Algebraic Structure and Analytic Solutions of a Moving V-type Three-Level Atom Interacting with a Two-Mode Cavity Field

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(Received April 2, 2012; Revised November 6, 2012)

In the framework of algebraic dynamics, the dynamics of a V-type three-level atom interacting with a two-mode of the quantized cavity field is considered with atomic center-of-mass motion included. The general analytical expressions for the linearized Hamiltonian and the reduced density operator are derived, which can be used to calculate the expectation values of any system operators. As an illustration, analytical expressions for the optical forces are obtained.

DOI: 10.6122/CJP.51.1241 PACS numbers: 42.50.Pq, 03.67.Mn, 42.50.Wk

I. INTRODUCTION

Studying a three-level atomic system under various configurations interacting with one- or two-mode quantized cavity fields is an active field of research. This is partly due to the facts that it describes the essential physics of the radiation-matter interaction and the enormous experimental progress in a series of interesting effects owing to the quantum interference between multiple atomic transition pathways, the flexibility resulting from the extended level structure, the large momentum changing from the enhanced atom-field interaction strength, and the large number of controlling parameters. Several studies have investigated the collapse and revival phenomena [1, 2], coherent population trapping [3, 4], laser cooling [5, 6], electromagnetically induced transparency [7, 8], chaos [9], quantum entanglement [10], statistical aspects [11], canonical transformations [12], etc. However, much earlier work was simply concentrated on the internal atomic transitions without regarding the external motion of the atom. As is well known, in a real experiment, the atoms are not stationary as are often assumed in the theoretical investigations. The consideration of the center-of-mass motion is crucial in studies of many fields, such as the laser cooling of atoms [13] and resonance fluorescence [14]. Recently, an interest has been developed to study the effects coming from the center-of-mass motion. For instance, the spatial motion of the atom inside the cavity in the resonant case can lead to a transition between topologically different solutions [15], the coupling of the center-of-mass motion to the intracavity field mode can be deleterious to nonclassical effects in photon statistics [16], and the interplay among the atomic center-of-mass motion along with the atomic collective spin and the cavity field will lead to a strong nonlinearity, resulting in multistable behavior [17]. These results motivate us to study the dynamics of the atom-cavity field system in detail.

Since the motion of the atom in a cavity field and inter-level transitions are connected to each other, the dynamics of the atom-cavity field system becomes much more
complicated. We enjoy a large number of theoretical methods for the study of dynamics in the three-level atom-cavity field system, which includes the Schrödinger equation method [11], the dressed state method [18], the Monte Carlo wave function method [19], etc. However these methods all have merits and defects. For example, using the dressed state method, the physical explanation of the stimulation radiation can be achieved, while there are some difficulties in finding a proper transformation for eliminating the complicated coupled terms of the Hamiltonian in a multi-level atom-cavity systems. The Monte Carlo wave function method is an effective way to treat this problem by employing wave functions, unfortunately, it cannot be used for a completely analytical solution. In order to obtain the analytical results, we shall employ an algebraic dynamical approach [20] to investigate the dynamics in the moving three-level atom-cavity field system. The key idea in this is to introduce a canonical transformation for the Hamiltonian and linearize it in terms of a set of Lie algebraic generators. As a result, linear algebraic dynamical systems are integrable, and thus solvable easily. Using the algebraic dynamics method, much work has been done. Ref. [21] discusses the dynamical behavior of entanglement, purity, and energy for two atoms in the presence of dissipation based on the algebraic dynamical method. Using the algebraic dynamical method, the entanglement dynamics of a moving multi-photon Jaynes-Cummings model in mixed states with and without a Kerr medium are both investigated in Ref. [22]. The laser cooling of a moving two-level atom coupled to a cavity field was also studied [23] by using the algebraic dynamical method. A Lie algebraic duality approach was used to analyze the nonequilibrium evolution of a closed dynamical systems and the thermodynamics of interacting quantum lattice models (formulated in terms of Hubbard-Stratonovich dynamical systems) [24]. With a combination of functional integral and Lie algebraic techniques, the entanglement dynamics of two independent oscillators coupled with their Markovian environment was investigated [25].

