CHINESE JOURNAL OF PHYSICS VOL. 33, NO. 3 JUNE 1995

Determining the Top Mass from $t\bar{t}$ Dilepton Decay Events

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(Received April 4, 1995)

A method for determining the top mass from $t\bar{t}$ dilepton decay events is proposed. Based on simulation, it is shown that an accuracy within 10 GeV/c^2 may be achieved with a few $t\bar{t}$ dilepton events.

PACS. 14.65.Ha – Top quarks.

Recently both CDF and DO announced the observation of top quark and published the measured top quark mass [1]. For CDF, the top quark mass is $176 \pm 8(\text{stat.})\pm 10(\text{syst.})$ GeV/c^2 while for DO it is $199\pm^{31}_{25}(\text{stat.})\pm 22(\text{syst.})$ GeV/c^2. These mass values were obtained by fitting the reconstructed $t\bar{t}$ lepton-jets decay events.

It is generally believed to be difficult to extract the top quark mass value from a limited number of dilepton decay events as there are two neutrinos undetected in such events. Dalitz and Goldstein [2] proposed a method to obtain a most probable value of top quark mass from even a single top dilepton event. Their method assumed that a parton and an anti-parton annihilate to produce the top and the anti-top and that the transverse momenta of the top and the anti-top exactly cancelled. These assumptions do not always conform to the experimental situation. Both simulation and data indicate that for top dilepton candidate events, the final states are usually more complicated than just two leptons and two jets.

In this work, we propose a new method to extract the top quark mass value from the dilepton events. We assume that the missing transverse energy ($\not{E}_\perp$) measured equal to the sum of the $E_\perp$ of the two neutrinos, when missing $E_\perp$ of the jets are taken into account by appropriate jet energy corrections. Applying our method to simulated data indicates that it is possible to obtain the top quark mass from 10 dilepton events with statistical error below 10 GeV/c^2. The main limiting factor of the method is the jet energy resolution.

The kinematics of top quark decay had been investigated by Dalitz and Goldstein [3]. Here we briefly describe the notation we used.

Given the momenta k_b and $k_{\bar{\ell}}$ of the b-quark and the anti-lepton which are decay products of the top quark, we may construct the following orthonormal frame:

$$\begin{aligned} \hat{c}_1 &= \frac{1}{\sin\theta_{b\bar{\ell}}}(\hat{k}_b - \cos\theta_{b\bar{\ell}}\,\hat{k}_{\bar{\ell}})\,, \\ \hat{c}_2 &= \frac{1}{\sin\theta_{b\bar{\ell}}}\,\hat{k}_{\bar{\ell}} \times \hat{k}_b\,, \\ \hat{c}_3 &= \hat{k}_{\bar{\ell}}\,, \end{aligned} \tag{1}$$

where $\theta_{b\bar{\ell}}$ is the angle between the b-jet and the anti-lepton. Similarly we denote by $\hat{\bar{c}}_1, \hat{\bar{c}}_2, \hat{\bar{c}}_3$ the orthonormal frame constructed from $k_{\bar{b}}$ and k_ℓ where the b-quark and the lepton are the decay products of the anti-top quark.

The neutrino direction $\hat{k}_\nu$ can be expressed in terms of the above orthonormal frame as follows:

$$\hat{k}_\nu = \sum_{i=1}^{3} e_i \hat{c}_i\,,$$

where

$$e_1 = \frac{1}{\beta_b \sin\theta_{b\bar{\ell}}}\left[1 - \frac{z-\kappa^2}{2xE_b} - \beta_b\cos\theta_{b\bar{\ell}}\left(1 - \frac{M_W^2}{2xE_{\bar{\ell}}}\right)\right], \tag{2}$$

$$e_3 = 1 - \frac{M_W^2}{2xE_{\bar{\ell}}}\,, \tag{3}$$

$$e_2^2 = 1 - e_1^2 - e_3^2\,, \tag{4}$$

$$x = E_\nu\,, \quad z = m_t^2\,, \quad \kappa^2 = M_W^2 + m_b^2 + 2E_{\bar{\ell}}E_b(1 - \beta_b \cos\theta_{b\bar{\ell}})\,. \tag{5}$$

