}^1$ and $\Gamma_{\text{Sullivan}}^1$ are defined in [15].</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{\text{bare}}^1$</td>
</tr>
<tr>
<td>Proton</td>
</tr>
<tr>
<td>Neutron</td>
</tr>
</tbody>
</table>
FIG. 2. The predicted proton and neutron spin structure function $xg_1^p(x, Q^2)$ and $xg_1^n(x, Q^2)$ at $Q^2 = 10.7$ GeV$^2$ plotted as a function of $x$, using the parametrization of EHLQ [10] as the input for $u_1(x)$ and $d_1(x)$. The data points are from EMC [12], as well as from the SLAC E142 with $Q^2 = 2$ GeV$^2$ [14].

In addition, we have also investigated semi-inclusive $\Lambda$ production with a high energy electron or muon beam [18]. The numerical results suggest that, in the kinematic region characterized typically by small $x$, large $z$, and a small specific range of $y$, the contribution from Sullivan processes is of numerical importance as compared to that due to the fragmentation process. We thus believe that Sullivan processes in general affect extraction of the fragmentation functions from the data.

III. QCD SUM RULES

It is widely accepted that quantum chromodynamics (QCD), an $SU(3)$ gauge theory constructed out of the exact color $SU(3)$ symmetry, describes strong interactions among quarks, antiquarks, and gluons. At high energies (i.e., large momentum transfer squared, $Q^2 >> 1$ GeV$^2$), QCD is asymptotically free, allowing perturbative treatment of physical processes involving hadrons. At low energies (i.e., $Q^2 \leq 1$ GeV$^2$), the nonperturbative physics dominates such that the physical ground state (i.e., the physical vacuum) differs in general from the trivial ground state (i.e., the trivial vacuum in which all field variables vanish identically). Indeed, hadrons, including baryons, mesons, and glueballs, all of which
are color-singlet objects consisting of quarks, antiquarks, and gluons, act as the effective degrees of freedom in hadron physics (strong interaction physics).

The study of hadron physics is essential for progresses in many research areas, such as nuclear physics, particle physics, collapse of heavy stars, and the hadron era or nucleosynthesis in the early universe. Our understanding of the structure of a nucleon, a proton or neutron, is very much deficient as compared to the structure of a hydrogen atom in atomic physics (where the nucleus can be treated to a very good approximation as a pointlike object). Yet, the behavior of a nucleon may vary from one environment to another. The nonperturbative feature of QCD has thus defied many attempts to find solutions to strong interaction problems, including the problem of the nucleon structure. To achieve desired accuracy in lattice QCD simulations, on the one hand, an incredible amount of computing power is needed. To model nucleons via a quark model of some sort, on the other hand, we often lose the linkage between the hadron structure and QCD. Therefore, the method of QCD sum rules, as originally proposed by Shifman, Vainshtein, and Zakharov [19], may well be the only efficient method, available to date, for extracting consequences of QCD in relation to hadron physics.

In short, the method of QCD sum rules is based on the idea of finding a $Q^2$ region ($\approx 1 \text{ GeV}^2$) where one may incorporate nonperturbative physics, through Wilson's operator product expansion, into the perturbative QCD treatment of physical processes involving hadrons. The ansatz is, on the one hand, to replace, in the evaluation of a certain correlation function (or Green's function) $\Pi(p)$, the free quark (or gluon) propagator by the one suitable in the case of the nontrivial vacuum while, on the other hand, to express, via dispersion relations, the same correlation function in terms of the variables in the meson-baryon picture. The result at the quark level is then equated with that obtained in the meson-baryon picture, yielding sum rules which allow for determination of the variables adopted in the meson-baryon picture.

### III-1. The Nucleon Axial Coupling Constants

The magnitude of the isovector axial vector coupling constant, $g_A^{(1)}$ or abbreviated simply as $g_A$, has long been of considerable interest. The prediction of the Goldberger-Treiman relation [20] that for a free nucleon $g_A = 1.25$, consistent with its experimental value, is a remarkable achievement of the theory of hadronic strong interactions. At momentum transfer $q^2 = 0$, this relation is $M_N g_A = f_p g_{\pi NN}$, so that the axial coupling constant is determined by the pion decay constant, $f_\pi$, the pion-nucleon coupling constant, $g_{\pi NN}$, and the nucleon mass, $M_N$.

It has also been of considerable interest to determine $g_A$ from the theories of electroweak interactions and QCD. Although it is not possible to use the QCD lagrangian...
directly for this purpose, since this would require the solution to the problem of determining the structure of a nucleon and the low-energy interaction of a nucleon with a gauge boson, it has been possible to use the methods of QCD sum rules [19] to determine the value of $g_A$. Following the method of introducing an external electromagnetic field [21] for the study of nucleon magnetic moments, studies of a nucleon in the presence of an external axial-vector field, have been carried out within the framework [22-24] of QCD sum rules, with results consistent with the experimental value of $g_A = 1.26$.