In the present paper, we shall investigate the dynamics of the V-type three-level atom-cavity system interacting with a two-mode quantized cavity field using the algebraic dynamical method with the center-of-mass motion included. According to algebraic dynamics, linear algebraic dynamical systems are integrable and thus solvable, and then their solutions are easy to handle. The analytical expressions for the linearized Hamiltonian and the reduced density operator are given, and analytical expressions for the optical forces are obtained as an example.

In Sec. II, we introduce the model and the dressed Hamiltonian is derived. Sec. III is devoted to giving the analytical solutions of the exact evolution. In Sec. IV, we study the optical forces, and the exact results of the optical forces are given. The main results are summarized in the conclusion.

II. THE EXACT TRANSFORMATION HAMILTONIAN

The scheme of the V-type moving three-level atomic system is depicted in Fig. 1. The two allowed transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ are mediated by photons of different modes of the cavity fields, which are characterized by the photon operators $a_1$ and $a_2$ with
corresponding frequencies $\omega_1$ and $\omega_2$, respectively. As the center-of-mass motion of the atom is included, the Hamiltonian for an atomic system interacting with the two-mode quantized cavity fields in the rotating approximation can be written as

$$
H = \frac{P^2}{2m} + \sum_{i=1}^{3} E_i \sigma_{ii} + \hbar \omega_1 a_1^\dagger a_1 + \hbar \omega_2 a_2^\dagger a_2
+ \hbar g_1 \left( \sigma_{13} a_1 e^{-i\Phi_1(R)} + h.c. \right) + \hbar g_2 \left( \sigma_{23} a_2 e^{-i\Phi_2(R)} + h.c. \right),
$$

where $\sigma_{ij}$ ($i, j = 1, 2, 3$) are the atomic level transition operators and $E_i$ ($i = 1, 2, 3$) is the energy of the three atomic levels. $P$ and $R$ are the atomic momentum and position operators, the coupling strengths $g_1$ and $g_2$ are assumed to be real. $\Phi_1$ and $\Phi_2$ are the effective phases operators of the two cavity modes. The Hamiltonian (1) treats all the atomic degrees of freedom quantum mechanically, so its solutions generally provide an accurate description of the atom’s dynamics. Of course, for simplicity, the model here is thus ideal and only applicable to a cavity system with rather small loss. Inclusion of cavity loss will result in increased complexity of the model; the dissipation due cavity loss is usually described by a non-Hermitian Hamiltonian which is difficult to deal with. Based on an algebraic dynamical analysis, besides the Hamiltonian itself, the system possesses three invariants

$$
N_1 = a_1^\dagger a_1 + \sigma_{11}, \quad N_2 = a_2^\dagger a_2 + \sigma_{22}, \quad P_i = P - \hbar k_1 \sigma_{11} - \hbar k_2 \sigma_{22}.
$$

$N_1$ and $N_2$ are the excitation number operators of the two cavity modes and $k_1$ and $k_2$ are their wavevectors. $P_i$ is the atomic momentum connected with the level $|3\rangle$. The kinetic energy operators of the atom can now be expressed as

$$
\frac{P^2}{2m} = \frac{1}{2m} \left( P_i + \hbar k_1 \sigma_{11} + \hbar k_2 \sigma_{22} \right)^2.
$$
We introduce the following generators by nonlinear transformations:

\[
A_{13}^r = N_1^{-1/2} \sigma_{13} a_1 e^{-i \Phi_1(R)}, \quad A_{23}^r = N_2^{-1/2} \sigma_{23} a_2 e^{-i \Phi_2(R)},
\]

\[
A_{12}^r = N_1^{-1/2} N_2^{-1/2} \sigma_{12} a_1 a_2 e^{-i[\Phi_1(R) - \Phi_2(R)]},
\]

\[
A_{ii}^r = \sigma_{ii}, \quad (A_{ij}^r)^\dagger = A_{ji}^r \quad (i = 1, 2, 3).
\]

