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Strong Electric-Field Effects on Doubly Excited Feshbach Resonant States of H^- below the N = 3 Threshold of Hydrogen Atom†

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Electric-field effects on the doubly excited resonances of H^- below the $H(N=3)$ threshold are investigated theoretically using a method of complex-coordinate rotation. Products of Slater orbitals are used to represent the two-electron wave functions, with $l_{max} = 5$ employed for individual electron. Block matrices with up to $L_{max} = 6$ (I-states) are used. Convergence behaviors for the resonance parameters (resonance energy and width) are examined by using different values of L_{max}. Results for the electric-field effects on the $M = 0$ components of the Feshbach $^1S^e(1), ^1P^o(1), ^1D^e(1)$, and "D"(1) resonances are given, as well as the effects on the $M = \pm 1$ components of the latter three states.

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I. INTRODUCTION

A hydrogen negative ion is a three-body system consisting of two electrons and a proton interacting through Coulomb forces. Such a non-trivial system has only one bound state, the so-called $1s^2\,^1S^e$ state in the notation for single-particle approximation. It does not have any singly-excited bound states. It has, however, rich resonance structures of doubly excited states, lying in the $e^- - H$ scattering continua, and in the vicinity of the excited states of the target hydrogen atoms. Measured from the $1s^2\,^1S^e$ ground state, such doubly excited states are at least 10.3 eV away. From the experimental side, they are usually observed in collisional experiments between electrons and hydrogen atoms [1]. Observations of doubly excited $^1P^o$ resonant states (the lowest $N = 2\ ^1P^o$ Feshbach resonance is 10.926

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eV away from the H^- ground state) in the photoionization measurements using a single-photon light source represent a great experimental challenge, as no tunable lasers exist in this wave length region ($\sim$ 1130 A). A unique experimental set up was carried out at the Los Alamos Meson Physics Facility (LAMPF) by researchers led by the New Mexico team [2]. By accelerating the H^- beam to reach an energy of about 800 MeV, the ion beam would travel at a speed of about 84% of the speed of light. The beam is then intersected with a laser beam with various intersecting angles. As a result, the energy of the laser photon, in the rest frame of the H^- beam, is shifted to a considerably higher value due to the Doppler effect. For example, using the fourth harmonic of a Nd:YAG laser (where YAG denotes yttrium aluminum garnet), the energy of the laser photon is about 4.66 eV in the laboratory. In the internal frame of the intersecting relativistic H^- ion beam, the energy of the photon is shifted up to 15.8 eV. Furthermore, by varying the intersecting angle between the H^- beam and the laser beam, a continuous (tunable) light source now becomes available, and with which a series of elegant experimental measurements of the doubly excited $^1P^o$ states have been carried out [2].

In addition to the observations of the $^1P^o$ states in the photoionization experiments, the New Mexico group also studied the strong electric-field effects on the doubly excited $^1P^o$ states. By applying a moderate external magnetic field to the relativistic H^- beam, the H^- ion, again in its own rest frame, would experience a strong electric field plus a moderate magnetic field, as a result of the Lorentz transformation [3]. For example, a laboratory magnetic field of 1200 Gauss would be transformed to a magnetic field of 2200 Gauss plus a strong electric field of 560 kV/cm. When we consider the effects on the H^- ions due to the magnetic field and the electric field, a simple calculation indicates that the energy of the latter field is about 230 times larger than that of the former, if the doubly excited states are in low-lying orbitals. Therefore, for practical purposes we only need to consider the electric field when we interpret the experimental results. Under such experimental set up, the strong electric-field effects on the $^1P^o$ states have been measured. In a series of publications, the Stark effect on the lowest $^1P^o$ Feshbach resonance of H^- below the N = 2 hydrogen threshold has been investigated [3], as well as the effects on the $^1P^o$ shape resonance above the N = 2 hydrogen threshold [4]. Furthermore, studies of field effects on the N = 3 Feshbach $^1P^o(1)$ state [5], and on the $^1P^o$ states associated with other high excitation thresholds have also been reported [6]. As for the Stark effect on the $N = 2$ $^1P^o$ Feshbach resonance, since the $^1P^o(1)$ resonance lies at a position very close to the second member of the $^1S^e$ state, even a weak external electric field will cause a strong mixing of these two states and result in splitting the $^1P^o(1)$ state into two components. A third weak component, which was also observed in the experiment, is a result of the mixing of the $^1P^o(1)$ state with a nearby $^1D^e(1)$ state.

