Meson-Exchange Model for πN Scatterings and Pion Photoproduction Reactions

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A meson-exchange model, based on a three-dimensional reduction of Bethe-Salpeter equation, for πN interaction and pion photoproduction reactions is presented. The model is able to describe πN phase shifts and pion photoproductions from nucleon at low energy. Rescattering effects are found to play important role in $\gamma\pi$ reaction in most multipole channels. The constructed meson-exchange potential is used to evaluate rescattering effects in the dipole amplitude E_{0^+} of π^0 photoproduction from proton near threshold. The results are close to the most recent Mainz data.

I. INTRODUCI ' ION

In the traditional picture of nucleus as a system of nucleons, one of the two basic ingredients of the microscopic nuclear theory is the nucleon-nucleon interaction. In pion-nucleus reactions, pion degree of freedom has to be treated explicitly and pion-nucleon interaction is needed.

Recent high precision experimental data have confirmed that the description of the long-range and medium-range NN interaction in terms of hadronic (nucleons, mesons and isobars) degrees of freedom is quantitatively very successful.' It has also been established that unified theoretical description of the πd–πd, nd-NN and NN–πNN reactions, in terms of hadronic degrees of freedom, at intermediate energies seems to be more or less in qualitative agreement with the extensive set of experimental data.[2] The πN models used in these unified theoretical studies are often parametrized in separable forms, mostly for ' computational simplicities. Although such a phenomenological approach has achieved significant successes,.recent advances have pointed to the need of a more rigorous treatment. For example, it has been demonstrated[3-6] that pion production/absorption on two-nucleon system are sensitive to the parameterizations of πN off-shell amplitudes. To make progress, it is necessary to develop a πN model which contains the mechanisms originated from some fundamental theory of strong interactions. Since models based on meson-exchange picture[1,7] have been very successful in describing the NN phase shifts and observables, it is therefore reasonable to expect that the πN dynamics at low and intermediate energies can be described by the same approach. The degree of success of such a meson-exchange πN model would depend on the consistency in the parameters employed, e.g., masses and coupling constants, as compared to those used in the meson-exchange NN model.

Pion photoproduction reactions are known to provide information on nuclear dynamics complementary to those extractable in pion-induced reactions. In pion nucleus scattering, the phase shifts depend only on the asymptotic behaviors of the wave function. On the other hand, the weakness of the electromagnetic interaction allows the photon to penetrate deep into the nucleus, contrary to the hadronic probe such as pion, which are strongly absorbed and see only the surface of the nucleus. The subsequent propagation of the pion and delta through the nucleus, once photoproduced deep inside, probe the short distance behavior of the wave function. It is hence of great interest to extend the above mentioned unified theoretical framework to electromagnetic reactions involving πNN systems. This requires model for photo- and electro-production of pion on nucleon which are consistent with the existing unitary πNN models. Such a unified meson-exchange model for hadronic as well as electromagnetic reactions involving πNN systems could possibly help to shed the light on the important question of transition from hadronic picture to quark-gluon picture in the nuclear phenomena.

In this talk, I will describe some of our recent efforts[8,9] in constructing the meson-exchange model for pion-nucleon interaction and pion photoproduction reactions.

II. MESON-EXCHANGE πN MODEL

We start from the Bethe-Salpeter (BS) equation for the πN scattering

$$T_{\pi N} = B_{\pi N} + B_{\pi N} G_0 T_{\pi N}, \tag{1}$$

where $B_{\pi N}$ is the sum of all irreducible two-particle Feynman amplitude and Go is the relativistic pion-nucleon propagator

$$G_0(q,P) = \frac{(\frac{1}{2}\not{P} + \not{q}) + m_N}{(\frac{1}{2}P+q)^2 - m_N^2 + i\epsilon} \cdot \frac{1}{(\frac{1}{2}P-q)^2 - m_\pi^2 + i\epsilon}. \tag{2}$$

q and P are respectively the relative and total four momentum of the πN system. The BS equation can be rewritten as

$$T_{\pi N} = \hat{B}_{\pi N} + \hat{B}_{\pi N} g_0 T_{\pi N}, \tag{3}$$

with

$$\hat{B}_{\pi N} = B_{\pi N} + B_{\pi N}(G_0 - g_0)\hat{B}_{\pi N} \tag{4}$$

If we choose

$$g_0(k,P) = \frac{1}{(2\pi)^3}\int \frac{d\sqrt{s'}(\frac{\not P'}{2}+\not q+m_N)}{\sqrt{s}-\sqrt{s'}+i\epsilon}\delta^{(+)}\left(\left[\frac{P'}{2}+q\right]^2-m_N^2\right) \cdot\delta^{(+)}\left(\left[\frac{P'}{2}-q\right]^2-m_\pi^2\right), \tag{5}$$