One can verify that the above operators span the Lie algebra \( su(3) \) and \( N_1 (N_2) \) commute with \( A_{ij}^r \). In the irreducible eigenspace of operators \( N_1, N_2, \) and \( P_i \), they form an associative matrix algebra isomorphic to that spanned by the operators \( \sigma_{ij} \). We can now write the Hamiltonian (1) in terms of the operators \( A_{ij}^r \)

\[
H = E_0 (N_1, N_2, P_i) + \Delta_1 (P_i) A_{i1}^r + \Delta_2 (P_i) A_{i2}^r + \tilde{g}_1 (N_1) (A_{13}^r + A_{31}^r) + \tilde{g}_2 (N_2) (A_{23}^r + A_{32}^r),
\]

where

\[
E_0 (N_1, N_2, P_i) = E_3 + \hbar \omega_1 N_1 + \hbar \omega_2 N_2 + \frac{P_i^2}{2m},
\]

\[
\hbar \Delta_1 (P_i) = \frac{\hbar^2 k_1^2}{2m} + \frac{\hbar P_i \cdot k_1}{m} + E_1 - E_3 - \hbar \omega_1,
\]

\[
\hbar \Delta_2 (P_i) = \frac{\hbar^2 k_2^2}{2m} + \frac{\hbar P_i \cdot k_2}{m} + E_2 - E_3 - \hbar \omega_2,
\]

\[
\tilde{g}_1 (N_1) = \hbar \sqrt{N_1} g_1, \quad \tilde{g}_2 (N_2) = \hbar \sqrt{N_2} g_2.
\]

Since the operators \( N_1, N_2, \) and \( P_i \) can be treated as constants in their common eigenspace, the Hamiltonian (5) is thus a linearized version of the Hamiltonian (1) in terms of the \( su(3) \) generators \( A_{ij}^r \), which is completely equivalent to Hamiltonian (1) without any approximation. The Hamiltonian (5) can now be diagonalized by the unitary transformation

\[
U' = U^{-1} H U_g,
\]

\[
U_g = \exp [\alpha (A_{12}^r - A_{21}^r)] \exp [\beta (A_{23}^r - A_{32}^r)],
\]

where \( \alpha \) and \( \beta \) are transformation parameters to be specified later. After some manipulation, we then get the exact transformed Hamiltonian:

\[
H' = E_0 + \sum_{i=1}^3 f_i A_{ii}^r + \sum_{i<j} f_{ij} (A_{ij}^r + A_{ji}^r).
\]

It is expected that the coefficients of \( A_{ij}^r \) and its Hermitian conjugate are identical in \( H' \), since the transformation (7) is unitary and \( H' \) shall remain Hermitian. This is also crucial for one to use the parameters \( \alpha \) and \( \beta \) to decouple the transformed Hamiltonian. The coefficients \( f_i \) (\( i = 1, 2, 3 \)) and \( f_{ij} \) (\( i, j = 1, 2, 3 \)) in Eq. (8) and other parameters are given
by

\[ f_1 = \Delta_1 \cos^2 \alpha + \Delta_2 \sin^2 \alpha, \]
\[ f_2 = (\Delta_1 \sin^2 \alpha + \Delta_2 \cos^2 \alpha) \cos^2 \beta - (\tilde{g}_1 \sin \alpha + \tilde{g}_2 \cos \alpha) \sin 2\beta, \]
\[ f_3 = (\Delta_1 \sin^2 \alpha + \Delta_2 \cos^2 \alpha) \sin^2 \beta + (\tilde{g}_1 \sin \alpha + \tilde{g}_1 \cos \alpha) \sin 2\beta, \]
\[ f_{12} = \frac{1}{2} (\Delta_1 - \Delta_2) \sin 2\alpha \cos \beta - (\tilde{g}_1 \cos \alpha - \tilde{g}_2 \sin \alpha) \sin \beta, \]
\[ f_{23} = \frac{1}{2} (\Delta_1 \sin^2 \alpha + \Delta_2 \cos^2 \alpha) \sin 2\beta + (\tilde{g}_1 \sin \alpha + \tilde{g}_2 \cos \alpha) \cos 2\beta, \]
\[ f_{13} = \frac{1}{2} (\Delta_1 - \Delta_2) \sin 2\alpha \sin \beta + (\tilde{g}_1 \cos \alpha - \tilde{g}_2 \sin \alpha) \cos \beta. \]

(9)

These equations describe an infinite set of unitary transformations. One might choose the transformation parameters \( \alpha \) and \( \beta \) so as to give the simplest transformed Hamiltonian and in particular to decouple the three level, i.e., they are chosen as follows:

\[ f_{12} = f_{23} = f_{13} = 0. \]

(10)