On the other hand, the polarized structure function of the proton $g_1^p(x)$ has been measured recently by the European Muon Collaboration (EMC) [12] for the Bjorken variable $x$ down to $x \approx 0.01$, making possible extraction of the first moment $\int_0^1 dx g_1^p(x)$ which turns out to be considerably smaller than the Ellis-Jaffe sum rule [25]. When combined with the known values for $g_A$ and the $F/D$ ratio [26], the EMC data yield a value for the isoscalar axial coupling, $g_A^{(0)}$ or simply $g_A^2$, which is unexpectedly small, i.e. $g_A^2 \approx 0.28 \pm 0.08$. In the context of the QCD sum rule approach, Belyaev, Ioffe, and Kogan [21'] were able, a few years before the EMC data came about, to predict $g_A^2 \approx 0.5$, a value already significantly below the naive value $g_A^2 \approx 1$. A slightly smaller value $g_A^2 \approx 0.35$ was obtained by Gupta, Murthy, and Pasupathy [28] using again the QCD sum rule approach (soon after the EMC data came out). With corrections to some numerical coefficients in the sum rule and with the addition of terms consistently up to dimension eight, we [24] have also re-investigated the problem of $g_A^2$ and obtained $g_A^2 = 0.13 \pm 0.08$ significantly lower than the previously reported theoretical values [27,28] but in approximate agreement with the EMC data.

Although the method of QCD sum rules as originally developed [19] was applied to the study of hadronic properties in the region of about 1 GeV, Ioffe and Smilga [21] developed techniques for embedding hadrons in an external field in order to derive static properties in terms of the condensates, including induced condensates which introduce new parameters. In Refs. [22] and [23] the method was applied to an external static axial-vector field. We briefly review the method here for an external axial field.

The starting point is the polarization function in an external axial field, which we call $Z_\mu$. The correlation operator, $\Pi(p)$, is defined as [21-24]

$$\Pi(p) \equiv i \int d^4x e^{ip\cdot x} \langle 0|T[\eta(x)\bar{\eta}(0)]|0\rangle,$$

where for the nucleon current we use a standard (but not unique) form [29]

$$\eta(x) = e^{abc} \{ u^a(x)^T C \gamma_\mu u^b(x) \} \gamma^\mu \gamma_5 \bar{d}(x),$$

$$\langle 0|\eta(0),N(p)\rangle \equiv \lambda_{N} v_{N}(p),$$

with $C$ the charge conjugation operator, $a$, $b$, $c$ color indices, and $v_N(p)$ the nucleon spinor normalized such that $\bar{v} v = 2 M_N$. Embedding the system in an external $2_i$ field and
introducing intermediate states one can express the polarization operator in the limit of a constant external field, $Z_\omega(x) = Z_\omega$, as [23,24]

$$\Pi(p) = -|\lambda_N|^2 \frac{1}{p - M_N} - |\lambda_N|^2 \frac{1}{p - M_N} g_A \vec{Z} \gamma_5 \frac{1}{p - M_N} + \cdots,$$

(11)

if we adopt the on-shell definition of the nucleon axial form factor:

$$\langle N(p', \lambda') | J^a_\mu(0) | N(p, \lambda) \rangle = \hat{u}_\lambda(p') \langle g_A(q^2) \gamma_\mu \gamma_5 + g_\rho(q^2) q_\mu \gamma_5 \rangle u_\lambda(p),$$

(12)

with $q_\mu \equiv p_\mu' - p_\mu$ and $\gamma_\mu \equiv \gamma_\mu \gamma^\mu$. The term shown in Eq. (11) corresponds to nucleon intermediate states, while the continuum contributions to $\Pi$ are also implied. The axial coupling constant $g_A$ in Eq. (11) is defined at $q^2 = 0$. Eq. (11) is the expression for the phenomenological form, in which $\Pi(p)$ is evaluated at the baryon level. When evaluating the polarization operator $\Pi(p)$ at the quark level and comparing it with Eq. (11), one is led to three sum rules involving $g_A$, which [23,24] may not be consistent among themselves although there is indeed one sum rule which seems most appropriate for $g_A$.

We now evaluate the polarization function at the quark level by evaluating $\Pi(p)$ via the quark propagators in the presence of gluonic and $Z$ fields. The starting point is the quark propagator,

$$iS^{ab}_q \equiv \langle 0 | T(q^a(x) q^b_\mu(0)) | 0 \rangle.$$

Following the method of Ref. [21], including terms up to the second order in the Taylor expansion, we find

$$iS^{ab} = \frac{\delta^{ab}}{2\pi^2 x^4} \left( i \vec{z} - g_\xi \cdot \vec{Z} \gamma_5 \right) + \frac{1}{32\pi^2 x^2} \theta^{\lambda\mu} \frac{\lambda^\mu \gamma^\lambda}{2} C^{\gamma}_\mu (\hat{z} \sigma^\mu \gamma_5 + \sigma^\mu \hat{z})$$

$$+ i \langle \bar{q} q \rangle \left\{ - \frac{1}{12} \left( 1 + \frac{1}{10} x^2 m_0^2 \right) + \frac{1}{12} g \chi \vec{Z} \gamma_5 + \frac{1}{216} g \nu \left( \frac{5}{2} x^2 \vec{Z} - \vec{x} \cdot \vec{Z} \right) \gamma_5 \right\} + \cdots,$$