where $P' = \sqrt{s'/s}\,P$ and $P = (\sqrt{s}, 0)$, and approximate $\hat{B}_{\pi N}$ of Eq. (4) by $B_{\pi N}$, then we arrive at the following Lippmann-Schwinger type equation for the πN scattering amplitude

$$T_{\pi N}(\mathbf{q}',\mathbf{q};\omega) = V_{\pi N}(\mathbf{q}',\mathbf{q};\omega) + \int d\mathbf{q}'' \frac{V_{\pi N}(\mathbf{q}',\mathbf{q}'';\omega)T_{\pi N}(\mathbf{q}'',\mathbf{q};\omega)}{\omega - E_N(\mathbf{q}'') - E_\pi(\mathbf{q}'') + i\epsilon}, \tag{6}$$

where q is the πN relative three-momentum, ω is the total energy in the center of mass frame, and $V_{\pi N}(\mathbf{q}', \mathbf{q}; \omega)$ is a πN potential which can be calculated directly from $B_{\pi N}(\mathbf{q}', \mathbf{q}, P)$

$$\begin{aligned} V_{\pi N}(\mathbf{q}',\mathbf{q},\omega) &= \frac{1}{(2\pi)^3}\frac{1}{\sqrt{4E_\pi(\mathbf{q}')E_\pi(\mathbf{q})}}\sqrt{\frac{m_N^2}{E_N(\mathbf{q}')E_N(\mathbf{q})}} \\ &\cdot\int dq_0'\delta[q_0' - \frac{1}{2}(E_N(\mathbf{q}') - E_\pi(\mathbf{q}'))] \\ &\cdot\int dq_0\delta[q_0 - \frac{1}{2}(E_N(\mathbf{q}) - E_\pi(\mathbf{q}))]\bar{u}(\mathbf{q}')B_{\pi N}(q',q,P)u(\mathbf{q}). \end{aligned} \tag{7}$$

We further approximate the Bethe-Salpeter kernel $B_{\pi N}$ by the sum of all lowest order tree diagrams evaluated from the following effective Lagrangian

$$\begin{aligned} L_{\pi NN} &= \frac{f_{\pi NN}}{m_\pi}\bar{N}\gamma_5\gamma_\mu\vec{\tau}\cdot\partial^\mu\vec{\pi}N, \\ L_{\pi N\Delta} &= \frac{g_{\pi N\Delta}}{m_\pi}\{\bar{\Delta}_\mu[g^{\mu\nu}-(Z+1/2)\gamma^\mu\gamma^\nu]\vec{T}_{\Delta N}N \\ &\quad +\bar{N}[g^{\mu\nu}-(Z+1/2)\gamma^\nu\gamma^\mu]\vec{T}^\dagger_{\Delta N}\Delta_\mu\}\cdot\partial_\nu\pi, \\ L_{\rho NN} &= -g_{NN\rho}\bar{N}\{\gamma_\mu\vec{\rho}^\mu + \frac{\kappa_V}{4m_N}\sigma_{\mu\nu}(\partial^\mu\vec{\rho}^\nu-\partial^\nu\vec{\rho}^\mu)\}\cdot\tau N, \\ L_{\rho\pi\pi} &= -g_{\rho\pi\pi}\vec{\rho}^\mu\cdot(\vec{\pi}\times\partial_\mu\vec{\pi}) - \frac{g_{\rho\pi\pi}}{4m_\rho^2}(\delta-1)(\partial^\mu\vec{\rho}^\nu-\partial^\nu\vec{\rho}^\mu)\cdot(\partial_\mu\vec{\pi}\times\partial_\nu\vec{\pi}), \\ L_{\rho\pi\pi} &= -g_{\rho\pi\pi}m_\pi(\vec{\pi}\cdot\vec{\pi})\sigma, \\ L_{\sigma NN} &= -g_{NN\sigma}\bar{N}\sigma N. \end{aligned} \tag{8}$$

The $\vec{T}_{\Delta N}$ is the isospin transition operator acting between a nuclean and a A. The resulting πN potential defined by Eq. (7) then contains the familiar direct and exchange nucleon terms, σ-exchange, p-exchange, and also the direct and exchange A terms. We introduce a dipole form factor of the form $[\Lambda^2/(\Lambda^2+\mathbf{q}^2)][\Lambda^2/(\Lambda^2+\mathbf{q}'^2)]$ for each term to regularize the matrix elements of πN potential at high momenta.