The phenomenon of atoms and ions in external electric fields can be studied from a resonance view point. Under the influence of the field the bound states of the atoms/ions become quasi-bound states, and the electrons will tunnel through the potential barrier formed by the combined Coulomb and external electric fields. As a result, the energies of such states become complex. The real part of a complex energy represents the shifted resonance position and the imaginary part can be related to the lifetime of the quasi-bound state by the usual uncertainty principle. The difference between the influences on the bound states and on the resonance states is that even without the external electric field, the resonance states would undergo autoionization. The opening of the new tunneling channels due to the external electric fields would provide an alternative route for autoionization processes.

Motivated in part by the experimental studies of the field effects on the $^1P^o$ resonances in H^-, we have recently carried out theoretical investigations of electric-field effects on doubly excited resonance states of H^-. The method of complex-coordinate rotation [7,8] has been used. We have reported results for the field effects on the $N=3\,^1P^o(1)$ resonance in Ref. [9]. The effects on the Feshbach resonances below the $H(N = 2)$ threshold were studied in Ref. [10]. Other theoretical works to study the field effects on the $N=2\,^1P^o$ shape resonance were performed by Wendoloski and Reinhardt [11], and by Du, Fabrikant, and Starace [12]. The Stark effects on the $N = 2$ Feshbach $^1P^o(1)$ and $^1S^e(2)$ states were investigated by Callaway and Rau [13] who used the stabilization method, and found qualitative agreement with the experimental results for the resonance positions. In the present work we employ the method of complex-coordinate rotation to investigate field effects on Feshbach resonances below the $N=3$ threshold of hydrogen atom. In particular, we show results for electric-field effects on the lowest lying $^1S^e(1), ^1P^o(1), ^1D^e(1)$, and $^1D^o(1)$ resonances. In the previous work [9], we reported results for the $M=0$ component of the $^1P^e(1)$ state. Here, in addition to the $M = 0$ components, results for the $M = \pm 1$ components for the latter three states are also given.

II. THEORY AND WAVE FUNCTIONS

The electric-field effects on atoms/ions can be investigated by using a method of complex-coordinate rotation [1,2]. The Hamiltonian of an atom in an external field is

$$H = T + V + \bar{F}\cdot\bar{r}, \tag{1}$$

where $\boldsymbol{F}$ is the external field, and $\boldsymbol{T}$ and $\boldsymbol{V}$ are the usual kinetic and potential operators, respectively. For the H^- system, $\boldsymbol{T}$ and $\boldsymbol{V}$ are given by

$$\boldsymbol{T} = -\nabla_1^2 - \nabla_2^2, \tag{2}$$

and

$$V = -\frac{2}{r_1} - \frac{2}{r_2} + \frac{2}{r_{12}}, \tag{3}$$

where r_1 and r_2 are the coordinates of the electrons with respect to the nucleus and $r_{12} = |\bar{r}_1 - \bar{r}_2|$. Atomic units are used in our work with energy in Rydberg units. The field strength in Rydberg units is 1 Ry $= 2.57 \times 10^9$ eV/cm.

In the method of complex-coordinate rotation, the radial coordinates are transformed by

$$r \rightarrow re^{i\theta}, \tag{4}$$

and the Hamiltonian can be written as

$$H(\theta) = Te^{-2i\theta} + Ve^{-i\theta} + \bar{F} \cdot \bar{r}e^{i\theta}. \tag{5}$$

Complex eigenvalues are obtained by diagonalizing the transformed Hamiltonian,

$$E = \langle \Phi | H(\theta) | \Phi \rangle / \langle \Phi | \Phi \rangle \tag{6}$$

and the complex resonance energy is given by

$$E_{res} = E_r - i\Gamma/2, \tag{7}$$

where E_r gives the shifted energy position, and Γ the resonance width with which the Stark broadening can be studied. The method is valid for isolated resonances and if the interaction with the background for such resonances is not too strong. This method was used by Chu and Reinhardt and coworkers to examine the Stark effect of hydrogen atoms [14]. Recently, by using a method of complex-coordinate rotation we have carried out investigations of electric field effects on the $N = 3\ ^1P^o(1)$ resonance of H^- [9], and on the $N = 2$ Feshbach resonances [10].