Similarly, we have for the anti-neutrino from the anti-top decay,

$$\hat{k}_{\bar{\nu}} = \sum_{i=1}^{3} f_i \hat{\bar{c}}_i\,,$$

where

$$f_1 = \frac{1}{\beta_{\bar{b}} sin\theta_{\bar{b}\ell}}\left[1 - \frac{z-\bar{\kappa}^2}{2yE_{\bar{b}}} - \beta_{\bar{b}}\cos\theta_{\bar{b}\ell}\left(1 - \frac{M_W^2}{2yE_\ell}\right)\right], \tag{6}$$

$$f_3 = 1 - \frac{M_W^2}{2yE_\ell}\,, \tag{7}$$

$$f_2^2 = 1 - f_1^2 - f_3^2\,, \tag{8}$$

$$y = E_{\bar{\nu}}\,, \quad \bar{\kappa}^2 = M_W^2 + m_{\bar{b}}^2 + 2E_\ell E_{\bar{b}}(1 - \beta_{\bar{b}} \cos\theta_{\bar{b}\ell})\,. \tag{9}$$

In these formulas, E_b represents the energy of the b-quark, β_b represents the velocity of the b-quark etc.

It is straightforward to derive the following kinematical limits for the three variables x, y, z that we used:

$$\frac{M_W^2}{4E_{\bar{\ell}}} \le x \le \frac{M_W^2}{4E_{\bar{\ell}}}\gamma_W^2\,; \quad \frac{M_W^2}{4E_\ell} \le y \le \frac{M_W^2}{4E_\ell}\gamma_{\bar{W}}^2\,, \tag{10}$$

$$z - \kappa^2 \ge \frac{m_b^2 M_W^2}{2E_{\bar{\ell}}E_b(1-\beta_b\cos\theta_{b\bar{\ell}})}\,; \quad z - \bar{\kappa}^2 \ge \frac{m_{\bar{b}}^2 M_W^2}{2E_\ell E_{\bar{b}}(1-\beta_{\bar{b}}\cos\theta_{\bar{b}\ell})}\,, \tag{11}$$

$$\frac{z-\kappa^2}{2E_b(1+\beta_b)} \le x \le \frac{z-\kappa^2}{2E_b(1-\beta_b)}\,; \quad \frac{z-\bar{\kappa}^2}{2E_{\bar{b}}(1+\beta_{\bar{b}})} \le y \le \frac{z-\bar{\kappa}^2}{2E_{\bar{b}}(1-\beta_{\bar{b}})}\,, \tag{12}$$

where $\gamma_W = E_W/M_W$, $\gamma_{\bar{W}} = E_{\bar{W}}/M_W$ are the Lorentz γ factors for W^+ and W^- respectively. Assuming that the measured missing transverse energy equals the sum of the transverse energy of the neutrino and the anti-neutrino, we have

$$\sum_{i=1}^{3}(x e_i \hat{c}_{i\perp} + y f_i \hat{\bar{c}}_{i\perp}) = \vec{\not{E}}_\perp\,, \tag{13}$$

where $\hat{c}_{i\perp}$ denotes the transverse component of $\hat{c}_i$, etc.

It follows from Eq. (13) that we have the relation

$$z = \frac{\bar{Q}_0 Q_2 - Q_0 \bar{Q}_2}{2(\bar{Q}_0 Q_1 - Q_0 \bar{Q}_1)} = \frac{2(\bar{Q}_1 Q_2 - Q_1 \bar{Q}_2)}{\bar{Q}_0 Q_2 - Q_0 \bar{Q}_2}\,, \tag{14}$$

where

$$Q_0 = \left(\frac{\hat{\bar{c}}_{2\perp} \times \hat{c}_{2\perp}}{2E_b\beta_b \sin\theta_{b\bar{\ell}}}\right)^2 + (\vec{B} \times \hat{\bar{c}}_{2\perp})^2\,, \tag{15}$$

$$\begin{aligned} Q_1 = {} & \frac{1}{2E_b}\left(\frac{\hat{\bar{c}}_{2\perp} \times \hat{c}_{2\perp}}{\beta_b \sin\theta_{b\bar{\ell}}}\right)^2 \left[(1-\beta_b\cos\theta_{b\bar{\ell}})x + \frac{M_W^2}{2E_{\bar{\ell}}}\beta_b\cos\theta_{b\bar{\ell}} + \frac{\kappa^2}{2E_b}\right] \\ & + (\vec{B} \times \hat{\bar{c}}_{2\perp})[(\vec{A} \times \hat{\bar{c}}_{2\perp})x + (\vec{\bar{A}} \times \hat{\bar{c}}_{2\perp})y + \vec{C} \times \hat{\bar{c}}_{2\perp}]\,, \end{aligned} \tag{16}$$