III. MESON-EXCHANGE MODEL FOR $\pi N \rightarrow \gamma N$

The task is to construct a transition potential operator $V_{\gamma\pi}$ to be used in the Hamiltonian

$$H = H_0 + V_{\pi N} + V_{\gamma\pi}, \tag{9}$$

where Ho is the sum of relativistic free-energy operators for N, π and γ and $V_{\pi N}$ is the pion-nucleon potential of Eq. (7). The transition matrix for $\gamma N \rightarrow \pi N$, to first order in e, is then given as

$$T_{\gamma\pi} = T_{\pi N} g_0 V_{\gamma\pi} + V_{\gamma\pi}. \tag{10}$$

For consistency, we will assume that the matrix elements of $V_{\gamma\pi}$ **is** given by the tree approximation of a Lagrangian which is derived from the Lagrangian of Eq. (8) by minimal coupling. In addition, the t-channel ω vector-meson exchange has to be added. There arises, however, the complication of preserving gauge invariance when the pion is off-shell. For detailed discussions, see Ref. **10.**

There are two important features in our formulation of Eqs. (9) and (10). Since pion rescatterings are included explicitly, as represented by the first term on the right hand side of Eq. (10), the pion photoproduction amplitude thus obtained is unitary and satisfies the Watson theorem. Furthermore, it also provides us with consistency in two- ' and many-body problems when we apply it to microscopic description of the pion photoproduction reactions from nuclei.

For physical multipole amplitude in channel a, multipole decomposition of Eq. (10) *gives*

$$T_{\gamma\pi}^{(\alpha)}(q_E, k; w + i\varepsilon) = \exp(i\delta^{(\alpha)}) \cos\delta^{(\alpha)} \Big(V_{\gamma\pi}^{(\alpha)}(q_E, k) + P \int_0^\infty dq' \frac{q'^2 R_{\pi N}^{(\alpha)}(q_E, q'; \omega) V_{\gamma\pi}^{(\alpha)}(q', k)}{w - E_\pi(\mathbf{q}') - E_N(\mathbf{q}')} \Big), \tag{11}$$

where $\delta^{(\alpha)}$ is the πN phase shift in channel *a*, $R_{\pi N}^{(\alpha)}$ *is* the πN reaction matrix and q_E is the pion on-shell momentum. One notes that $\gamma\pi$ amplitude in Eq. (11) manisfestly satisfies the Watson theorem and depends on the half-off-shell behaviors of $R_{\pi N}^{(\alpha)}$.

IV. RESULTS AND DISCUSSIONS

IV-l. πN phase shifts

We have carried out a fitting procedure to the pion-nucleon scattering phase shifts in S-waves and the dominant P_{33} channel at low energies. The other weaker P_{13} and P_{31} channels are then predicted from the resulting parameters. The results are shown in Figs. 1 and 2. The

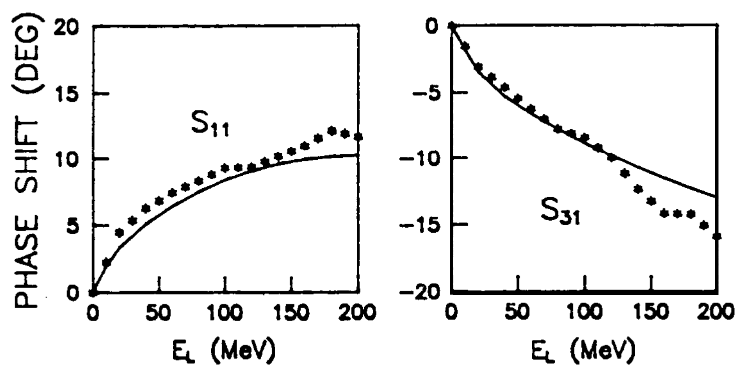

FIG. 1. Our fits to the πN phase shifts in S-waves. The data points represent the Karlsruhe phase shifts (Ref. 22).