In the present work, we use products of Slater orbitals to represent the two-electron wave functions. The products of Slater orbitals are the following:

$$\Phi = A \sum_{la,lb} \sum_{ij} C_{a_i,b_j} \eta_{a_i}(r_1) \eta_{b_j}(r_2) Y^{LM}_{la,lb}(1,2) S(\sigma_1, \sigma_2), \tag{8}$$

where

$$\eta_{a_i}(r) = r^{n a_i} \exp(-\xi_{a_i} r). \tag{9}$$

In Eq. (8), A is the antisymmetrising operator, S is a two-particle spin eigenfunction and the η are individual Slater orbitals. Y is eigenfunction of the total angular momentum L,

$$Y^{LM}_{la,lb}(1,2) = \sum_{m_{la}} \sum_{m_{lb}} C(la, lb, L; m_{la}, m_{lb}, M) Y_{la,m_{la}}(1) Y_{la,m_{lb}}(2) \tag{10}$$

with C the Clebsch-Gordan coefficients.

III. CALCULATIONS AND RESULTS

We now discuss our calculations and results for the electric-field effects on the doubly excited Feshbach resonances of H^- lying below the N = 3 threshold of hydrogen atom. First we construct the wave functions for the field-free case that could produce resonance parameters comparable to the other accurate results in the literature. We employ orbitals with 9 s-type, 8 p-type, 7 d-type, 6 f-type, 5 g-type, and 4 h-type to construct the two-electron wave functions. Using such orbitals the expansion sets for various angular momentum states are 155 $^1S^e$ states, 220 $^1P^o$ states, 280 $^1D^e$ states, 270 $^1F^o$ states, 258 $^1G^e$ states, 196 $^1H^o$ states, and 137 $^1I^e$ states, respectively. They are all coupled together to form a basis set of 1516 terms by the external electric field for the M = 0 components. We first obtain field-free resonance parameters (E_r and Γ) for the $^1S^e(1), ^1P^o(1), ^1D^e(1)$, and $^1D^o(1)$ Feshbach resonances. The resonance positions and widths for these states are shown in Table I. They are compared with other theoretical results [15,16] and with our earlier complex-coordinate calculations using elaborate Hylleraas functions [17-20]. The agreements are generally good. It is noted, however, that the complex-coordinate results for the widths of the $^1P^o(1)$ and $^1D^o(1)$ states are somewhat narrower than those of the R-matrix calculation [16]. In Fig. 1 we show the field-free energy levels of the states that are under the present investigation.

Throughout this work block matrixes with up to L_{max} = 6 (I-states) are used (see Fig. 2). In order to examine the convergence behaviors for the resonance parameters, we

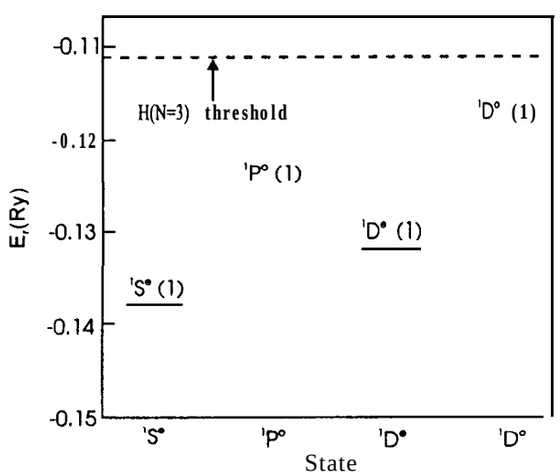

FIG. 1. Energy levels for the doubly-excited Feshbach resonances below the $H(N=3)$ threshold.

TABLE I. Resonance energies and widths for the field-free Feshbach resonances below the $H(N=3)$ threshold.

Ref.	$^1S^e(1)$	$^1P^o(1)$	$^1D^e(1)$	$^1D^o(1)$
a	E_r = -0.1380155	-0.12542891	-0.13192655	-0.11886249
	Γ = 0.0028187	0.00236098	0.00333538	0.00049051
b	E_r = -0.13801	-0.12543	-0.131901	
	Γ = 0.00286	0.00239	0.00314	
c	E_r = -0.138013	-0.125425	-0.13191	-0.118819
	Γ = 0.00301	0.00251	0.00327	0.000551
d	E_r = -0.1380116	-0.1254335	-0.1319066	-0.118862014
	Γ = 0.00283778	0.002383	0.003316	0.000499802

a: present calculation.
b: Callaway [15]; algebraic close coupling.
c: Pathak ***et al.*** [16]; 15-state R-matrix calculation.
d: complex-coordinate rotation with Hylleraas functions. ' S " (1) from Ref. [17]; ' P " (1) from Ref. [18]; $^1D^e(1)$ from [19]; and $^1D^o(1)$ from [20].