$$\begin{aligned} Q_2 = {} & \left(\frac{\hat{\bar{c}}_{2\perp} \times \hat{c}_{2\perp}}{\beta_b \sin\theta_{b\bar{\ell}}}\right)^2 \left[(1-\beta_b\cos\theta_{b\bar{\ell}})x + \frac{M_W^2}{2E_{\bar{\ell}}}\beta_b\cos\theta_{b\bar{\ell}} + \frac{\kappa^2}{2E_b}\right]^2 \\ & + [(\vec{A} \times \hat{\bar{c}}_{2\perp})x + (\vec{\bar{A}} \times \hat{\bar{c}}_{2\perp})y + \vec{C} \times \hat{\bar{c}}_{2\perp}]^2 \\ & - (\hat{\bar{c}}_{2\perp} \times \hat{c}_{2\perp})^2 \frac{M_W^2}{E_{\bar{\ell}}}\left(x - \frac{M_W^2}{4E_{\bar{\ell}}}\right)\,, \end{aligned} \tag{17}$$

$$\bar{Q}_0 = \left(\frac{\hat{c}_{2\perp} \times \hat{\bar{c}}_{2\perp}}{2E_{\bar{b}}\beta_{\bar{b}} \sin\theta_{\bar{b}\ell}}\right)^2 + (\vec{B} \times \hat{c}_{2\perp})^2 , \tag{18}$$

$$\begin{aligned}\bar{Q}_1 = & \frac{1}{2E_{\bar{b}}}\left(\frac{\hat{c}_{2\perp} \times \hat{\bar{c}}_{2\perp}}{\beta_{\bar{b}} \sin\theta_{\bar{b}\ell}}\right)^2 \left[(1-\beta_{\bar{b}}\cos\theta_{\bar{b}\ell})y + \frac{M_W^2}{2E_\ell}\beta_{\bar{b}}\cos\theta_{\bar{b}\ell} + \frac{\bar{\kappa}^2}{2E_{\bar{b}}}\right] \\ & +(\vec{B} \times \hat{c}_{2\perp})[(\vec{A} \times \hat{c}_{2\perp})x + (\vec{\bar{A}} \times \hat{c}_{2\perp})y + \vec{C} \times \hat{c}_{2\perp}],\end{aligned} \tag{19}$$

$$\begin{aligned}\bar{Q}_2 = & \left(\frac{\hat{c}_{2\perp} \times \hat{\bar{c}}_{2\perp}}{\beta_{\bar{b}} \sin\theta_{\bar{b}\ell}}\right)^2 \left[(1-\beta_{\bar{b}}\cos\theta_{\bar{b}\ell})y + \frac{M_W^2}{2E_\ell}\beta_{\bar{b}}\cos\theta_{\bar{b}\ell} + \frac{\bar{\kappa}^2}{2E_{\bar{b}}}\right]^2 \\ & +[(\vec{A} \times \hat{c}_{2\perp})x + (\vec{\bar{A}} \times \hat{c}_{2\perp})y + \vec{C} \times \hat{c}_{2\perp}]^2 \\ & -(\hat{\bar{c}}_{2\perp} \times \hat{c}_{2\perp})^2 \frac{M_W^2}{E_\ell}\left(y - \frac{M_W^2}{4E_\ell}\right),\end{aligned} \tag{20}$$

and

$$\begin{aligned}\vec{A} = & \left(\frac{1-\beta_b\cos\theta_{b\bar{\ell}}}{\beta_b \sin\theta_{b\bar{\ell}}}\right)\hat{c}_{1\perp} + \hat{c}_{3\perp}, \quad \vec{\bar{A}} = \left(\frac{1-\beta_{\bar{b}}\sin\theta_{\bar{b}\ell}}{\beta_{\bar{b}} \sin\theta_{\bar{b}\ell}}\right)\hat{\bar{c}}_{1\perp} + \hat{\bar{c}}_{3\perp}, \\ \vec{B} = & \frac{1}{2E_b\beta_b\sin\theta_{b\bar{\ell}}}\hat{c}_{1\perp} + \frac{1}{2E_{\bar{b}}\beta_{\bar{b}}\sin\theta_{\bar{b}\ell}}\hat{\bar{c}}_{1\perp}, \\ \vec{C} = & \frac{1}{\beta_b\sin\theta_{b\bar{\ell}}}\left(\frac{\kappa^2}{2E_b} + \frac{M_W^2}{2E_{\bar{\ell}}}\beta_b\cos\theta_{b\bar{\ell}}\right)\hat{c}_{1\perp} - \frac{M_W^2}{2E_{\bar{\ell}}}\hat{c}_{3\perp} \\ & + \frac{1}{\beta_{\bar{b}}\sin\theta_{\bar{b}\ell}}\left(\frac{\bar{\kappa}^2}{2E_{\bar{b}}} + \frac{M_W^2}{2E_\ell}\beta_{\bar{b}}\cos\theta_{\bar{b}\ell}\right)\hat{\bar{c}}_{1\perp} - \frac{M_W^2}{2E_\ell}\hat{\bar{c}}_{3\perp} - \vec{\not{P}}_\perp .\end{aligned} \tag{21}$$

