Low Energy Anomaly Processes in the Baryon Sector

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We construct an anomaly Lagrangian involvingbaryons, mesons and photons in the Skyrme model. The most simple process in this Lagrangian is the neutral pion photoproduction on nucleon. The value of electric dipole amplitude E_{0+} is calculated and compared with the experimental data. At the pion production threshold we obtain $E_{0+}{}^{(anomaly)}$ = 2.99 X 10^{-3}fm = 2.12 X $10^{-3}/m_{\pi^+}$. This value together with the low energy theorem prediction $E_{0+}{}^{(L.E.T.)}$ = -2.4 X $10^{-3}/m_{\pi^+}$ gives E_{0+} = -0.28 X $10^{-3}/m_{\pi^+}$, which is in good agreement with the measured value of E_{0+} = -0.5 $\pm$ 0.3 X $10^{-3}/m_{\pi^+}$. This may reconcile the longstanding difference between theory and experiment. Other anomaly processes such as the Compton scattering $\gamma P \to \gamma P$ and $\pi^- P \to \pi^0 \pi^- P$ are also discussed.

I. REVIEW ON NEUTRAL PION PHOTOPRODUCTION

I-l. Low Energy Theorem Prediction and Recent Experimental Data

It has been reported that the electric dipole amplitude E_{0+} of the neutral pion photoproduction deviates from the theoretical prediction of low energy theorem.[1] The Feynman diagrams involved in this process can be classified into four categories: the Born terms (Fig. 1), final-state interactions (F.S.I.) (Fig. 2), vector meson emission, and the A intermediate state. The last two kind of interactions are negligible at the pion production threshold and hence can be safely omitted in the discussion of electric dipole amplitude.

The low energy theorem, based upon PCAC, predicts that the Born term contribution for the electric dipole amplitude of the neutral pion production on the proton is of the form[3]

$$E_{0+} = -\frac{e}{4\pi}\frac{g_{\pi NN}}{2M_N}\left[\frac{m_\pi}{M_N} - (2+\mu_P)\left(\frac{m_\pi}{M_N}\right)^3\right] + \cdots \tag{1}$$

The electric dipole amplitude is related to the differential cross section at the threshold in the following manner

$$\left(\frac{d\sigma}{d\Omega}\right)\bigg|_{Th} = \left|\frac{P_\pi}{E_\gamma}\right| |E_{0+}|^2, \tag{2}$$

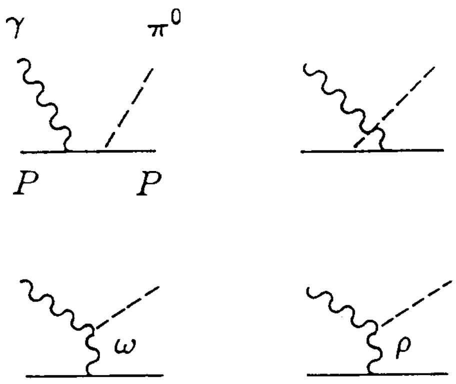

FIG. 1. Born terms which contribute to the $\gamma P \to \pi^0 P$.

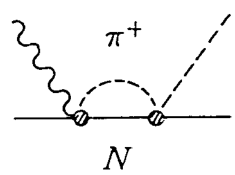

FIG. 2. Final state interactions: the bulb stands for the effects of all the Born terms and Kroll-Ruderman term in $\gamma P \to \pi^+ N$.

where P_π and E_γ are the momentum of the pion and the energy of photon respectively in the c.m. frame. The F.S.I. contribution to the electric dipole amplitude can be calculated by means of the K-matrix approach.[4] The numerical evaluation gives $E_{0+}^{(L.E.T.)} = -2.4 \times 10^{-3}/m_{\pi^+}$ and $E_{0+}^{(F.S.I.)} = -1.0 \times 10^{-3}/m_{\pi^+}$.

Recent experiments[1,2] measured $E_{0+} = -1.5 \times 10^{-3}/m_{\pi^+}$. If we adapt the K-matrix approach for the $E_{0+}^{(F.S.I.)}$ then we obtain $E_{0+} = -0.5 \times 10^{-3}/m_{\pi^+}$ for the Born term contribution, which is in serious conflict with the low energy prediction.