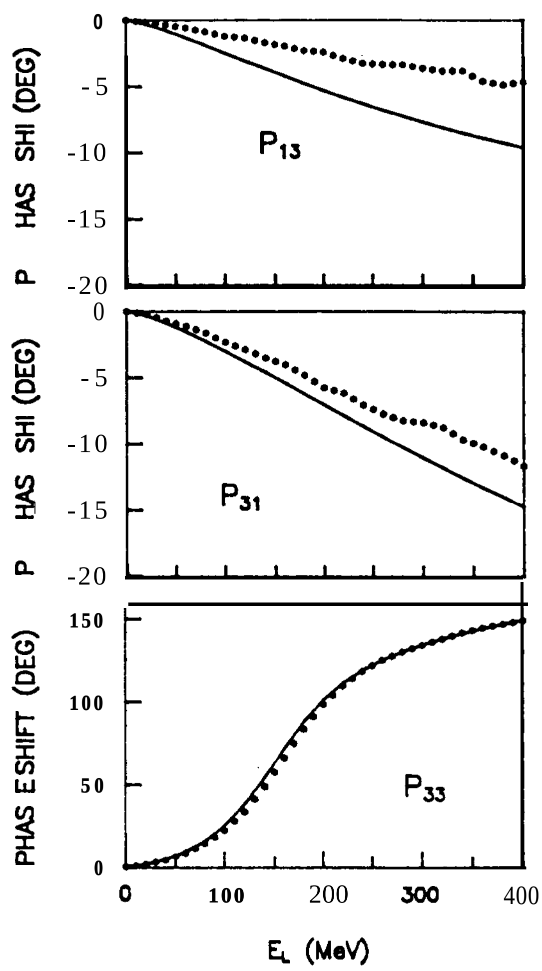

FIG. 2. Our fits to the πN phase shifts in P-waves. The data points represent the Karlsruhe phase shifts (Ref. 22).

results for P_{11} channel are not shown because Pearce and Afnan[11] have shown that a reasonable description of P_{11} phase shifts would require a proper treatment of the renormalization of mass and coupling constant in nucleon-pole diagram. To begin with, however, we shall sidestep the question of renormalization and leave it for later improvements. The agreements with experimental data are very good up to pion laboratory energy of about 200 MeV in S-waves and 400 MeV in P_{33}. It is comforting to see that the predicted phase shifts in P_{13} and P_{31} are in qualitative agreement with the experimental data. The resulting parameters are: Λ_N = 1000 MeV, m_σ = 644.8 MeV, $m_\pi g_{\sigma NN} g_{\sigma\pi\pi}$ = -2478.45 MeV, Λ_σ = 1339 MeV, $g_{\rho NN} g_{\rho\pi\pi}$ = -39.91, δ = 0.17, κ_V = 3.7, Λ_ρ = 1424 MeV, m_Δ = 1400 MeV, $g_{\pi N\Delta}$ = 1.535 and AA = 1015 MeV. Our values of the bare A mass m_Δ and bare $\pi N\Delta$ coupling constant $g_{\pi N\Delta}$ are very close to that obtained by Tanabe and Ohta.[12] Parameters associated with the σ meson are almost identical to those used in Bonn potential.

IV-2. $\gamma\pi$ **multipole amplitudes**

In order to suppress the high momentum contributions in the integral term of Eq. (11), we include a dipole cut-off factor $(\Lambda_{\gamma\pi}^2 + k_E^2)^2/(\Lambda_{\gamma\pi}^2 + k^2)^2$ with the $V_{\gamma\pi}$. We use the same cut-off form factor and with the same value of $\Lambda_{\gamma\pi}$ in all multipole channels. As a first step, we perform calculations for $\gamma\pi$ multipole amplitudes given in Eq. (11) with phenomenological separable πN potentials employed in Ref. (13), while thepny and $\omega\pi\gamma$ couplings are taken from Ref. 14.

The predicted multipole amplitudes in all three isospin channels with final pion state in S- and P-waves are shown in Figs. 3-5, respectively. Dashed curves represent the contribution from $V_{\gamma\pi}$ when only the nucleon Born terms in pseudovector are included. Solid and dot-dashed curves correspond to the results of full calculations whenp and ω contributions are included and final state πN interactions are taken into account. The differences between solid and dot-dashed curves come from the different values $\Lambda_{\gamma\pi}$ used. For solid curves, $\Lambda_{\gamma\pi}$ = 476 MeV and $\Lambda_{\gamma\pi}$ = 800 MeV for dot-dashed curves.

The agreements with the experimental data[15,16] are, in general, very encouraging. It is seen that the contributions of final state interactions are important in all channels except $E_{1^+}(1/2)$ of Fig. 3. In particular, they play an essential role in bringing $M_{1^+}(3/2)$ and $E_{1^+}(3/2)$ in Fig. 5 into close agreement with the experiments. We also see that the results in $E_{0^+}(0)$, $E_{0^+}(3/2)$, M,+(O), and $M_{1^-}(1/2)$ are sentive to the value of cut-off parameter $\Lambda_{\gamma\pi}$. Our prediction in $M_{1^-}(1/2)$ is in total disgreement with the experiments. Since the final πN state in $M_{1^-}(1/2)$ is P_{11}, it may be related to the fact that renormalization of nucleon-pole has not been carried out yet. It should be studied further together with the use of meson-exchange πN interaction model constructed in Sec. II. This is currently in progress.