Since all the vectors $\vec{A}, \vec{\bar{A}}, \vec{B}, \vec{C}, \hat{c}_{i\perp}$ and $\hat{\bar{c}}_{i\perp}$ are transverse, their cross products lie in the z-direction and can be treated as scalars. This is to be understood in Eqs. (15)-(20).

The top quark mass is determined in the following way. We first search for the solution of Eq. (14) in the x-y plane, i.e., we compute the two expressions involving Q_i, $\bar{Q}_i$ and require that they be equal within a certain accuracy according to the jet energy resolution and the Breit-Wigner width of the top. A curve on the s-y plane is obtained and a top-quark mass value z is associated with each point on the curve through Eq. (14). We show some examples of such curve in Fig. 1. Thus for each top dilepton event, the allowed values of top-quark mass will cover a certain region of the z-axis. If there are several dilepton events, we can obtain a histogram according to how many times a given interval on the z-axis is covered. Alternatively, we may just choose the central value of the allowed region for each event and obtain a histogram as illustrated in Fig. 2(a) for simulated top dilepton events. This allows us to determine the mean value and the standard deviation of the top-quark mass.

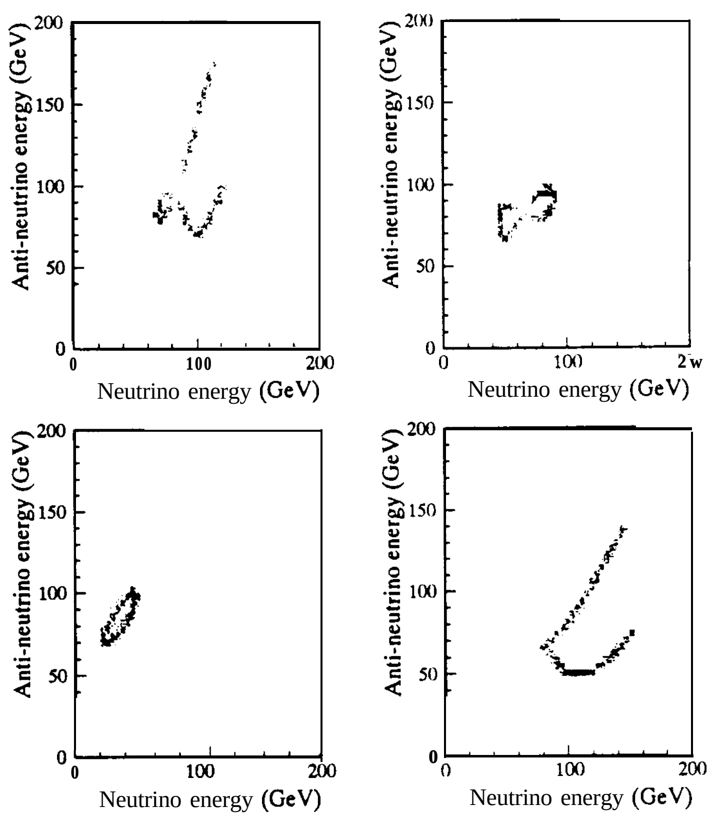

FIG. 1. Examples of the curve on the z-y plane obtained from Eq. (14).