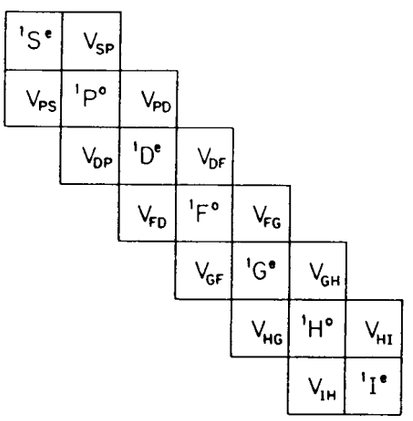

FIG. 2. Block matrices are coupled together by the external electric field. Here we consider $M = 0$ components and block matrices up to $L_{max} = 6$ are shown here.

TABLE II. Resonance parameters for different $L_{\max}$ and l_{max} valules. Results for the $M=0$ components are shown here for the external electric-field strength of $F=9\text{x}\,10^{-4}$ Ry.

state\[$L_{\max}$, $l_{\max}$, terms]		[4,5,1183]	[5,5,1379]	[6,5,1516]
$^1S^e(1)$	E_r(Ry)	-0.14257903	-0.14257877	-0.14257898
	Γ(Ry)	0.00269556	0.00269585	0.00269410
$^1P^o(1)$	E_r(Ry)	-0.12780841	-0.12769450	-0.12770667
	Γ(Ry)	0.00485859	0.00505827	0.00497169
$^1D^e(1)$	E_r(Ry)	-0.13707354	-0.13703113	-0.13703688
	Γ(Ry)	0.00449897	0.00449243	0.00448480

have performed calculations with different $L_{\max}$ values. In general, when the field strength is increased, states with higher angular momentum would become more important since more channels are open by the external field. For a strong field strength of 9.0 x 10^{-4} Ry (2.31 MeV/cm), we carry out calculations for $L_{\max}$ = 4, $L_{\max}$ = 5, and $L_{\max}$ = 6. Results are shown in Table II, and it is seen that the resonance parameters are converged when $L_{\max}$ = 6 is used. From such a test we estimate that the uncertainties in widths are within 2% for the above field strength. The uncertainties are somewhat larger for stronger field strengths, and smaller for weaker field strengths. For practical purposes we have not extended the basis sets beyond $L_{\max}$ = 6.

We show our results for the ' S "(1) state in Table III and in Fig. 3. It is seen that initially when the field strength is increased, the width decreases slightly, and reaches a minimum at about F = 7.0 x 10^{-4} Ry. After that, the width starts to increase, and in a more rapid fashion. Also in Fig. 3 the resonance energy is found shifted downward when the field strength is increased. It is further noted that at low field strengths, the energy is shifted down only slightly. But when the field strength is higher then approximately 4 x 10^{-4} Ry, the downward shift becomes more pronounced. The initial decrease in width for the N = 3 $^1S^e(1)$ state is a common finding for doubly excited intrashell states under the influence of external electric fields. In an earlier calculation for the field effects on the N = 3 ' P "(1) state, similar behavior for its width was also observed [9]. In an analysis made by Lin [21] when he investigated the field effects on the N = 2 $^1P^o$ shape resonance by solving the effective potential curve in the hyperspherical coordinates, he found that the

TABLE III. Electric-field effects on the $N = 3\ ^1S^e(1)$ resonance of H^-.

F (10^{-5} Ry)	E_r (Ry)	Γ (Ry)
0	-0.13801554	0.00281870
1	-0.13801607	0.00281851
2.5	-0.13801860	0.00281803
5	-0.13802761	0.00281628
7.5	-0.13804265	0.00281337
9.336	-0.13805753	0.00281049
10	-0.13806372	0.00280929
12	-0.13808495	0.00280518
14	-0.13811008	0.00280031
16.3	-0.13814383	0.00279378
20	-0.13820911	0.00278110
23	-0.13827209	0.00276883
26.84	-0.13836608	0.00275046
30.73	-0.13847690	0.00272865
32.5	-0.13853260	0.00271762
35	-0.13861717	0.00270081
37.5	-0.13870860	0.00268250
40.0	-0.13880722	0.00266264
45.902	-0.13906971	0.00260955
50	-0.13927830	0.00256813
55	-0.13956514	0.00251501
60	-0.13989020	0.00246583
65	-0.14025510	0.00243040
70	-0.14065733	0.00242017
76.63	-0.14124341	0.00245353
80	-0.14156155	0.00249327
85.0	-0.14205708	0.00257137
90	-0.14257877	0.00269585
91.8	-0.14277352	0.00274390
100.0	-0.14369610	0.00302232