Substituting Eq. (9) into Eq. (10) and simplifying it straightforwardly, we arrive at

\[ 2 (\tilde{g}_1 \cos \alpha - \tilde{g}_2 \sin \alpha) \sin \beta = (\Delta_1 - \Delta_2) \sin 2\alpha \cos \beta, \]
\[ (\Delta_1 \cos^2 \alpha + \Delta_2 \sin^2 \alpha) \sin 2\beta = -2 (\tilde{g}_1 \sin \alpha + \tilde{g}_2 \cos \alpha) \cos 2\beta, \]
\[ 2 (\tilde{g}_1 \cos \alpha - \tilde{g}_2 \sin \alpha) \cos \beta = -(\Delta_1 - \Delta_2) \sin 2\alpha \sin \beta. \]

(11)

The transformed Hamiltonian (8) now has the simple form

\[ H' = E_0 + f_1 A_{11}^r + f_2 A_{22}^r + f_3 A_{33}^r, \]

(12)

where the parameters \( f_i \) (\( i = 1, 2, 3 \)) are still given by Eqs. (9). Obviously the decoupled Hamiltonian is efficient for arbitrary \( \Delta_1 \) and \( \Delta_2 \). The parameters \( \alpha \) and \( \beta \) can be calculated by solving the Eqs. (11). The general analytical solution to Eq. (11) is extremely complicated and not very instructive. A typical physically useful solution is the case of \( \Delta_1 = \Delta_2 = \Delta \). We can obtain the following solution:

\[ \alpha = \arctan \frac{\tilde{g}_1}{\tilde{g}_2}, \quad \beta = -\frac{1}{2} \arctan \frac{\sqrt{\tilde{g}_1 + \tilde{g}_2}}{\Delta}, \]

(13)

Substituting the above results into Eq. (9), the coefficient \( f_i \) (\( i = 1, 2, 3 \)) is greatly simplified and has the form

\[ f_1 = \Delta, \quad f_{2,3} = \frac{\Delta}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 + \tilde{g}_1^2 + \tilde{g}_2^2}. \]

(14)

Here ‘\( \pm \)’ is for \( f_2 \) and \( f_3 \), respectively.
III. THE DENSITY OPERATOR

We have obtained the exact transformed Hamiltonian of a V-type three-level atom interacting with a two-mode quantized cavity with the center-of-mass motion included. Because the Hamiltonian is linearized in terms of Lie algebraic generators, the algebraic dynamical structure of the system appears quite clear and its solution is easy to handle by the algebraic dynamical method. In the following, the transformed Hamiltonian can be used to obtain the exact solution for the density operator by using the algebraic dynamical method. The evolution operator of the system can now be calculated as

$$U (t) = e^{-iHt/\hbar} = U_q^{-1} e^{-iH_qt/\hbar} U_q = \sum_{i,j=1}^{3} U_{ij} (N_1, N_2, P_i) A_{ij}^r,$$

where

$$
egin{align*}
U_{11} &= e^{-iE_{0t}/\hbar} \left( e^{-i\beta t/\hbar} \sin^2 \beta \sin^2 \alpha + e^{-i\Delta t/\hbar} \cos^2 \beta \sin^2 \alpha + e^{-i\Delta t/\hbar} \cos^2 \alpha \right), \\
U_{22} &= e^{-iE_{0t}/\hbar} \left( e^{-i\beta t/\hbar} \sin^2 \beta \cos^2 \alpha + e^{-i\Delta t/\hbar} \cos^2 \beta \cos^2 \alpha + e^{-i\Delta t/\hbar} \sin^2 \alpha \right), \\
U_{33} &= e^{-iE_{0t}/\hbar} \left( e^{-i\beta t/\hbar} \cos^2 \beta + e^{-i\Delta t/\hbar} \sin^2 \beta \right), \\
U_{13} &= e^{-iE_{0t}/\hbar} \sin \beta \cos \beta \sin \alpha \left( e^{-i\beta t/\hbar} - e^{-i\Delta t/\hbar} \right), \\
U_{23} &= e^{-iE_{0t}/\hbar} \sin \beta \cos \beta \cos \alpha \left( e^{-i\beta t/\hbar} - e^{-i\Delta t/\hbar} \right), \\
U_{12} &= e^{-iE_{0t}/\hbar} \sin \alpha \cos \left( e^{-i\beta t/\hbar} \sin^2 \beta + e^{-i\Delta t/\hbar} \cos^2 \beta + e^{-i\Delta t/\hbar} \right).
\end{align*}
$$