(13)

The first three terms in Eq. (13) are the perturbative free quark propagator, and the quark propagator with a $Z$ and a $g_{\text{gluon}}$, depicted in Figs. 3a-c. The next five nonperturbative terms, proportional to $\langle \bar{q} q \rangle$, are the quark condensate and this same condensate in the presence of gluonic and external $Z$ fields, depicted in the five diagrams of Figs. 3d-h. The other quantities appearing in Eq. (13) are the $Z$-quark coupling constant ($g = g_n - g_d$ for the isovector axial coupling $g_A$ or $g = g_n + g_d$ for the isoscalar axial coupling $g_A^S$) and the condensate parameters defined by
We have expressed the quark propagator (13) as a power series in $\tau$, with the first term derived from the Fourier transform of $i/(p - m)$ (the free propagator in momentum space) in the limit of $m = 0$ and small $x_\perp$. This is an example of Wilson's short-distance operator product expansion (OPE), through which we have incorporated terms from quark condensates (and, at a later stage, gluon condensate as well). On the derivation, we refer an interested reader to Ref. [30]. The fact that it is a short-distance expansion suggests that it is valid for the $Q^2$ which is large enough.

In addition to the quark and gluon condensates, one has the parameter $m_0^2$ and the two susceptibilities $\kappa$ and $\chi$. Numerically, we find that there is only a very weak dependence on $\kappa$ and $\chi$. The parameter $m_0^2$ does enter the mass sum rule, which we use to derive the $g_A - 1$ sum rule.

$$
\begin{align*}
(0|\bar{q}\gamma_\mu q\gamma_\nu G_{\mu\nu}|0) &= -m_0^2\langle \bar{q}q \rangle, \\
(0|\bar{q}\gamma_\mu \tilde{G}_{\mu\nu} \gamma_\nu q|0) &= g\kappa Z_\mu \langle \bar{q}q \rangle, \\
(0|\bar{q}\gamma_\mu \gamma_\nu sq|0) &= g\chi Z_\mu \langle \bar{q}q \rangle.
\end{align*}
$$

FIG. 3. Diagrammatic representation of the quark propagator as given by Eq. (13).
As in Refs. [22] and [23], we find it most useful to use the sum rule which is derived with the Borel transformation [19] of the coefficients of the covariant $p \cdot Z\hat{p}\gamma_5$ after the Fourier transform of both the phenomenological and the quark-level forms of the polarization function $\Pi(p)$. The processes which enter the calculation are shown in Figs. 4a-h. These diagrams can readily be evaluated by using the relationship

$$i\Pi(p) = \int d^4 x e^{ip \cdot x} e^{i\delta \gamma} e^{i\gamma_5} T \{ iS(x)\gamma_\nu C iS(x)\gamma_\mu \}$$

$$\gamma_\nu S(x)\gamma_\mu$$

(15)

Note that Figs. 4b and 4h are evaluated with the aid of the identity for the gluon condensate:

$$(g^2 G_{\mu\nu} G^m_{\alpha\beta}) = \frac{\delta^{nm}}{96} (g_{\rho\sigma} g_{\mu\beta} - g_{\rho\sigma} g_{\mu\alpha}) (g^2 G^2)$$

(16)

This is the place where we begin to incorporate effects due to gluon condensate. On the other hand, Fig. 4f is evaluated using the relation:

$$(g^2 G_{\mu\nu} G^m_{\alpha\beta}) = \frac{\delta^{nm}}{96} (\gamma_5 \sigma_{\mu\nu} + \sigma_{\mu\nu} \gamma_5) \gamma_\beta \gamma_5$$

(17)

with $\gamma_5 \gamma_\beta \gamma_5$. Note that, in Eqs. (16) and (17), all the field operators are evaluated at $x = 0$.

In addition to terms included in Refs. [22,23,27], and [28], we have in Ref. [24] added Fig. 4h and others so that contributions are included consistently up to dimension eight ($d = 8$). Note that Figs. 4a-h enter the sum rules when the coefficients of $p \cdot Z\hat{p}\gamma_5$ and $\hat{Z}\gamma_5$ are compared.

After taking into account the continuum contribution through a perturbative QCD method [27,29] (which describe contributions from the excited states on the r.h.s. in a very efficient manner), we find for the $g_A$ sum rule:

$$\frac{M_B^4 E_2}{8 L^{1/2}} + \frac{1}{32 L^{1/2}} (g^2 G^2) E_0 - \frac{1}{18 L^{3/2}} \kappa_0 E_0 + \frac{5}{18 M_B^4} a^2 L^{1/2}$$

$$+ \frac{5}{288 L^{1/2} M_B^4} \kappa_0 (g^2 G^2)$$

$$= \beta_5^2 \left( \frac{g_A}{M_B} + A \right) \exp(-M_B^2/M_B^2)$$

(18)

where $a = -(2\pi)^2 \langle q \bar{q} \rangle$ and $L = 0.621 \ln(10 M_B)$, corresponding to $\Lambda_{QCD} = 0.1$ GeV with the Borel mass, $M_B$, in GeV and $\beta_5^2 \equiv (2\pi)^4 \lambda_B^2 / 4$. The most important terms on the left hand side are the first term and that proportional to Figs. 4(a) and 4(c) and (g), respectively.