There are two complications. First, there is a two-fold ambiguity in associating a given lepton to the corresponding b-jet. The amibugities increase if there are more than two jets in the event. If we chose the wrong combination, we would be computing a different Lorentz invariant z' than the top-quark mass-squared z. With this ambiguity, for each event, we may get two or more curves on the z-y plane. Example of the histogram for top mass distribution from the wrong combinations is shown in Fig. 2(b). The wrong combinations are often rejected by kinematical cuts. Those passing the kinematical cuts will contribute as background, as illustrated in Fig. 2(c). Second, in looking for solutions of Eq. (14) on the z-y plane, we need only limit our search to the region defined by Eq. (10). In order to choose a proper cut on the maximum value of $\gamma_W(\gamma_{\bar{W}})$, we simulate proton-anti-proton collisions at 1.8 TeV to produce $t\bar{t}$ pairs with the top-quark mass ranging from 150 GeV/c^2 to 220 GeV/c^2. From the simulated result, we apply a cut of γ_W = 2.

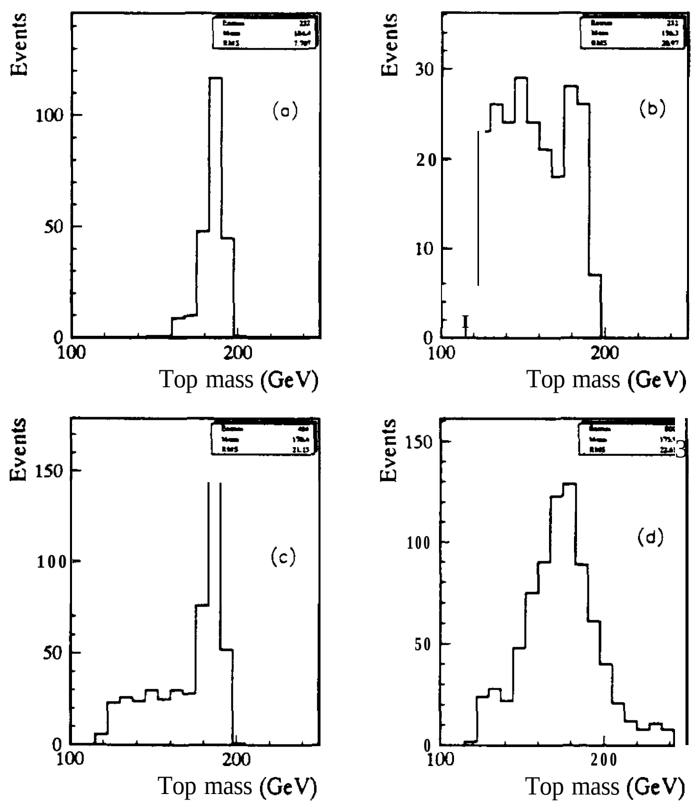

FIG. 2. Top mass distribution from simulated events: (a) choosing the right combination of leptons and b-jets, the width reflects the intrinsic ambiguities of the method; (b) same as in (a) but choosing the wrong combination of leptons and b-jets; (c) superposition of (a) and (b); (d) PYTHIA level simulation including effects of initial and final state radiation, calorimeter resolution etc.

The method is tested on simulated $t\bar{t}$ dilepton events. For the simulation, we use PYTHIA 5.7, including initial and final state radiation and assuming, for hadronic calorimeter, the conservative estimate of the energy resolution $\sigma_E/E = 100\%/\sqrt{E}$. An example of the simulated result is given in Fig. 2(d). The distribution of the top mass is Gaussian-like with the standard deviation σ_M = 22.6 GeV. For N top dilepton events, the expected statistical error on the top mass measurement is thus $\sigma_M/\sqrt{N}$. The simulation results clearly indicate that the method works.

Our method is insensitive to initial state gluon radiation. It is also insensitive to final state gluon radiation by top quarks. The final state gluon radiation from b-quark may be taken into consideration by properly adjusting the definition of b-jet. The neutrinos that may be present in the b-jets will contribute to the missing transverse energy and proper corrections are needed.

In summary, we have proposed a method of determining the top-quark mass from a few top-dilepton events. The only assumption being that a top-quark and an anti-top quark are produced and that the observed missing transverse energy equals the sum of the transverse energies of the two neutrinos from $t\bar{t}$ decay. Applying the method to simulated data indicates that the top mass may be determined to within an accuracy of 10 GeV with only 10 top-dilepton events. We expect the energy resolution of the jets and the jet combinatorial background will be the limiting factor in the accuracy of mass determination when applying the method to real data. There may still be room for improvement in our method and the investigation is underway.

ACKNOWLEDGMENTS

This work is supported in part by a grant from the National Science Council, Taiwan, R.O.C. under the contract number NSC84-2112-M-001-001-PE. One of us (S.-C. Lee) would like to thank Dr. H. C. Kao for checking some of the formulas.

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