Assume that at \( t = 0 \) the internal state of the atom is in the ground state, and the density operator of the system can be factorized into three parts:

$$
\rho (0) = |\Psi_F (0)\rangle \langle \Psi_F (0)| \otimes |\Psi_{p_x} (0)\rangle \langle \Psi_{p_x} (0)| \otimes |3\rangle \langle 3|,
$$

where

$$
|\Psi_F (0)\rangle = \sum_{n_1', n_2'} F_{n_1', n_2'} |n_1', n_2'\rangle, \quad |\Psi_{p_x} (0)\rangle = \int dp_x' C (p_x') |p_x'\rangle.
$$

In order to explore the dynamics of the atom, we introduce the reduced density operator, which describes the internal state of the moving V-type three-level atom:

$$
\rho^A (t) = \int dp_x \sum_{n_1, n_2} \langle n_1, n_2, p_x | \rho (t) | n_1, n_2, p_x \rangle.
$$

Here \( n_1 \) and \( n_2 \) denote the photon numbers of the two cavity modes. \( \rho (t) \) is the density operator of the atom-cavity system, and it evolves according to the formula

$$
\rho (t) = U (t) \rho (0) U^\dagger (t).
$$
Using Eq. (21), the analytical expression of the optical force can be obtained as follows:

$$\langle F \rangle = \langle -\nabla H_{\text{int}} \rangle = -Tr (\rho \nabla H_{\text{int}}) ,$$

(22)

where

$$\nabla H_{\text{int}} = \hat{g}_1 (A_{13}^r + A_{31}^r) - i \hat{g}_1 \Phi_1 (R) (A_{13}^r - A_{31}^r)$$

$$+ \hat{g}_2 (A_{23}^r + A_{32}^r) - i \hat{g}_2 \Phi_2 (R) (A_{23}^r - A_{32}^r).$$

(23)

Using Eq. (21), the analytical expression of the optical force can be obtained as follows:

$$\langle F \rangle = -\int dp_x \sum_{n_1,n_2} |C (p_x)|^2 |F_{n_1,n_2}|^2$$

$$\times \left[ \frac{U_{13} U_{33}^\dagger}{\sqrt{n_1}} a_1^\dagger + \frac{U_{33} U_{13}^\dagger}{\sqrt{n_1}} a_1 \right] \hat{g}_1 + i \Phi_1 (R) \hat{g}_1 \left( \frac{U_{13} U_{33}^\dagger}{\sqrt{n_1}} a_1^\dagger - \frac{U_{33} U_{13}^\dagger}{\sqrt{n_1}} a_1 \right)$$

$$+ \left( \frac{U_{23} U_{33}^\dagger}{\sqrt{n_2}} a_2^\dagger + \frac{U_{33} U_{23}^\dagger}{\sqrt{n_2}} a_2 \right) \hat{g}_2 + i \Phi_2 (R) \hat{g}_2 \left( \frac{U_{23} U_{33}^\dagger}{\sqrt{n_2}} a_2^\dagger - \frac{U_{33} U_{23}^\dagger}{\sqrt{n_2}} a_2 \right).$$

(24)
Treating the cavity field classically: \( a_1 \to \alpha_1 e^{-i\omega_1 t} \) and \( a_2 \to \alpha_2 e^{-i\omega_2 t} \), and assuming the amplitudes are identical with the conjugated amplitudes respectively \([26]\), namely \( \alpha_1 = \alpha_1^* \), \( \alpha_2 = \alpha_2^* \), the optical force becomes

\[
\langle F \rangle = -\int dp_x \sum_{n_1, n_2} |C(p_x)|^2 |F_{n_1, n_2}|^2 \\
\times \left[ \left( U_{13}U_{33}^\dagger e^{i\omega_1 t} + U_{33}U_{13}^\dagger e^{-i\omega_1 t} \right) \tilde{g}_1 + i\Phi_1(R) \left( U_{13}U_{33}^\dagger e^{i\omega_1 t} - U_{33}U_{13}^\dagger e^{-i\omega_1 t} \right) \tilde{g}_1 \\
+ \left( U_{23}U_{33}^\dagger e^{i\omega_2 t} + U_{33}U_{23}^\dagger e^{-i\omega_2 t} \right) \tilde{g}_2 + i\Phi_2(R) \left( U_{23}U_{33}^\dagger e^{i\omega_2 t} - U_{33}U_{23}^\dagger e^{-i\omega_2 t} \right) \tilde{g}_2 \right]. \tag{25}
\]

