Screening and Variable Range Hopping Conduction in Silicon MOSFETs

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We re-analyze earlier published data on two dimensional electron system Si-MOSFETs sample of a two dimensional electron system in n-channel Si-MOSFETs down to 2K at zero magnetic field. The system is of low electron densities and near the metal-insulator transition point from the insulating side. Our results show the existence of a crossover, from Efros-Shklovskii variable range hopping (ES-VRH) characterized with the existence of a Coulomb gap and
\[ \rho = \rho_0 \exp(T_{ES}/T)^{1/2}, \]
down to the Mott regime where \[ \rho = \rho_0 \exp(T_{M}/T)^{1/3}. \]
With \( \rho_0 \) is a pre-exponential factor that is found to be close to \( 2(h/e^2) \), and this crossover occurs when the value of temperature oscillates around 1 K.

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I. INTRODUCTION

The crossover from Mott to Efros-Shklovskii variable range hopping (VRH) as the temperature decreases has been observed in many localized systems [1–3]. VRH conduction is possible when the temperature is sufficiently low, where an activated type of conduction is not possible. ES-VRH is due to the Coulomb interaction between carriers, which produces a Coulomb gap around the Fermi energy \( E_F \) and the energy state density \( N(E-E_F) \) is proportional to \( |E-E_F| \) for two-dimensional (2D) systems. However, at higher temperature Mott conventional hopping occurs [4]. In the Mott regime, the resistivity \( \rho \) is proportional to \( (1/T)^{1/(D+1)} \); \[ \rho \propto (1/T)^{1/(D+1)} \] [5], where \( T \) is the temperature and \( D \) the dimensionality. Mott derives this law by assuming a constant density of states (DOS) at the Fermi energy. In this model, Mott ignores the Coulomb interactions in two dimensional systems and the hopping resistivity is given by
\[ \rho = \rho_0 \exp(T_{M}/T)^{1/3}. \] (1)

In contrast, in the Efros-Shklovskii model, the hopping resistivity with a Coulomb gap is given by [6]
\[ \rho = \rho_0 \exp(T_{ES}/T)^{1/2}, \] (2)
where \( T_M \) and \( T_{ES} \) are the hopping energies in each region, and the pre-exponential factor \( \rho_0 \) can be temperature independent [7] or temperature dependent [8, 9], depending on the

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type of scattering [10] and the interaction mechanism. The crossover from Mott to ES VRH is not established for all two dimensional systems: in a few of the systems it has been found that the ES to Mott VRH crossover is due to filling of the Coulomb gap in the higher temperature ranges [11]. Timp et al. [12] investigated the conductivity as a function of the temperature and the electric field in sodium doped silicon MOSFETs and found no evidence for a Coulomb gap. In contrast, a Coulomb gap behaviour has been observed in a GaAs/AlGaAs heterostructure [13, 14].

In this paper, we report on experimental studies of electrical resistivity as a function of the temperature of high-mobility silicon MOSFETs: Si-62. The data show the existence of a crossover due to screening from Mott to Efros-Shklovskii VRH as the temperature decreases.

II. EXPERIMENTAL DETAILS

In this work, we re-analyzed the data taken by Pudalov and co-workers [15]. In short, measurements were performed on the Si-MOSFETs: Si-62 with maximum mobility $\mu = 4.5 \text{ m}^2/\text{Vs}$, and with different carrier densities $n_s$ in the range $0.7156 - 0.8464 \times 10^{11} \text{ cm}^{-2}$, in steps of $0.0218 \times 10^{11} \text{ cm}^{-2}$, the critical density where the metal-insulator transition occurs for this sample is $n_c \approx 1 \times 10^{11} \text{ cm}^{-2}$. The experiments are carried out at low temperatures up to 4 K.

III. RESULTS AND DISCUSSION

Fig. 1 displays the temperature dependence of the electrical resistivity $\rho$ near the critical point $n_c$ of the metal insulator transition (MIT) from the insulating side. The value of the critical density where the transition happens is $n_c \approx 10^{11} \text{ cm}^{-2}$. For densities smaller than $n_c$ the electrical resistivity decreases with increasing temperature $T$.