height of the inner part of the potential curve actually increases at low field, before it is suppressed by the external electric field and decreased in height at higher field. As a result, the potential barrier turns out to be thickened somewhat as the field strength starts to increase from zero. It will hence take longer time for the tunneling effect to take place, and the width becomes narrower. When the applied field becomes sufficiently large, the thickness of the potential barrier becomes smaller, and the resonance width broader. There is however a difference between our finding and that of Lin's for the $N = 2$ shape resonance. In his finding the resonance energy was found blue-shifted initially before it was

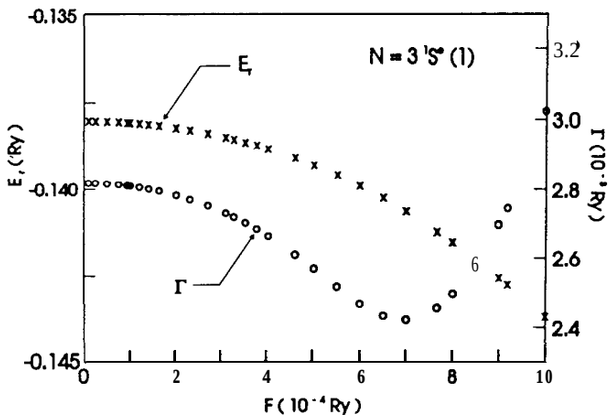

FIG. 3. Electric-field effects on the energy and width of the $N=3\,{}^1S^e(1)$ state.

shifted downward at higher field. Our finding shows the energy level is shifted down all the way, although at lower field it is shifted only slightly.

We next show our results for the ${}^1D^e(1)$ state in Table IV and in Fig. 4. Again it is seen that initially when the external field is turned on, the width starts to decrease slightly, until it reaches a minimum at field strength of about F = 5.0 x 10^{-4} Ry. After that, the width starts to increase, and in a more rapid fashion when the field strength is further increased. Table IV also shows results for the ${}^1D^e$ resonance state with $M=$ fl. In the case for $M=\pm1$, the ${}^1S^e$ states are no longer relevant. We add 80 ${}^1P^e$ states, 148 ${}^1D^o$ states ,159 ${}^1F^e$ states, and 160 ${}^1G^o$ states. The total number of terms in the basis set is therefore 1771 when $L_{\max}$ = 5 is used. When we compare the results for different M values in Table IV. It is seen that for low fields the $M=0$ and $M=\pm1$ results are nearly the same, with the widths for the M = 0 are less than those of the $M=\pm1$ counterparts. The differences are smaller than 1% for field strengths up to F = 5 x 10^{-4} Ry. Such a difference increases for increasing field strengths. It reachs to 2.2% for $F=6$ x 10^{-4} Ry, and gradually to 7.2% when F is increased to 9 x 10^{-4} Ry.

In the present investigation, we have not considered the $M=\pm2$ components for the ${}^1D^e$ case, as such M components do not apply to the ${}^1P^o(1)$ state. We should mention that under the influence of the external electric fields, while the total angular momentum of the atomic system is no longer a good quantum number, the M value remains a good quantum number. Furthermore, the energy levels for the $\pm M$ components remain degenerate, as the electric-field effects on the atomic system do not change when the direction of the electric

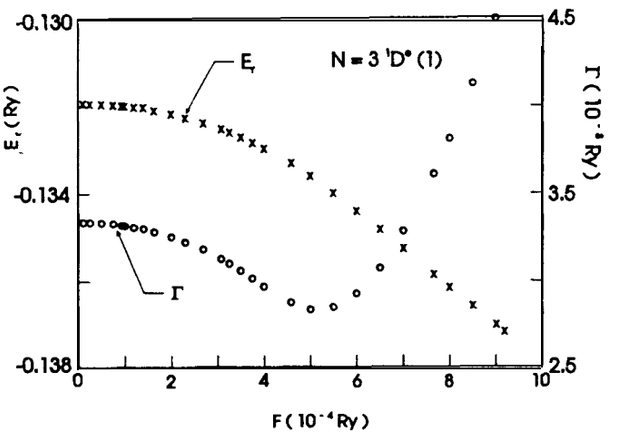

FIG. 4. Electric-field effects on the energy and width of the $N = 3\,{}^1D^e\,(1)$ state. The $M = 0$ results are shown here.

field is reversed. Of course, an external magnetic field will split the $\pm M$ components. However, for the reason we have discussed earlier in the text, we do not consider the magnetic-field effects here.