To improve further the $Q^2$ range of the validity of the derived QCD sum rules, it is useful to incorporate the $Q^2$ dependence of the various terms using the renormalization
FIG. 4. Processes included in the polarization function leading to the sum rule of Eqs. (18) and (20).

group (RG) equation. In particular, it is useful to multiply each term in the operator product expansion by a coefficient \( \left[ \frac{\ln \frac{Q^2}{\Lambda^2}}{\ln \frac{\mu^2}{\Lambda^2}} \right]^{-2\gamma_\eta+\gamma_\Delta} \), where \( \mu \) is the renormalization point taken to be 0.5 GeV, \( \Lambda \) is the QCD scale parameter taken to be 0.1 GeV, \( \gamma_\eta \) is the anomalous dimension of the current \( \eta \), and \( \gamma_\Delta \) is the anomalous dimension of the operator under
consideration $O_n$. (Note that, after the Borel transformation, the dependence on $Q^2$ is translated into a dependence on the Borel mass, $M_B$.) Here the anomalous dimensions may be taken from the literature [31-33,19].

$$\eta(x) : \frac{2}{9}$$
$$\bar{q}q : \frac{4}{9}$$
$$\alpha_s G_{\mu\nu}^a G^{a\mu\nu} : 0$$
$$m_1 : \frac{4}{9}$$
$$\bar{q}g \cdot Gq : 0.$$ (19)

The factors $E_0 = 1 - e^{-x}$, $E_1 = 1 - (1 + x)e^{-x}$, and $E_2 = 1 - (1 + x + \frac{1}{2} x^2)e^{-x}$, with $x \equiv W^2/M_B^2 = (2.3 \text{ GeV}^2)/M_B^2$ (see Ref. [21]) are used to correct the sum rule to obtain consistent $M_B^2$ dependence for contributions from excited states through perturbative QCD techniques [27,31]. They also serve to restrict the range of the integration and increase the weight given to the nucleon. We have thus made the usual assumption in Eq. (18). The constant $A$ is introduced to represent the residual continuum contribution to the dispersion integral. Note that only the standard quark and gluon condensates and the susceptibilities $\kappa$ and $\chi$ enter, and that the latter is numerically small.

On the same footing, we may obtain the sum rule for $g_A^S$

$$-\frac{M_B^4 E_2}{8L^{4/9}} + \frac{1}{32L^{4/9}}(g_A^2 G^2)E_0 + \frac{1}{6L^{4/9}} \chi a M_B^2 E_1 - \frac{1}{18L^{8/9}} \kappa a E_0$$
$$- \frac{1}{18M_B^2} \alpha L^{4/9} + \frac{1}{288L^{4/9} M_B^2} \chi a (g_A^2 G^2)$$
$$= \beta^S \left( \frac{g_A^S}{M_B^2} + A^S \right) \exp(-M_B^2/M_B^2).$$ (20)

This is the sum rule for the "isoscalar" axial coupling constant $g_A^S$. It is assumed that the susceptibilities and $W^2$ are identical to those for the isovector case. This assumption can be investigated, but we adopt here for simplicity. The most important terms on the left hand side are the first term and that proportional to $\chi a M_B^2$, corresponding to Figs. 4(a) and 4 (d-1) and (d-2), respectively. Note that the susceptibility $\chi$ is very important in the sum rule for $g_A^S$ but only makes a small correction to $g_A$.

We may obtain a sum rule for $(g_A - 1)$ making use of a Belyaev-Ioffe sum rule [27] for the nucleon mass:

$$-\frac{M_B^4 E_2}{8L^{4/9}} + \frac{2M_B^4}{32L^{4/9}}(g_A^2 G^2)E_0 + \frac{1}{6} \alpha^2 L^{4/9} - \frac{1}{24M_B^2} \alpha^2 m_0$$
$$= \beta^S \exp(-M_B^2/M_B^2).$$ (21)
Note that the first two terms in the left-hand side of the two sum rules, Eqs. (18) and (20), are equal. By subtracting Eq. (20) from Eq. (18), one obtains a sum rule involving the condensates $a$, $m_0^2$, and the susceptibilities $\kappa$ and $X$. These parameters have been estimated to be \[ a \approx 0.55 \text{ GeV}^3, \]
\[ \kappa a \approx 0.140 \text{ GeV}^4, \]
\[ Xa \approx 0.70 \text{ GeV}^2. \]
\[ (g_2^2 G^2) \approx 0.47 \text{ GeV}^4, \]
\[ m_0^2 \approx 0.8 \text{ GeV}^2. \]