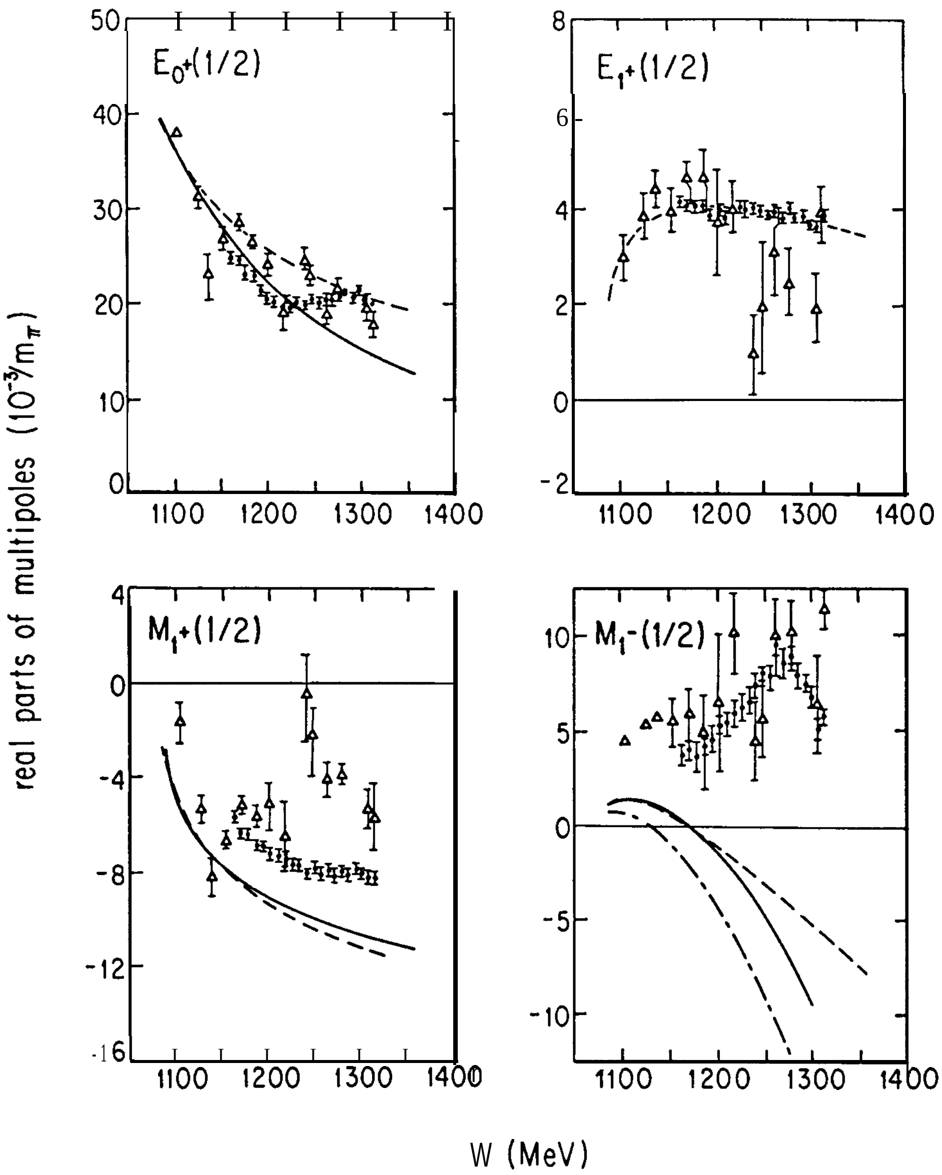

FIG. 3. Real parts of isospin-1/2 multipoles. Dashed curves include only the contribution from Born terms in pseudovector coupling. Dot-dashed curves and solid curves are our model predictions obtained with cut-off parameter $\Lambda_{\gamma\pi}$ = 476 and 800 MeV, respectively. Data are from Refs. 15 and 16.