We next show our results for the ${}^1P^o(1)$ state. The results for the $M = 0$ component were discussed in a previous publication [9]. For completeness we also show them here in Table V, and in Figs. 5 and 6. Our results were in good agreement with the experimental

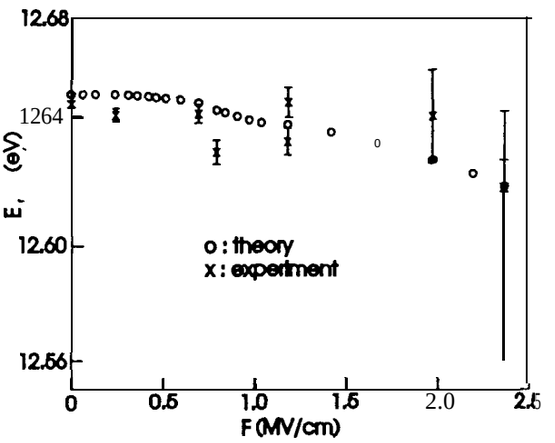

FIG. 5. Comparison of the resonance energy of ${}^1P^o(1)$ state with experimental measurements [5]. The bars represent experimental standard deviations.

TABLE IV. Electric-field effects on the $N = 3\ ^1D^e(1)$ resonance.

	$M = 0$		$M = \pm 1$	
$F(10^{-4}$ Ry)	E_r (Ry)	Γ (Ry)	E_r (Ry)	Γ (Ry)
0	-0.13192655	0.00333534	-0.13192655	0.00333534
0.2	-0.13192907	0.00333447	-0.13192897	0.00333455
0.4	-0.13193663	0.00333191	-0.13193621	0.00333218
0.6	-0.13194923	0.00332765	-0.13194829	0.00332823
0.8	-0.13196687	0.00332168	-0.13196522	0.00332267
1.0	-0.13198954	0.00331398	-0.13198700	0.00331551
1.2	-0.13201724	0.00330454	-0.13201367	0.00330671
1.4	-0.13204997	0.00329334	-0.13204523	0.00329623
1.6	-0.13208773	0.00328035	-0.13208172	0.00328405
1.8	-0.13213053	0.00326552	-0.13212319	0.00327011
2.0	-0.13217839	0.00324881	-0.13216969	0.00325434
2.2	-0.13223135	0.00323015	-0.13222128	0.00323665
2.4	-0.13228944	0.00320947	-0.13227807	0.00321695
2.6	-0.13235276	0.00318667	-0.13234017	0.00319512
2.8	-0.13242142	0.00316167	-0.13240775	0.00317102
3.0	-0.13249559	0.00313440	-0.13248090	0.00314460
3.2	-0.13257550	0.00310484	-0.13256059	0.00311551
3.4	-0.13266145	0.00307306	-0.13264639	0.00308409
3.6	-0.13275380	0.00303929	-0.13273910	0.00305038
3.8	-0.13285300	0.00300402	-0.13283931	0.00301484
4.0	-0.13295951	0.00296805	-0.13294766	0.00297834
4.2	-0.13307374	0.00293264	-0.13306477	0.00294228
4.4	-0.13319600	0.00289943	-0.13319113	0.00290863
4.6	-0.13332637	0.00287043	-0.13332693	0.00287989
4.8	-0.13346488	0.00284659	-0.13347198	0.00285876
5.0	-0.13361068	0.00283220	-0.13362561	0.00284785
5.5	-0.13400050	0.00284342		
6.0	-0.13441694	0.00292758	-0.13448071	0.00299097
6.5	-0.13484229	0.00307057		
7.0	-0.13527613	0.00328148	-0.13539811	0.00342143
8.0	-0.13614946	0.00381317	-0.13634420	0.00404002
8.5	-0.13659406	0.00412407		
9.0	-0.13703113	0.00449243	-0.13731536	0.00481751
10.0	-0.13791992	0.00531427		