I-2. Anomaly **Effect**

A well-known yet educational example for the failure of low energy prediction is the prediction of the Sutherland-Veltman theorem on $\pi \to \gamma\gamma$. The Sutherland-Veltman theorem relates the three-current correlation functions

$$\Gamma_{\mu\nu\lambda}(k_1, k_2, q) \equiv \int d^4x d^4y e^{ik_2 - iqx} < 0|T(A^3_\lambda(x) J^{e.m.}_\nu(y) J^{e.m.}_\mu(0))|0> \tag{3}$$

and

$$\Gamma_{\mu\nu}(k_1, k_2, q) \equiv \frac{-ie^2(-q^2+m_\pi^2)}{f_\pi m_\pi^2} \int d^4x d^4y e^{ik_2 - iqx} \cdot <0|T(\partial^\lambda A_\lambda^3(x) J_\nu^{e.m.}(y) J_\mu^{e.m.}(0))|0> \tag{4}$$

by

$$g^\lambda \Gamma_{\mu\nu\lambda}(k_1, k_2, q) = \frac{f_\pi m_\pi^2}{e^2(m_\pi^2 - q^2)} \Gamma_{\mu\nu}(k_1, k_2, q). \tag{5}$$

Obviously, the left hand side of Eq. (5) vanishes in the $m_\pi \to 0$ limit. But it failed to give the correct decay rate of the $\pi^0 \to \gamma\gamma$. It is necessary to modify Eq. (5) in order to be in compliance with experiment. We add the anomaly term to Eq. (5) and obtain

$$g^\lambda \Gamma_{\mu\nu\lambda}(k_1, k_2, q) = \frac{f_\pi m_\pi^2}{e^2(m_\pi^2 - q^2)} \Gamma_{\mu\nu}(k_1, k_2, q) - \frac{i}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} k_{1\sigma} k_{2\rho}. \tag{6}$$

The second term in Eq. (6), which represents the triangle anomaly, accounts for almost all the decay rate for $\pi \to \gamma\gamma$. The lessons we learn from this case are

1. Whenever the PCAC prediction fails, consider the anomaly contribution first.
2. The anomaly effect could be the dominant contribution.
3. Anomaly non-anomaly contributions are of opposite signs.

The third observation may not be that general, but it is true for both $\pi \to \gamma\gamma$ and $\gamma P \to \pi^0 P$, as we shall see later. The above lessons lead to the consideration of the anomaly contribution in $\gamma P \to \pi^0 P$.

II. ANOMALY LAGRANGINS IN THE SKYRME MODEL

II-l. Baryon-Photon System

It is helpful to gain some insight of anomaly effects in the baryon sector if we consider the anomaly Lagrangian for baryon-photon system first.

Consider the meson-photon anomaly Lagrangian[5]

$$\begin{aligned} L_{\pi\gamma} = & \frac{-3e}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} A_\mu Tr[Q(\partial_\nu U U^{-1})(\partial_\alpha U U^{-1})(\partial_\beta U U^{-1}) \\ & + Q(U^{-1}\partial_\nu U)(U^{-1}\partial_\alpha U)(U^{-1}\partial_\beta U)] \\ & + \frac{i\alpha}{2\pi} \epsilon^{\mu\nu\alpha\beta} \partial_\mu A_\nu A_\alpha Tr[Q^2(\partial_\beta U)U^{-1} + Q^2 U^{-1}(\partial_\beta U) \\ & + \frac{1}{2} QUQU^{-1}(\partial_\beta U)U^{-1} + \frac{1}{2} QU^{-1}QU(\partial_\beta U^{-1})U], \end{aligned} \tag{7}$$

where

$$U = \exp \frac{i}{f_\pi} \lambda \cdot \phi \tag{8}$$

is the nonlinear realization of meson fields, Q is the quark charge matrix, and A is the photon field. In order to incoporate baryons into the Lagrangian, we replace the U by

$$\Sigma \equiv A(t)\Sigma_0 A(t)^+, \tag{9}$$

where

$$\begin{aligned} \Sigma_0 &= \exp \frac{i}{f_\pi} F(r)\hat{\mathbf{r}} \cdot \lambda \\ &= \cos F + i \sin F(\hat{\mathbf{r}} \cdot \lambda) \end{aligned} \tag{10}$$

is the soliton solution of the baryon[7] and A(t) = $a_0 + i\mathbf{a} \cdot \lambda$ are the collective coordinates in isospin space,[6] which obey

$$a_0^2 + |\mathbf{a}|^2 = 1. \tag{11}$$

Since U and Σ_0 share the same chiral transformation properties, the Lagrangian automatically satisfies the consistency equation.