Because $\kappa$ is less well known than the other condensates, we also consider $\kappa a \approx -0.140 \text{ GeV}^4$ in order to estimate (roughly) the error of the sum rule method. The parameter $\beta^2_{\kappa}$ has been determined through the mass sum rule to be $\beta^2_{\kappa} \approx 0.26 \text{ GeV}^6$. In Eq. (22) we use the standard value $\beta_{\kappa}$ of the quark condensate. Subtracting Eq. (20) from Eq. (18), we obtain a sum rule very similar to one obtained by Belyaev and Kogan:
\[ \frac{1}{2} a^2 L^{4/9} + \frac{1}{24} \frac{a^2 m_0^2}{M_B^2} - \frac{1}{18} \frac{\kappa a M_B^4}{L^{68/81}} E_0 + \frac{1}{288 L^{5/9}} \kappa a (g_2^2 G^2) \]
\[ = \beta^2_{\kappa} ((g_A - 1) + A M_B^2) \exp(-M_B^2/M_0^2). \]

Equations (23a) and (23b) are our main result for the nucleon axial couplings. It is clear from Eqs. (22), (23a) that, for the isovector axial coupling $g_A$, the quark condensate, represented by $a$, dominates and that the induced condensate (proportional to the susceptibilities $X$ and $\kappa$) are not important. This is not so for the “isoscalar” $(g_2^2)$ sum rule, and it causes greater uncertainty in our results for this quantity.

In our numerical analysis, after moving the factor $e^{-M_B^2/M_0^2}$ to the l.h.s., we compare the l.h.s. to a straight-line approximation $C + D M_B^2$. In practice, for a given Borel mass $M_B$, we may determine the straight line which goes through the points $M_B \pm \delta M_B$ (with, say, $\delta M_B \approx 0.1 \text{ GeV}$) and then compare the values of the l.h.s. and r.h.s. of the sum rule at $M_B$. When both sides agree with the desired accuracy, the sum rule is said to hold to that accuracy and it allows for extraction of the constants $C$ and $D$. Numerically, we obtain
There are a number of points to note in understanding the significance of the result shown in Eqs. (24a) and (24b). First, the errors shown are based on variations in the parameter $A$ (used to represent the residual continuum contribution) and uncertainties in the quark susceptibility $\kappa$. This method yields an uncertainty of approximately 30% in $(g_A - 1)$. We have introduced other parametrizations of the continuum, such as those discussed in Ref. [23], with no significant alteration in our result. A most satisfactory aspect of our result is that we obtain a value of $g_A$ consistent with experiment with a value of the quark condensate parameter $\langle \overline{c}c \rangle$ which gives rise to the correct magnetic moments of nucleons [21]. In addition, the value for $g_A^S$ is also in approximate agreement with the EMC data. Note that the EMC data, together with an analysis of strange baryon decays, yields [12,26]

$$g_A^S = \Delta u + \Delta d = 0.28 \pm 0.08. \quad (25)$$

III-2. Future Prospects

The method of QCD sum rules has often been employed as a means of taking into account strong interaction effects in physical phenomena in which strong interactions, as described by quantum chromodynamics (QCD) among quarks and gluons, play an important role. In the above, we have used the determination of the isovector and isoscalar axial coupling constants, $g_A$ and $g_A^S$, via the method of QCD sum rules to illustrate the method. The determination of the neutron-proton mass difference from the method serves as another interesting example [34]. Our present efforts focus on the possibility of employing the method to treat nonleptonic weak interaction problems, including parity-violating $\pi NN$, $\rho NN$, and $\omega NN$ couplings, decays of heavy mesons or of heavy baryons, etc.

IV. SOLITON STARS AND QUARK STARS

Stable stellar configurations, i.e., the end products of stellar evolutions, are believed to be assemblages of electrically neutral objects. During the old days (before the present-day particle physics is universally accepted), electrons, protons, and neutrons (bound in stable light nuclei) are regarded as the fundamental building blocks of matter. Thus, dwarfs (stellar configurations consisting of hydrogen and/or helium atoms), neutron stars (consisting primarily of neutron matter, which is nuclear matter of a particular kind), and black holes (gravitationally collapsed physical systems) are commonly accepted as the only three possible end products for evolution of the various stellar objects.
In the standard model of the present-day particle physics, however, the building blocks of matter include:

1. quarks (and antiquarks) of six distinct flavors (up, down, strange, charm, bottom, and top) and of three possible colors;
2. leptons (electrons, muons, r-leptons, and associated neutrinos) and antileptons;
3. gauge bosons or mediators of fundamental forces (y, W±, Z0, and gluons); and
4. scalar fields (which may account for the origin of masses).

It is therefore natural to ask whether there exist stable stellar configurations other than dwarfs, neutron stars, and black holes.