IV-3. Neutral pion photoproduction on proton at threshold

Recently, absolute measurements of π^0 photoproduction on proton in the threshold region have been performed at Saclay[17] and Mainz[18] and values of (-0.5 ± 0.3) and -0.35 x $10^{-3}/m_{\pi^+}$ (these units will be used hereafter) are extracted for the dipole amplitude $E_{0^+}(p\pi^0)$, respective-

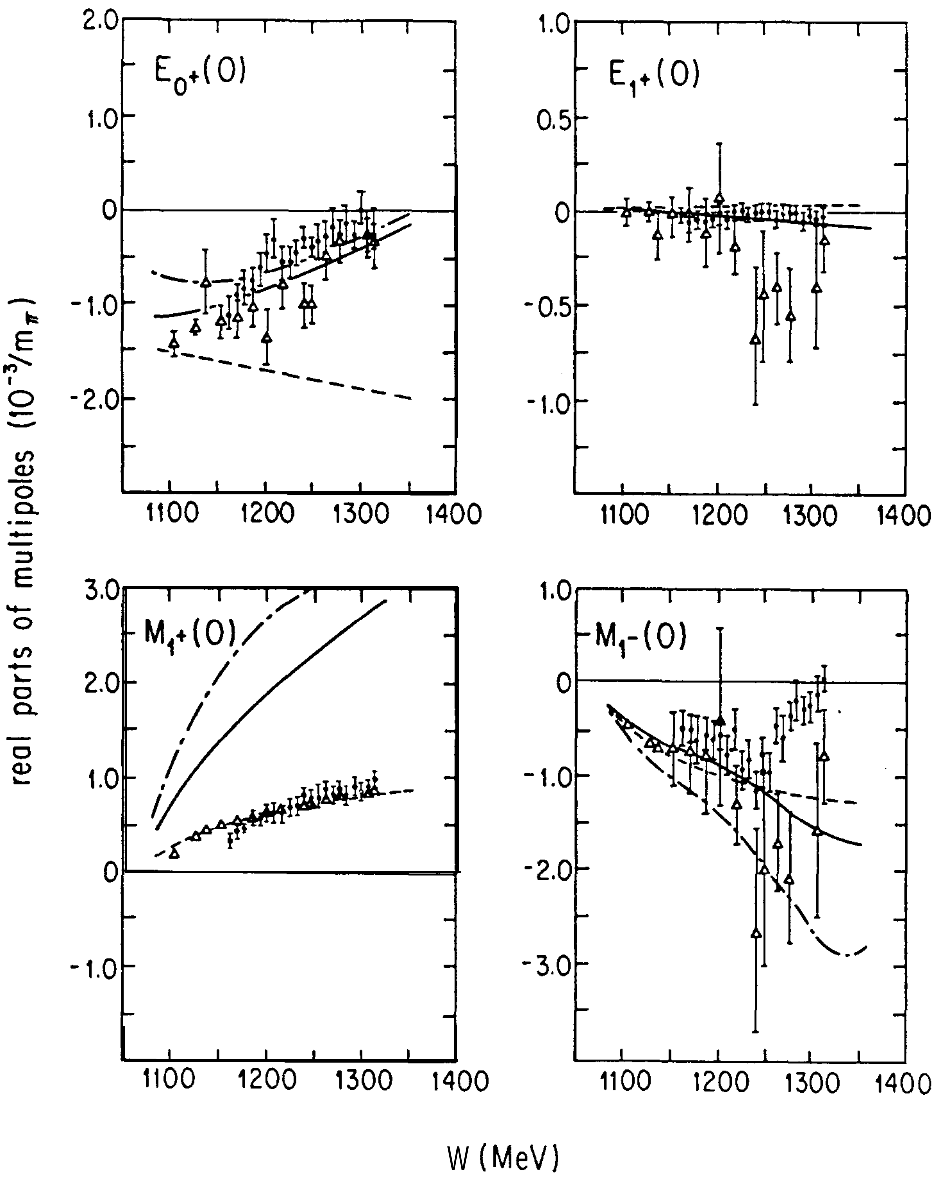

FIG. 4. Real parts of isospin-0 multipoles. Notations same as in Fig. 3.

ly. Both strongly disagree with the values of -2.47 predicted by the low energy theorems[19] and the previously inferred experimental result[20] of (-1.8 ± 0.6). This large discrepancy between the low-energy-theorem prediction and the latest experimental results may indicate a large rescattering effects" in the E_{0^+} amplitude. Since we unitarize $\gamma\pi$ amplitude by explicitly including πN recattering into the model, it would be of interest to compare our prediction with the ex-