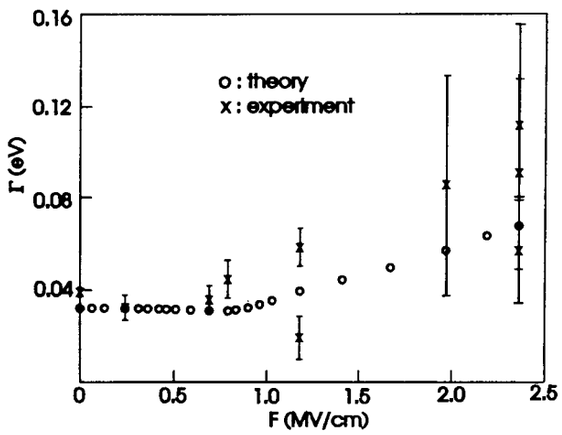

FIG. 6. Comparison of the autoionization width of $^1P^o(1)$ state with experimental measurements [5]. The bars represent experimental standard deviations.

measurements to within the experimental errors. For a detailed discussion of the comparison with experimental measurements, readers are referred to Ref. [9]. In Fig. 7 and in Table V, we show the electric-field effects on the $M = 0$ and $M = \pm 1$ components of the $^1P^o(1)$ state. The $M = \pm 1$ results are new calculations. It is seen at first that the energy position is shifted downward slightly for increasing field strengths until it reaches to $F = 2.0 \times 10^{-4}$

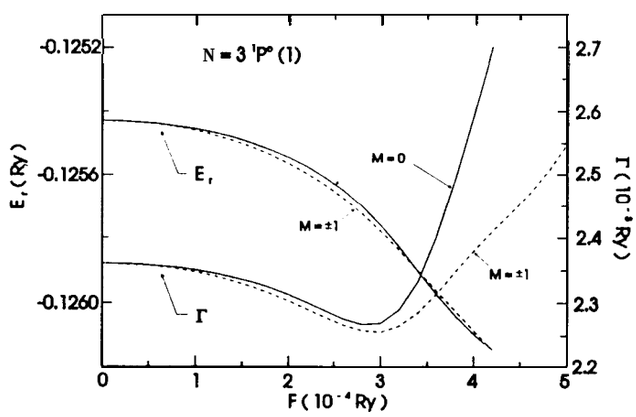

FIG. 7. Electric-field effects on the energy and width of the $N = 3$ $^1P^o(1)$ state. Both $M = 0$ (solid line) and $M = \pm 1$ (dashed line) results are shown here.

TABLE V. Electric-field effects on the $N = 3\,^1P^o(1)$ resonance.

	$M = 0$		$M = \pm 1$	
$F(10^{-4}$ Ry)	E_r (Ry)	Γ (Ry)	E_r (Ry)	Γ (Ry)
0	-0.12542891	0.00236098	-0.12542891	0.00236098
0.2	-0.12542993	0.00236054	-0.12543016	0.00236047
0.4	-0.12543305	0.00235939	-0.12543392	0.00235894
0.6	-0.12543828	0.00235744	-0.12544023	0.00235637
0.8	-0.12544570	0.00235463	-0.12544913	0.00235269
1.0	-0.12545539	0.00235086	-0.12546070	0.00234785
1.2	-0.12546751	0.00234603	-0.12547505	0.00234176
1.4	-0.12548225	0.00233998	-0.12549235	0.00233434
1.6	-0.12549988	0.00233255	-0.12551282	0.00232550
1.8	-0.12552080	0.00232356	-0.12553676	0.00231520
2.0	-0.12554557	0.00231291	-0.12556456	0.00230354
2.2	-0.12557496	0.00230071	-0.12559675	0.00229083
2.4	-0.12561004	0.00228756	-0.12563394	0.00227781
2.6	-0.12565215	0.00227513	-0.12567678	0.00226596
2.8	-0.12570268	0.00226692	-0.12572569	0.00225768
3.0	-0.12576233	0.00226894	-0.12578094	0.00225574
3.2	-0.12582998	0.00228888	-0.12583958	0.00226472
3.4	-0.12590199	0.00233300	-0.12590125	0.00228454
3.6	-0.12597316	0.00240247	-0.12596285	0.00231373
3.8	-0.12603887	0.00249244	-0.12602319	0.00234729
4.0	-0.12609667	0.00259450	-0.12608324	0.00238013
4.2	-0.12614659	0.00270022	-0.12614527	0.00241021
4.4	-0.12619023	0.00280257	-0.12621136	0.00243907
4.6	-0.12623027	0.00289784	-0.12628240	0.00246993
4.8	-0.12626650	0.00297871	-0.12635820	0.00250583
5.0	-0.12630751	0.00305520	-0.12643798	0.00254873