The central density of a typical neutron star is often greater than several times the nuclear matter density \( \rho_{n.m.} (= 2 \times 10^{14} \text{gm/cm}^3) \). At such high density, neutron matter may fuse into quark matter consisting of up (u) and down (d) quarks. In light of the fact that the Fermi energies \( \varepsilon_d \) and \( \varepsilon_u \) for such quark matter are higher than the mass of the strange quark s, such (ud) quark matter in turn converts it into a (u&d)-symmetric quark matter (the "strange matter") via the weak reactions such as

\[
\begin{align*}
\text{u} + \text{d} &\rightarrow \text{u} + \text{s}, \\
\text{u} &\rightarrow \text{e}^+ + \nu_e, \\
\text{d} &\rightarrow \text{u} + \nu_e. 
\end{align*}
\]

It is sometime argued that transition into strange matter softens the equation of state, making it more likely for supernovae explosion to take place.

Should a quark star (i.e. a star consisting of quark matter of a certain kind) exist or a dense (neutron) star with a quark core exist, the structure of the star is not expected to differ dramatically from that of a neutron star, making it very difficult, if not impossible, to distinguish such star from a neutron star. On the other hand, the neutron star candidates (X-ray pulsars) may well be quark stars rather than neutron stars, unless a clever way to distinguish between the two scenarios can be found.

Scalar fields, or composites of Lorentz scalar in nature, are indispensable building blocks of matter, which may explain why objects have masses. In the present-day particle dynamics, such fields interact with the matter fields very strongly at very high temperature (e.g., greater than \( TeV \)) but such interaction becomes only very feeble at ordinary temperatures. It is therefore plausible to conjecture that scalar nuggets (likely of topological nature), produced in the very early universe, may expand and cool, becoming very weakly interacting with the rest of the universe. In other words, soliton stars synthesized in the early universe could well be the candidate for "cold dark matter".

T. D. Lee and his colleagues[35] were among the first ones who considered the possible existence of soliton stars.

IV-1. General Ideas

To study possible stellar configurations, we are interested in setting up the framework
to describe the behavior of a system of interacting scalar and fermion fields in curved space-time.

The curved space-time manifold may be characterized by the spherical coordinates $(t, \tau, \theta, \phi)$:

$$ds^2 = g_{\mu \nu} dx^\mu dx^\nu = -e^{2u} dt^2 + e^{2v} d\tau^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2).$$

(26)

For the sake of the present talk (in which we consider only stable spherical star solutions), we assume that both $u$ and $v$ are functions of the radial coordinate $r$. However, please note that one primary objective of this talk is to formulate the problem such that we could also study the time dependence (evolution) of quark stars, of soliton stars, or of solitonic quark stars.

The background field $X(x^\mu)$ has the same quantum numbers as the physical vacuum. It is Lorentz scalar in nature. It is described by the lagrangian density

$$\mathcal{L}_X = -\frac{1}{2} X^\mu X_\mu - U(X),$$

$$U(X) = B + \frac{a}{2} \chi^2 + \frac{b}{6} \chi^3 + \frac{c}{24} \chi^4, \quad B > 0.$$}

(27a, 27b)

Here $B$ is the bag constant. The potential $U(X)$ is chosen that it has the absolute minimum $U = 0$ at $X = X_\infty$ and another local minimum $U = B$ at $X = 0$. Such choice of the potential admits the (non-topological) soliton solutions which look like bubbles ("bags") with $X = X_0$ inside the bubble and $X = X_\infty$ outside.

To study the question of soliton stars; we may introduce, on top of the background scalar field $X$, the complex scalar fields $\phi(x^\mu)$,

$$\mathcal{L}_\phi = -\phi^\dagger \mu \phi_{\mu} - U_\phi(\phi^d \phi),$$

$$U_\phi(\phi^d \phi) = \mu^2 \phi^d \phi + \mu_1^2 \left( \frac{X}{X_\infty} \right)^2 \phi^d \phi,$$

$$\mu_1^2 \gg \mu^2 \geq 0.$$}

(28a, 28b, 28c)

Here the choice (28c) is used to ensure that the fields $\phi$ exist only inside a bubble as they get extremely heavy when they get outside.

In general, the fields $\phi$ may be endowed with some internal group structure, such as the case that they may transform like a doublet (or like other multiplet) under $SU(2)$ of the Glashow-Salam-Weinberg $SU(2) \times U(1)_{\text{em}}$ weak theory. This aspect makes it possible to create scalar nuggets (of nontrivial global topology in nature) during the very
early universe. Of course, the fields $\phi$ may also be gauged, yielding the so-called "Higgs mechanism" (on top of describing their interaction with gauge fields).