The insulating behaviour of the MIT can be described by conventional variable range hopping (VRH) conduction in the presence of a coulomb gap by Eq. (2). $T_{ES}$ is a measure of the Hartree interaction strength and depends on the electron density $n_s$ in a 2D conducting channel. In a single-particle hopping mechanism, $T_{ES}$ is related to the localization length $\xi$ by

$$T_{ES} = C_{ES} e^2 / k_B \varepsilon \xi,$$

where $C_{ES}$ is a constant characterizing the single-particle hopping amplitude: $C_{ES} = 2.8$ in tridimensional systems (3D) and $C_{ES} = 13.8$ in two dimension systems (2D) [16, 17], $\varepsilon$ is the dielectric constant, $e$ is the electron charge, and $k_B$ is the Boltzmann constant. Nevertheless there is an ambiguity about the way of this hopping, and an important question is raised: is the model of a single-particle sufficient? or is the correlated hopping picture required? In reality, in several studies [8, 16, 18, 19] the existence of a correlated hopping model has been verified by observing much smaller values of $T_{ES}$ than the value expected for a
single-particle.

In some references [18, 19], researchers have found very high values of the localization length (unreasonable lengths), unless the constant $C_{ES}$ is renormalized by a constant whose value varies from one sample to others to make reasonable values of the localization lengths, for example, Nam-Jung Kim [20] found $A \approx 27$. Theoretical [18] and experimental results [10, 21] show that there is a crossover from Efros-Shklovskii VRH to Mott hopping due to screening around 1 K. In the present work, we report on the experimental results showing the existence of a crossover from ES to Mott VRH.

Resistivity versus $T^{-1/2}$ for several electron densities, from top to bottom: 0.7156; 0.7374; 0.7592; 0.781; 0.8028; 0.8246; 0.8464 $\times 10^{11}$ cm$^{-2}$ are depicted in Fig. 2. Our experimental data follows the form of Eq. (2) at low temperature, this behaviour is consistent with the existence of a Coulomb gap.

Fig. 2 displays curves representing the resistivity $\rho$ plotted on a log scale versus $T^{-1/2}$, and shows a Coulomb gap hopping region identified by the straight lines for several densities below the critical density $n_c$ represented by various symbols. The values of $T_{ES}$ are deduced from the slopes for each of the curves in the region where they are linear, therefore each density has its own $T_{ES}$.

Similarly, we plot log $(\rho)$ versus $T^{-1/3}$ from Eq. (1) defining the screening of Mott, and from the slopes of each curves, we get the values of $T_M$ for each density. In Table I we have collected the results of $T_{ES}$ and $T_M$ for each density. Fig. 3a and 3b show respectively $T_{ES}$ and $T_M$ as a function of the electron density $n_s$ from the data shown in Table I.

Fig. 4 shows $\rho/\rho_0$ as a function of $(T_{ES}/T)^{1/2}$ for the Si-62 sample. With the temperature scaled by $T_{ES}$, the resistivity data from the various electron densities collapses
FIG. 2: Resistivity vs $T^{-1/2}$ for the sample Si-62 on the insulating side for various densities from Fig. 1.

TABLE I: Values of $T_{ES}$ (K) (ES temperature) and $T_M$ (K) (Mott temperature) derived from Fig. 2.

<table>
<thead>
<tr>
<th>$n_s \times 10^{11}$ cm$^{-2}$</th>
<th>$T_{ES}$ (K)</th>
<th>$T_M$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7156</td>
<td>19.63</td>
<td>371.7</td>
</tr>
<tr>
<td>0.7374</td>
<td>16.16</td>
<td>252.5</td>
</tr>
<tr>
<td>0.7592</td>
<td>13.54</td>
<td>183.25</td>
</tr>
<tr>
<td>0.781</td>
<td>12.18</td>
<td>120.55</td>
</tr>
<tr>
<td>0.8028</td>
<td>9.92</td>
<td>82.88</td>
</tr>
<tr>
<td>0.8246</td>
<td>8.82</td>
<td>44.36</td>
</tr>
<tr>
<td>0.8464</td>
<td>7.13</td>
<td>20.12</td>
</tr>
</tbody>
</table>

onto a single curve when plotted as a function $(T_{ES}/T)^{1/2}$. We note that this is a straight line (or almost, because of errors committed during measurement) for different values of the densities on the insulating side. The prefactor $\rho_0$ is deduced from Fig. 2.