The Lagrangian so constructed describes the anomaly interaction between the baryon and photon. It contributes to the Compton scattering $\gamma P \to \gamma P$ and gives predictions about the electromagnetic form factors and polarizability of the proton. A detailed calculation will be shown elsewhere.*

It is not difflcult to understand the necessity of such anomaly contribution in $\gamma P \to \gamma P$ from the point of view of the current-algebra analysis. In $\pi^0 \to \gamma\gamma$, the divergence of the axial current receives contributions from the pion mass term and the anomaly:

$$\partial^\mu j_\mu^5 = f_\pi m_\pi^2 \pi^0 + \frac{e^2}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}. \tag{12}$$

If we define

$$\begin{aligned} j_\mu^{5,\pi} &= f_\pi \partial_\mu \pi^0 \\ j_\mu^{5,\gamma} &= \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta \end{aligned} \tag{13}$$

for the meson and photon axial currents respectively, we can construct an effective Lagrangian

$$L_{\pi\gamma} \propto f_\pi \partial_\mu \pi^0 \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta \tag{14}$$

which can be identified immediately as a part of the well-known W.Z.W. Lagrangian. Repeating the same procedure, we find

$$\partial^\mu j_\mu^5 = 2iM_N\bar{\psi}\gamma_5\psi + \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}. \tag{15}$$

for the divergence of the baryon axial current. We also define

$$j_\mu^{5,N} = \bar{\psi}\gamma_\mu\gamma_5\psi \tag{16}$$

for the baryon axial current. Then we derive the effective Lagrangian for the anomaly baryon-photon interaction:

$$L_{N\gamma} \propto \bar{\psi}\gamma_\mu\gamma_5\psi\epsilon^{\mu\nu\alpha\beta}A_\nu\partial_\alpha A_\beta. \tag{17}$$

This Lagragian is not gauge invariant. However, we can add a single photon term to make it gauge invariant. The physics implied by this Lagrangian is quite fruitful:

1. In the Compton scattering $\gamma P \rightarrow \gamma P$, the anomaly effect indeed exists.
2. This effect is of order w^2 (photon energy) and hence should be taken into account in the analysis of proton polarizability.
3. The single photon piece, which is not wirtten down explicitly in Eq. (17), may give predictions about the electromagnetic form factors of the proton.

1X-2. **Baryon-Meson-Photon System**

In order to complete the construction of the anomaly Lagrangian for the baryon-meson-photon system, we add the pion fluctuation by substituting $A(t)\Sigma_0 A(t)^+$ by $U_\pi A(t)\Sigma_0 A(t)^+ U_\pi$,[9] where

$$U_\pi = U^{\frac{1}{2}}. \tag{18}$$

We get

$$\begin{aligned} L_{N\pi\gamma} = & \frac{-3e}{48\pi^2}\epsilon^{\mu\nu\alpha\beta}A_\mu Tr\{Q[\partial_\nu(U_\pi\Sigma U_\pi)(U_\pi\Sigma U_\pi)^{-1}] \\ & \cdot[\partial_\alpha(U_\pi\Sigma U_\pi)(U_\pi\Sigma U_\pi)^{-1}][\partial_\beta(U_\pi\Sigma U_\pi)(U_\pi\Sigma U_\pi)^{-1}] \\ & +Q[(U_\pi\Sigma U_\pi)^{-1}\partial_\nu(U_\pi\Sigma U_\pi)][(U_\pi\Sigma U_\pi)^{-1}\partial_\alpha(U_\pi\Sigma U_\pi)] \\ & \cdot[(U_\pi\Sigma U_\pi)^{-1}\partial_\beta(U_\pi\Sigma U_\pi)]\} + \frac{i\alpha}{2\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu A_\nu A_\alpha Tr \\ & \cdot\{Q^2[\partial_\beta(U_\pi\Sigma U_\pi)](U_\pi\Sigma U_\pi)^{-1} + Q^2(U_\pi\Sigma U_\pi)^{-1}[\partial_\beta(U_\pi\Sigma U_\pi)] \\ & +\frac{1}{2}Q(U_\pi\Sigma U_\pi)Q(U_\pi\Sigma U_\pi)^{-1}[\partial_\beta(U_\pi\Sigma U_\pi)](U_\pi\Sigma U_\pi)^{-1} \\ & +\frac{1}{2}Q(U_\pi\Sigma U_\pi)^{-1}Q(U_\pi\Sigma U_\pi)[\partial_\beta(U_\pi\Sigma U_\pi)^{-1}](U_\pi\Sigma U_\pi)\}. \end{aligned} \tag{19}$$