Quarks may be described as quantized Dirac fields in curved space-time manifold. One may introduce the space-time dependent $\gamma$ matrices:

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu},$$

$$\gamma_\mu = g_{\mu\nu} \gamma^\nu,$$

$$[\gamma_\mu, \Gamma_\nu] \equiv \gamma_\mu \Gamma_\nu - \Gamma_\nu \gamma_\mu = \frac{\partial \gamma_\mu}{\partial x^\nu} - \Gamma_{\mu \nu} \gamma_\nu,$$

$$\Gamma_{\mu \nu}^\alpha = \frac{1}{2} g^{\alpha \beta} \{g_{\beta \nu, \mu} + g_{\beta \mu, \nu} - g_{\mu \nu, \beta}\},$$

so that the lagrangian density for a Dirac field $\psi(x^\mu)$ may be given by

$$L_f = \frac{1}{2} \left( \frac{\partial \bar{\psi}}{\partial x^\mu} \gamma^\mu \psi - \bar{\psi} \gamma^\mu \frac{\partial \psi}{\partial x^\mu} \right) - \frac{1}{2} \bar{\psi} \left( \gamma^\mu \Gamma_\mu + \Gamma_\mu \gamma^\mu \right) \psi$$

$$- (m + f(x)) \bar{\psi} \psi,$$

$$f x_\infty >> m_p.$$ 

Here we have chosen the quark mass outside a bubble as $m + f x_\infty$ which is much greater than, e.g., the proton mass $m_p$, a typical hadron energy scale. Thus, again, quarks always choose to stay inside a bubble (b bag) in order to minimize the total energy of the system. (If such bubble does not exist, quarks will first dig a bubble and then stay inside it.)

To take into account Pauli exclusion principle for fermions, we may adopt the Thomas-Fermi approximation to treat contributions from fermion fields. In other words, the system at a given space-time point may be treated locally as an ideal Fermi gas. We may then employ the many-body technique to improve such approximation. The method enables us to compute the pressure $P_f$ due to fermion fields as a function of the fermion energy density $\rho_f, P_f = P_f(\rho_f)$.

It is a routine task to compute the Einstein tensor $G_{\mu\nu}$ from the given metric $g_{\mu\nu}$ and, on the other hand, to express the overall energy-momentum tensor $T_{\mu\nu}$ in terms of the various fields. To illustrate the issue, we consider the stable spherical star solutions and, to this end, we assume that both $\chi$ and $\phi$ depend only on the radial coordinate $r$. The $(tt)$-component of the energy-momentum tensor allows us to define the overall energy density $\rho(r)$:

$$\rho(r) = \rho_f + U(x) + U_\phi + e^{-2\nu} \left\{ \frac{1}{2} \left( \frac{\partial \chi}{\partial r} \right)^2 + \left( \frac{\partial \phi}{\partial r} \right) \left( \frac{\partial \phi}{\partial r} \right) \right\}.$$ 

$$f x_\infty >> m_p.$$
so that, if we define the overall mass within the radius \( r \),
\[
m(r) = \frac{r}{2}(1 - e^{-2\nu}),
\]
(32)

the \((tt)\)-component of the Einstein equation becomes
\[
\frac{dm(r)}{dr} = 4\pi r^2 \rho(r, m).
\]
(33)

Note that we may use Eq. (32) to eliminate \( \nu \) in favor of \( m \) so that the energy density \( \rho \) also depends on \( m(r) \).

Along the same line, we use the \((\tau\tau)\)-component to define the pressure density \( P(r, m) \):
\[
P(r, m) = P_f - U(\chi) - U_\phi + \left( \frac{2m}{\tau} \right) \left\{ \frac{1}{2} \left( \frac{\partial \chi}{\partial r} \right)^2 + \left( \frac{\partial \phi}{\partial r} \right) \left( \frac{\partial \phi}{\partial r} \right) \right\}.
\]
(34)

The \((\tau\tau)\)-component of the Einstein equation then yields
\[
\frac{du}{dr} = \frac{m + 4\pi r^3 P}{\tau (\tau - 2m)},
\]
(35)

In the interior of a star (excluding the surface region), the energy-momentum conservation, \( T^{\alpha\beta}_{;\beta} = 0 \), yields
\[
(\rho + P) \frac{du}{dr} = -\frac{dP}{dr}.
\]
(36)

We have, from Eqs. (35) and (36),
\[
\frac{dP}{dr} = -\frac{(\rho + P)(m + 4\pi r^3 P)}{\tau (\tau - 2m)},
\]
(37)

which is the generalized Tolman-Oppenheimer-Volkoff (TOV) equation.

In the exterior of a star, we have \( \rho = 0 \) and \( P = 0 \) so that the Schwarzschild metric
\[
ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2),
\]
(38)

is the solution.

### IV-2. Spherical Stable Solutions

By suitably generalizing the energy density \( \rho(r) \) and the pressure density \( P(r) \), we have shown that the stable spherical stars are governed by the standard set of equations: the mass equation, the generalized TOV equation, and the equation of state (EOS). For a given central energy density \( \rho_c \), this set of equations can easily be integrated out until \( P(\tau = R) = 0 \), which defines the surface of the star. The star mass \( M \) is given by \( M = m(\tau = R) \). This standard approach differs from what has been adopted by T. D. Lee and his colleagues [35].

