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Radiation of Super-phase Velocity

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A classical treatment of the radiation of super-phase velocity of scalar and vector fields is presented in this paper with a hope that it could be extended to the case of multiple meson production in a nucleus by nucleons of a very high energy.

WHEN a wave source moves with a velocity greater than the phase velocity of the wave itself, a particular radiation would occur, and we call this the radiation of super-phase velocity. Examples are the bow wave produced by the motion of a ship, the Mach wave associated with supersonic velocities and, the well-known "Cherenkov radiation" by the charged particle of super-light speed moving through a medium. In recent years, the development of high energy physics enables us to use the protons of energies higher than 10^{12} ev with cosmic rays. In this case the wave length of protons ($\sim 10^{-17}$ cm) is much less than the dimension of nucleus ($\sim 10^{-12}$ cm). Therefore the behaviour of proton inside nucleus may be considered phenomenologically as a wave process, and we can expect that multiple meson production process similar to the Cherenkov radiation would occur when the proton passes through nucleus(2).

In this paper, we present a classical treatment of the radiation of super-phase velocity of scalar and vector fields with a hope that the present treatment could be extended in the near future to the case of multiple meson production when nucleons pass through a nuclens with a very high energy. Throughout this paper we use the system of units in which $\hbar = c = 1$.

I. Scalar *Field*

For simplicity, we use the following simplifying assumtions:

(1) The source is assumed to move at constant velocity greater than the phase velocity of field in the medium.

(2) The medium with "index of refraction n" is finite.

Suppose the source of field moves in the direction of the z-axis. The equation of scalar field in interaction with the source will have the form,

$$\Delta\varphi-n^2\left(\frac{\partial^2}{\partial t^2}+m^2\right)\varphi=-\frac{g}{n^2}\sqrt{1-v^2}\,\delta(x)\delta(y)\delta(z-vt),$$

where $\delta(x)$ is the Dirac δ function, m is the rest mass of the scalar field, and v is the velocity of the source.

Using the Green function

$$G(x)=\frac{1}{(2\pi^4)}\int e^{i\vec{k}\cdot\vec{r}-iEt}\left\{\frac{1}{k^2-n^2(E^2-m^2)}+i\pi\frac{E}{|E|}\delta[k^2-n^2(E^2-m^2)]\right\}d^3kdE$$

and relation

$$\int_0^{2\pi}\exp(ipR\,\cos\theta)d\theta=2\pi J_0(pR),$$

we can get φ as

$$\varphi=\frac{g}{2\pi}\,\frac{\sqrt{1-v^2}}{n^2}\int e^{i(z-vt)k}\left\{i\frac{k}{|k|}J_0(R\sqrt{a^2k^2-n^2m^2})-N_0(R\sqrt{a^2k^2-n^2m^2})\right\}dk,$$

where J_0 and N_0 are the zero-order Bessel function of the first and second kind.

The energy flow through cylindrical surface is

$$dw=\int G\cdot ds=\int\left(\frac{\partial\varphi}{\partial R}\right)\left(\frac{\partial\varphi}{\partial t}\right)^*2\pi Rdz.$$

In consideration of the relation

$$J_0'(x)N_0(x)-J_0(x)N_0'(x)=\frac{2}{\pi}\,\frac{1}{x},$$

we obtain the energy dw emitted per unit time in the energy range between E and $E+dE$,

$$dw=\frac{g}{4\pi}\,\frac{1}{n^4}\,\frac{1-v^2}{v}\,EdE.$$

The angular distribution of radiation is obtained explicitly as

$$\varphi=\begin{cases}0 & vt<z+aR\\[2mm] -\dfrac{g}{2\pi}\,\dfrac{1-v^2}{n^2}\,\dfrac{\cos\frac{nm}{a}\sqrt{(vt-z)^2-a^2R^2}}{\sqrt{(vt-z)^2-a^2R^2}} & vt>z+aR.\end{cases}$$

Of course at $t=0$, the equation of equipotential surface is

$$z^2-a^2R^2=0.$$

The angle of radiation is given by

$$\cos\,\theta=\frac{1}{nv},$$

or quantum mechanically by

$$\cos\theta = \frac{1}{nv}\frac{E}{K},$$

where $K^2 = p^2 + k^2$ and $E^2 = K^2 + m^2$.

II. *Vector Field*

The vector field equation with the source moving in the direction of z-axis is given by

$$\left[\triangle - n^2\left(\frac{\partial^2}{\partial t^2} + m^2\right)\right]A_4 = -i\frac{g}{n^2}\sqrt{1-v^2}\,\delta(x)\delta(y)\delta(z-vt)$$

with

$$A_1 = A_2 = 0 \text{ and } A_3 = -in^2vA_4.$$

By the same way as in the case of scalar field, we get

$$A_4 = \frac{ig}{8\pi}\sqrt{1-v^2}\int\frac{1}{n^2}e^{i(z-vt)k}\left\{i\frac{k}{|k|}J_0(R\sqrt{a^2k^2-n^2m^2}) - N_0(R\sqrt{a^2k^2-n^2m^2})\right\}dk,$$
$$(a^2k^2 - n^2m^2 > 0).$$

In the subsequent part, we use the cylindrical coordinates;

	$\hat{R}$	$\hat{\varphi}$	$\hat{z}$
A	0	0	AS
E	$\partial_R(iA_4)$	0	$-a^2\partial_z(iA_4)$
H	0	$\partial_R(A_3)$	0

The energy flow through the cylindrical surface is then

$$dw = \int(\boldsymbol{E}\times\boldsymbol{H}^* + m^2(-iA_4)\boldsymbol{A})d\boldsymbol{s}$$
$$= \int(-E_z\cdot H_\varphi{}^* - im^2A_4A_R)2\pi R dz$$
$$= \frac{g^2}{4\pi}(1-v^2)\int_m^{E_{max}} v\left(1-\frac{1}{n^2v^2}\right)EdE.$$

The angle of radiation is given by the same relation

$$\cos\theta = \frac{1}{nv}\frac{E}{K}.$$

For a special case. when $m=O$, *we* arrive at the Cherenkov radiation in an electromagnetic field, namely,

$$dw = \frac{e^2}{4}(1-v^2)\int v\left(1-\frac{1}{n^2v^2}\right)\nu d\nu,$$

which agrees exactly with the classical results.

In the case of production of p-meson and p-meson in a nucleus by high energy nucleons the index of refraction n must be calculated from the optical model and the dispersion relation, and, in addition, another contribution from transition radiation must be considered in the nuclear surface.

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(2) W. Czyz, T. Ericson and S. L. Glashow, Nuclear Physics **13**, 516 (1959).