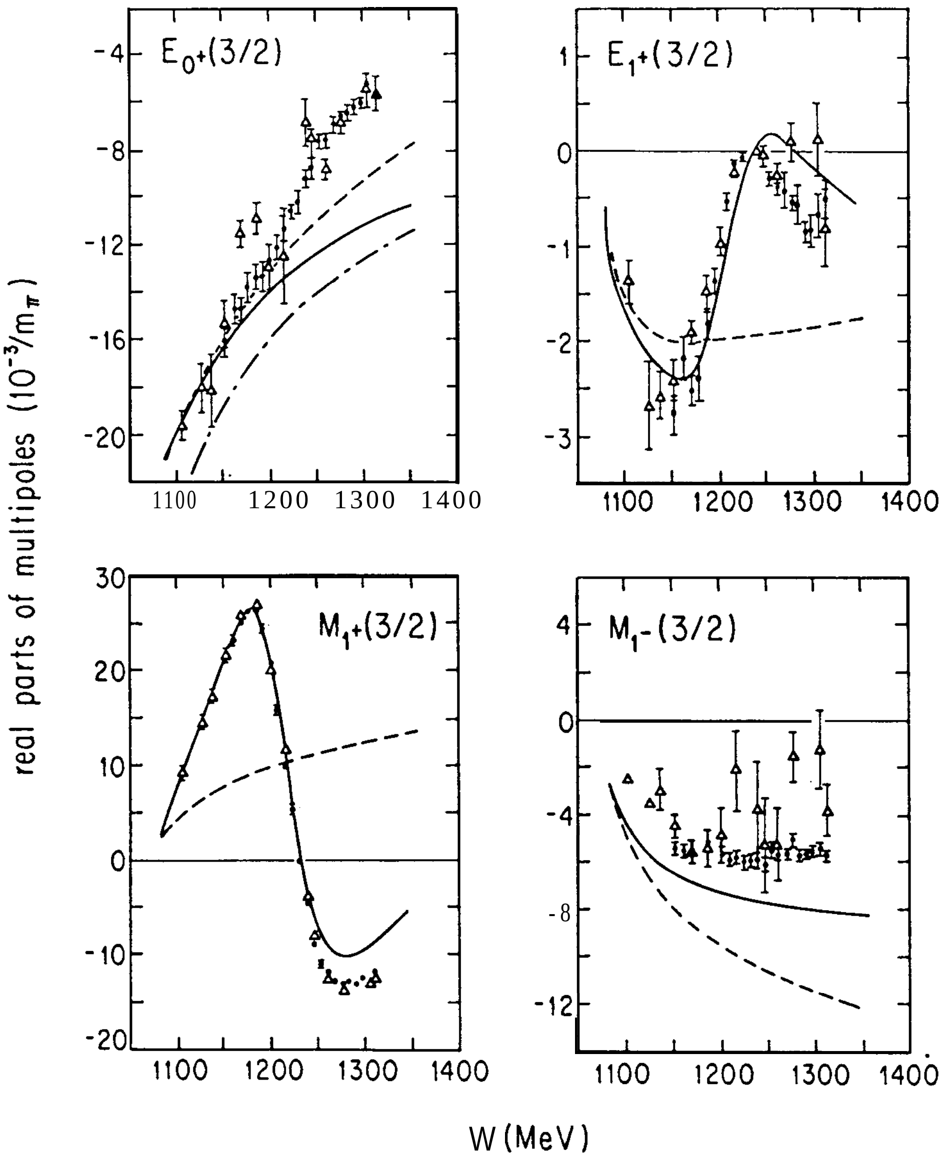

FIG. 5. Real parts of isospin-3/2 multipoles. Notations same as in Fig. 3.

periments.

The rescattering effects are reprsented by the second term in the right-hand side of Eq. (11). As pointed out before, it depends on the πN half-off-shell R-matrix elements. For E_{0^+}, only the πN S_{11} and S_{31} interactions are relevant. In Fig. 6, we compare the half-off-shell R-matrix elements calculated from the meson-exchange model (solid curves) described in Sec. IV-1 and the separable models employed in the calculations of Ref. 21. The main feature of the