This Lagrangian describes the anomaly interaction among baryons, mesons and photons. The gauge invariance of this Lagrangian can be easily checked. Note that if we set $\Sigma = 1$, namely, no soliton excitation, then we recover the original meson-photon anomaly Lagrangian. This consistency check ensures that Eq. (19) indeed satisfies the consistency equation of the anomaly.[10]

Proceeding standard Skyrmion calculation, we obtain the contribution of anomaly Lagrangian to the differential cross section in various photoproducton processes:

- $\gamma P \rightarrow \pi^0 P$

$$\frac{da}{d\Omega} = \frac{M_P}{4E_\gamma(2\pi)^2}|K|^2 \frac{(3M_\pi^2 + 2|\mathbf{P}_\pi|^2)|\mathbf{P}_\pi|}{2(E_\gamma + Mp - \frac{E_\gamma E_\pi \cos\theta}{|\mathbf{P}_\pi|})}$$

- $\gamma N \rightarrow \pi^0 N$

$$\frac{d\sigma}{d\Omega} = \frac{M_N}{4E_\gamma(2\pi)^2}|K|^2 \frac{(3M_\pi^2 + 2|\mathbf{P}_\pi|^2)|\mathbf{P}_\pi|}{2(E_\gamma + M_N - \frac{E_\gamma E_\pi \cos\theta}{|\mathbf{P}_\pi|})}$$

- yP+n+N

$$\frac{d\sigma}{d\Omega} = 0$$

All the dynamical variables are in the laboratory frame. The numerical constants in the above expression are defined as follows

$$C_1 = \int_0^\infty \sin^2 F dr,$$

$$C_2 = \int^\infty r^2 \left(\frac{\sin F \cos F}{r} \frac{dF}{dr} - \frac{\sin^2 F}{r^2} \right) dr, \tag{20}$$

$$K \equiv \frac{-ie}{216\pi f_\pi}(C_1 + \frac{2}{3}C_2)$$

We adapt the following approximation of F(r) for numerical evaluation

$$F(r) = \begin{cases} \pi - 3.087r + 1.285r^2 & (r < 1) \\ \frac{0.62}{r^2} & (r > 1) \end{cases} \tag{21}$$

Two important results can be read off directly from the differential cross sections. First, we note that the anomaly contribution to $\gamma P \rightarrow \pi^+ N$ is zero, in agreement with the experimental fact that no violation of the low energy theorem is observed. The vanishing anomaly amplitude of this process can be understood as a direct consequence of the structure of the axial current. The anomaly only contributes to the neutral axial current, hence processes involving charged baryon current receive no contribution from anomaly. Secondly, we note that the differential cross section of $\gamma N \to \pi^0 N$ is roughly the same as the differential cross section of $\gamma P \to \pi^0 P$. But the electromagnetic form factor of the neutron is much smaller than that of the proton, so the Born terms of $\gamma N \to \pi^0 N$ give little contribution to the electric dipole amplitude. This make the anomaly contribution become the dominant one.

11-3. Numerical Evaluation

Converting the differential cross section to E_{0+}, we find

$$E_{0+}^{(\mathrm{anomaly})} = 2.12 \times 10^{-3}/m_\pi^+ . \tag{22}$$

If we add this amplitude to the Born term and F.S.I. contribution, it yields

$$E_{0+}^{(total)} = -1.28 \times 10^{-3}/m_\pi^+ , \tag{23}$$

which is in good agreement with the experimental value

$$E_{0+}^{(anomaly)} = -1.5 \pm 0.3 \times 10^{-3}/m_\pi^+ . \tag{24}$$

This result strongly supports our conjecture of the anomaly contribution.

III. DISCUSSION

It has been over twenty years since the discovery of the anomaly, yet only one physical process $\pi \to \gamma\gamma$ has been closely examined. The experiment of neutral pion photoproduction on proton offers us another evidence of anomaly. This is the first anomaly process found in the baryon sector. There are lots of more processes worth exploring. The following is a partial list for the processes to which anomaly contributes.

meson-photon $\pi^0 \to \gamma\gamma, \gamma \to \pi^+\pi^-\pi^0$.
baryon-photon $\gamma P \to \gamma P, \gamma N \to \gamma N$.
baryon-meson-photon $\gamma P \to \pi^0 P, \gamma N \to \pi^0 N$.
pure **mesonic** reaction $K^+K^- \to \pi^+\pi^-\pi^0$.
baryon-meson $P\pi^- \to \pi^0\pi^- P$[11].

Experiments are called for in order to confire the anomaly effects in these processes.

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