Choosing suitable parameters for the potential \( U(\chi) \), we may adopt the approximate solution for \( \chi_0 \).
\begin{equation}
\chi(r) = \frac{1}{2} \left( 1 + \tanh \frac{m_X}{2} (r - R) \right) \chi_\infty, \tag{39a}
\end{equation}
\begin{equation}
m_X R \gg 1. \tag{39b}
\end{equation}

In the interior of the star, we have $X \approx 0$ so that
\begin{equation}
m_\phi^2 \approx \mu^2, \quad m_\psi \approx m. \tag{40a}
\end{equation}

In the exterior of a star, we have $X = X_e$. We may choose the couplings which yield
\begin{align*}
m_\phi^2 &= \mu^2 + \mu_1^2 \gg \mu^2, \\
m_\psi &= m + f \chi_\infty \gg m_p. \tag{40b}
\end{align*}

For pure quark stars, we consider only the $X$ field and the quark fields $(u, d, s, \text{and } c)$. The Thomas-Fermi approximation is used to generate the equation of state needed for the integrating-out. With slight modification, we may consider a neutron star with a quark core. The equation of state for neutron matter may be taken from the nuclear force studies. It is found that the volume energy plays a much more important role than the surface energy, so that results from our earlier studies [37] are reproduced almost quantitatively despite introduction of the surface energy from the outset.

In Figs. 5(a) and 5(b), we show that there may be two solutions for the solitonic quark star for a given central density $\rho_c$, as the surface pressure $P_s$ due to the $X$ field may have two intercepts with the pressure $P$ due to the (other) matter fields. For the critical central density $\rho_c^0 = 6.53 \times 10^{15} \text{g/cm}^3$, there is only one solution; for $\rho_c < \rho_c^0$, there is no solution. Specifically, we show in Fig. 5(a) the pressure $P$ as a function of the distance $r$ (from the center) for the two solutions of the solitonic quark stars and in Fig. 5(b) the star mass $M$ shown as a function of its central density $\rho_c$.

Note that the star solutions in the second branch, shown in a dashed curve in Fig. 5(b), are not stable, as a small perturbation to squeeze the star to a higher central density would make the system collapse indefinitely. Similarly, those stars in the first branch, which has a central density beyond the critical density corresponding to the first maximum star mass (the Chandrasekhar limit [38]) but below the valley region, are also not stable. The stable star with its central density below the Chandrasekhar limit behaves very much like a neutron star but, due to the introduction of the surface energy, the Chandrasekhar limit becomes quite arbitrary, contrary to conventional neutron stars or quark stars. Note that adoption of the equation of state for quark matter also leads to a new stable region which consists of solitonic quark stars with a central density about $10^2\text{--}10^4$ times those in the first stable region. We believe that search of quark stars in this region could offer exciting new possibilities.
FIG. 5. There may be two solutions for the solitonic quark star for a given central density $\rho_c$, as the surface pressure $P_s$ due to the $\chi$ field may have two intercepts with the pressure $P$ due to the (other) matter fields.

With some modification, we may consider a neutron star with a quark core. The equation of state for neutron matter may be taken from the nuclear force studies. In Fig. 6, we show the star mass as a function of its central density $\rho_c$ for stars which consist of
strange matter and neutron matter (whichever is more stable). $\rho_{ph.t.}$ is the density which the phase transition from neutron matter into strange matter occurs. There is little doubt that the possibility for the existence of pure quark stars, or of neutron stars with a quark core, is significantly enhanced, should we allow the surface energy as an important factor for separating the two phases.

IV-3. Outlook

We have shown, by introducing quantized scalar and fermion fields in curved space-time, that generalized mass and Tolman-Oppenheimer-Volkoff (TOV) equations may be obtained as a means of incorporating scalar and fermionic matter fields (such as quark matter or neutron matter). This provides a convenient framework to treat the problem of solitonic quark stars, stars consisting of quark matter confined to a phase which is separated from the true ground-state phase (for, e.g., neutron matter) by both the volume energy and the surface energy. Our numerical results demonstrate that solitonic quark stars dominated by the volume energy reduce to quark stars in the conventional sense [5] while those dominated by the surface energy reduce to fermionic soliton stars obtained by T. D. Lee and his colleagues [1]. In addition, the Chandrasekhar limit [6] becomes quite arbitrary and the possibility for the existence of quark stars is significantly enhanced, should we allow the surface energy as an important factor for separating the two phases.

**FIG. 6.** The star mass versus its central density $\rho_c$ for stars which consist of strange matter and neutron matter (whichever is more stable). $\rho_{ph.t.}$ is the density which the phase transition from neutron matter into strange matter occurs.
ACKNOWLEDGMENTS

The author would like to acknowledge J. Speth (at Jülich) for collaborating on the project related to parton distributions (Sec. II), E. M. Henley (Seattle) and L. S. Kisslinger (Pittsburgh) on the project related to QCD sum rules (Sec. III), and H.-Y. Chiu (NASA) on solitonic quark stars (Sec. IV). These gentlemen, together with talented students at Taida, are the real source for my learning and growth over the last few years. This work was supported in part by the National Science Council of the Republic of China (NSC84-2112-M002-021Y).

REFERENCES


K.-C. Yang, W.-Y. P. Hwang, E. M. Henley, and L. S. Kisslinger, Phys. Rev. D47, 3001 (1993). Note that the two figures, Figs. 8 and 9, should be interchanged.