For two plane waves modes with the same wave number but with two orthogonal polarizations, exciting transitions to different Zeeman sublevels, we obtain the optical force as follows:

\[
\langle F \rangle = -\int dp_x \sum_{n_1, n_2} |C(p_x)|^2 |F_{n_1, n_2}|^2 \\
\times \left[ \left( U_{13}U_{33}^\dagger e^{i\omega_1 t} + U_{33}U_{13}^\dagger e^{-i\omega_1 t} \right) \tilde{g}_1 + \left( U_{23}U_{33}^\dagger e^{i\omega_2 t} + U_{33}U_{23}^\dagger e^{-i\omega_2 t} \right) \tilde{g}_2 \right]. \tag{26}
\]

Since we have obtained the exact solution of optical forces, it is worthwhile to analyze the variation of the optical forces with the controlling parameters. We assume that the moving V-type three-level atom is moving in one dimension for simplicity, and we tune the wavenumbers of the two cavity modes as \( k_{l2} = -k_{l1} = k_j x \), along with carrying out a two-photon resonance process to make the cavity frequencies consistent as \( \omega_{l1} = \omega_{l2} = \omega_j \), and the coupling constants identical as \( \tilde{g}_1 = \tilde{g}_2, \tilde{g}_{10} = \tilde{g}_{20} = h g_0 \) (here, \( k_j = j \pi/L, \omega_j = c k_j \), with \( j = 1, 2, 3, \ldots \)). \( L \) is the length of the cavity, and \( c \) is the speed of light in vacuum. If we consider the field in a large but finite cubic cavity of side \( L \), and regard the cavity merely as a region of space with no specific boundaries, the quantized field in free-space can be obtained). The normalization is processed to the coefficient of the radiation force as \( \int dp_x \sum_{n_1, n_2} |C(p_x)|^2 |F_{n_1, n_2}|^2 = 1 \). Thus the force can be written as

\[
\langle F \rangle = g_0 h k \sin k x \left[ \left( U_{13}U_{33}^\dagger + U_{23}U_{33}^\dagger \right) e^{i\omega t} + \left( U_{33}U_{13}^\dagger + U_{33}U_{23}^\dagger \right) e^{-i\omega t} \right]. \tag{27}
\]

\( \alpha \) and \( \beta \) simplify as \( \arctan 1 \) and \(-0.5 \arctan(2\sqrt{2}g_0 h \cos k x / \Delta) \), so

\[
\sin \alpha = \cos \alpha = \sqrt{2}/2, \\
\cos 2\beta = \Delta / (\sqrt{\Delta^2 + 8h^2 g_0^2 \cos^2 k x}), \\
\sin 2\beta = -2\sqrt{2}g_0 h \cos k x / (\sqrt{\Delta^2 + 8h^2 g_0^2 \cos^2 k x}). \tag{28}
\]

Defining \( f_2 - f_3 \equiv f_0 \), and substituting them into Eq. \( (27) \), we can get

\[
\langle F \rangle = \frac{2h^2 g_0^2 k \sin 2k x}{\sqrt{\Delta^2 + 8h^2 g_0^2 \cos^2 k x}} \left( \sin \frac{f_0 t}{h} \sin \omega t - \frac{2\Delta \cos \omega t \sin^2 \frac{f_0 t}{2h}}{\sqrt{\Delta^2 + 8h^2 g_0^2 \cos^2 k x}} \right). \tag{29}
\]
From Eq. (29), it can be seen that the force is dependent on the wavenumbers, coupling constant, detuning, and the atomic position. Obviously we can change the optical force by regulating the system parameters.

V. CONCLUSIONS

In this paper, within the framework of the algebraic dynamics method, we have studied the quantum dynamics of a V-type three-level atom-cavity systems as the atomic motion is included. We have given a clear description of the dynamical algebraic structure of the system and obtained its analytical solution. Based on this, we have presented the analytical expression for the optical forces under certain conditions. We hope that the current study can help readers to investigate the considered system in the design of quantum optical experiments.

Acknowledgments

This work was supported by NSFC under Grant No. 20377020.

References