The hopping resistivity is described by the formula [20]

$$\rho = \rho_0 \exp(r_c/\xi),$$

(4)
where $r_c$ is the characteristic hopping length scale, which has a universal form across the Efros-Shklovskii and the screened Mott hopping regions [17]:

$$ r_c/d = (\xi/d) \ln[(\rho(T)/\rho_0)] = [C_{ES}f(x)/x]^{1/2}, $$

(5)

where $x = T/T_\xi$ and $T_\xi = e^2\xi/\varepsilon k_B d^2$, $k_B$ is Boltzmann’s constant, $e$ is the carrier charge, and $\varepsilon$ denotes the dielectric response of the 2D gas and the oxide; $d$ is the gate oxide thickness.
FIG. 4: Temperature scaled by $T_{ES}$ for the Si-62 sample, the resistivity data from various electron densities can be scaled onto one single straight line. ($\rho_0 \approx 2(h/e^2)$).

The localization lengths are obtained from both the high temperature and low-temperature limits, respectively, as

$$\xi = C_{ES} e^2/4\pi\varepsilon k_B T_{ES},$$  \hspace{1cm} (6)$$

$$\xi = (\beta e^2d/4\pi\alpha\varepsilon k_B T_M)^{1/2},$$ \hspace{1cm} (7)

where $T_{ES}$ and $T_M$ are the hopping energies in each region, and the single-electron hopping constants $C_{ES}$ and $\beta$ are 6.2 and 13.8 [17], and $\alpha = 0.1$. In Ref. [20] Nam-Jung Kim found unreasonable values of the localization lengths unless $C_{ES}$ is renormalized. In other work [18] $C_{ES}$ was reduced by about one order of magnitude when multi-electron hopping was dominant. The prefactor $A$ is determined by assuming that the localization lengths estimated from two neighboring regions should be equal at a given electron density. In Ref. [20], Kim et al. had found that the prefactor $A \approx 27$, and thus reduces the hopping amplitudes for justifying the correlating hopping behavior of the electron.

Using the same method, we found that the prefactor $A \approx 1$ for the sample Si-62. The values of $A$ for each density are represented in Table II.

In contrast with the result found in [20] by Nam-Jung Kim, where $A \approx 27$, our result is much smaller ($A \approx 1$), the explanation of this unexpected result is that the single-particle hopping picture is sufficient, and the correlated hopping model is not required to explain the hopping. This is due maybe to the high electron mobility of the Si-62 sample, so it’s a clean system compared to the samples studied in Ref. [20].
TABLE II: The value of the prefactor $A$ for each density for the Si-62 sample.

<table>
<thead>
<tr>
<th>$n_s \times 10^{11}$ cm$^{-2}$</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.715</td>
<td>0.96461</td>
</tr>
<tr>
<td>0.737</td>
<td>0.96689</td>
</tr>
<tr>
<td>0.759</td>
<td>0.99955</td>
</tr>
<tr>
<td>0.781</td>
<td>0.81259</td>
</tr>
<tr>
<td>0.802</td>
<td>0.84222</td>
</tr>
</tbody>
</table>

In Fig. 5, we plot $r_c/d$ against $(T_\xi/T)^{1/2}$, in this figure, the crossover from Mott to ES VRH with a Coulomb gap is marked by arrows at $(T_\xi/T)^{1/2} \approx 3.6$, which separates the two regimes: the Coulomb gap for $(T_\xi/T)^{1/2} < 3.6$ and that of Mott for $(T_\xi/T)^{1/2} > 3.6$. In some previous works, authors have observed that in 3D systems [16, 18] and in 2D [7, 8, 19] the scaling behaviour is required, even if multi-electron hopping is included.

Fig. 6 shows $r_c/d - (C_{ES}T_\xi/T)^{1/2}$ as a function of $(T_\xi/T)^{1/2}$ for the sample Si-62 for several electron densities. This figure is to confirm the crossover observed in Fig. 4, the crossover from the Coulomb gap to the Mott regime is marked by the arrow. The value of $r_c/d - (C_{ES}T_\xi/T)^{1/2}$ as a function of $(T_\xi/T)^{1/2}$ for each density remains constant before dropping at $(T_\xi/T)^{1/2} \approx 3.6$; it is at this level where the crossover between the ES VRH
and the Mott screening occurs.

\[ \text{FIG. 6: } rc/d - (C_{EST}/T)^{1/2} \text{ as a function of } (T/T)^{1/2}. \]

**IV. CONCLUSION**

In summary, the data presented in this paper shows the existence of a crossover from Efros-Shkovskii to Mott variable range hopping, which is consistent with a few previous studies cited below. A second result found through this investigation is that the single-particle hopping picture is sufficient to explain this kind of crossover, in contrast with many studies that assumed that the correlated hopping model is required.

**Acknowledgments**

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**References**