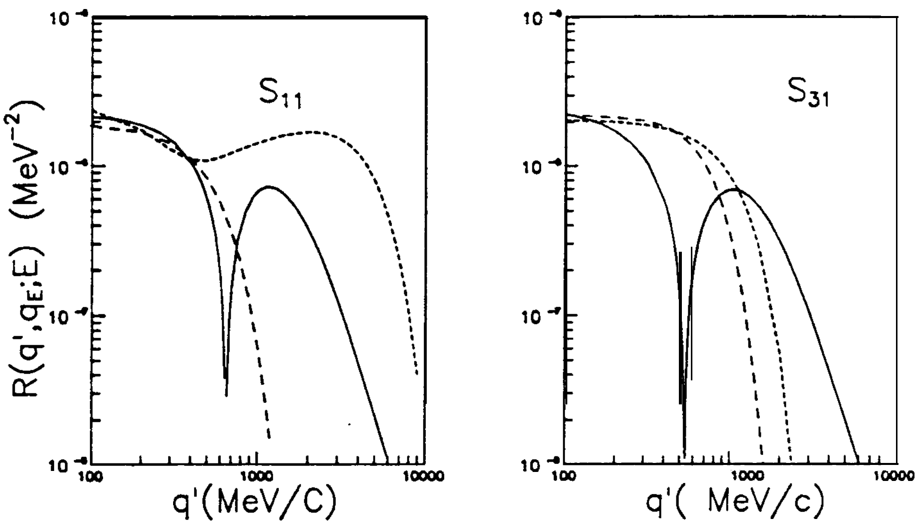

FIG. 6. Half-off-shell matrix elements $|R_{\pi N}(q', q_E; E)|$ of the πN reaction-matrix for energy 3 MeV above threshold given by our meson-exchange model (solid curves) and hvo different πN separable potentials developed by Betz and Lee[23] (dotted curve) and Ernest and Johnson[13] (dashed curve).

meson-exchange dynamics is the oscillatory behavior which has little resemblance with the simple separable models. It is therefore not surprising that the calculated rescattering effect is significantly different from that of the separable model. This is illustrated in Table I, where we also list the values from the analysis of the most recent Mainz data.” It is interesting to note that our meson-exchange results are very close to the Mainz values. This suggests that a large part of the difference between the Mainz value and the LET prediction can be accounted for

TABLE I. Results obtained for threshold value of $E_{0+}(p\pi^0)$ with S-wave πN interactions taken from the phenomenological separable potentials of Betz and Lee[23] (BL) and Ernst and Johnson[13] (EJ), and our meson-exchange model (MEM). Results obtained without including the final state interaction (FSI) and the Mainz data[18] are also listed for comparison.

E_γ (MeV)	No FSI	BL	EJ	MEM	Mainz
151.4	-2.39	-1.92	0.64	-0.53	-0.56
153.7	-2.37	-1.94	0.50	-0.51	-0.26
156.1	-2.35	-1.95	0.36	-0.52	-0.35

by the meson-exchange mechanisms. However, further work is needed to determine whether the meson-exchange model developed in this work can provide a detailed understanding of the deviations from the LET

V. SUMMARY

In summary, we have developed a meson-exchange πN model starting from an effective Lagrangian. It consists of using the nucleon, crossed nucleon, A, crossed A and σ- and p-exchange as driving terms in a three-dimensional reduction of the Bethe-Salpeter equation. The model gives excellent fits to the πN S-waves and P_{33} phase shifts. The predicted phase shifts in the weaker P_{13} and P_{31} channels are also in qualitative agreements with the data. The remaining discrepancies, as seen in Fig. 2, are perhaps mostly due to the neglect of a proper treatment of the nucleon mass renormalization arising from the nucleon pole mechanism. This will be pursued in the future, along with the extension of the approach to consider different reduction schemes and other meson-exchange mechanisms.

The approach has also been applied to construct a meson-exchange model for pion photoproduction on a single nucleon. It is unitary and gauge invariant. The driving terms are derived from an effective Lagrangian obtained from introducing minimal coupling into the Lagrangian used in developing the πN model. In this first attempt, the multipole amplitudes are evaluated with phenomenological separable πN potentials and agree well with the experiment. The final state interaction is seen to give important contributions in most channels, especially in 33 channels. The discrepancy seen in $M_{1^-}(1/2)$ is probably connected with the renormalization of nucleon-pole term. This is currently under investigation. Calculations of multipole amplitudes with meson-exchange πN interaction in the final state is also in progress.

We also find that the neutral pion photoproduction amplitude on proton at threshold strongly depends on the S_{11} πN interaction. The good S-waves results obtained with meson-exchange model, as shown in Fig. 1 allows a realistic estimate of the πN rescattering effects in the dipole amplitude E_{0^+} of pion photoproduction on the nucleon near threshold. Our results strongly suggest that the difference between the LET prediction and the recent Mainz value[18] can be understood to a very large extent by the meson-exchange mechanisms. Further theoretical and experimental studies are, however, needed to resovle this important question.

The meson-exchange model for πN interaction and pion photoproduction reactions on a nucleon developed above, when combined with the meson-exchange model of nucleon-nucleon interaction, could provide us with a unified microscopic theoretical framework for hadronic and electromagnetic nuclear reactions at low and intermediate energies. Such a theoretical model which involves hadronic degrees of freedom only, could possibly help to shed light on the important question of transition from hadronic picture to quark-gluon picture in the nuclear phenomena.

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