Ry approximately. The energy is then shifted down in a more pronounced manner. As for the width, initially it is decreased slightly when the field is turned on. It then reaches a minimum at about $F = 2.8 \times 10^{-4}$ Ry. After that, the width is increased rapidly for increasing field strengths. Also when we compare the $M = 0$ and $M = \pm 1$ results it is seen that the widths for the $M = \pm 1$ components have slightly lower values (less than 1%) than the $M = 0$ counterparts for the field strengths up to $F = 3.0 \times 10^{-4}$ Ry. When the field strength is larger than 3×10^{-4} Ry, the differences for the $M = 0$ and $M = \pm 1$ cases start to widen for increasing field strengths. For example, when $F = 3.4 \times 10^{-4}$ Ry, the $M = \pm 1$ widths are about 2% narrower than the $M = 0$ case. The difference increases to 8% for

$F = 4.0 \times 10^{-4}$ Ry and gradually to 17% for $F = 5.0 \times 10^{-4}$ Ry.

We now discuss the result for the $M = \pm 1$ components of the $^1D^o$ state. From Table VI and Fig. 8, it is seen that when the external electric field is turned on the energy is decreased only slightly until the field strength reaches to $F = 2 \times 10^{-4}$ Ry. After that, the energy is shifted downward more rapidly for increasing field strengths. As for the width, it stays nearly constant for the field strengths from zero to about 1.0×10^{-4} Ry. The width is then increased gradually when the field is further increased. It is also noted that around $F = 2 \times 10^{-4}$ Ry the width exhibits a small oscillation on top of the overall increasing background. The resonance energy position also exhibits a small oscillation over a smooth background. Such oscillations are due to the interference effects coming from a nearby $^1P^o(2)$ state. In the field-free case, the $^1P^o(2)$ states lies at E_r = -0.1171436 Ry, about 0.0017 Ry above the $^1D^o(1)$ state. As the field strength is increased, the former state catches up in energy with the latter around $F = 2.0 \times 10^{-4}$ Ry. These two states subsequently interfere with each other in such a way that their autoionization probabilities, and hence their widths, would exhibit oscillatory behaviors.

In the present work we have not considerd other M components for the $^1D^o(1)$ state. The $M = 0$ component of the $^1D^o$ state does not couple with the $^1P^o$ (M = 0) state by the electeic-field dipole operator. As for the $M = \pm 2$ components, since the $^1P^o(1)$ state does not have such components, we do not consider them here.

So far we have investigated electric-field effects on four doubly excited Feshbach resonances below the $N = 3$ hydrogen threshold. The field effects on the autoionization widths seem to have a similar trend. At low fields, the widths all stay nearly constant,

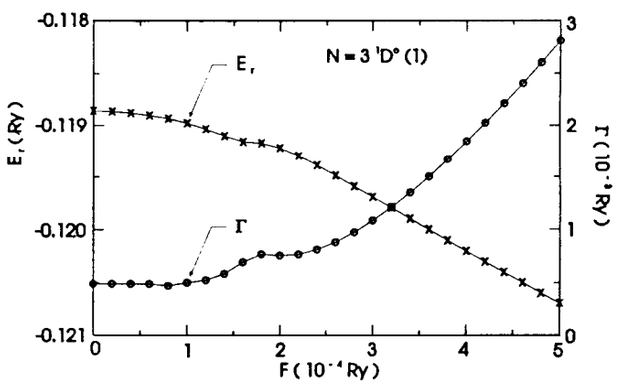

FIG. 8. Electric-field effects on the energy and width of $N = 3$ $^1D^o(1)$ state. The $M = \pm 1$ results are shown here.

TABLE VI. Electric-field effects on the $^1D^o$ resonance of H^-.

	$M = \pm 1$	
$F\,(10^{-4}$ Ry)	E_r (RY)	Γ (RY)
0	-0.11886249	0.00949051
0.2	-0.11886673	0.00049031
0.4	-0.11887964	0.00048986
0.6	-0.11890185	0.00048978
0.8	-0.11893448	0.00049168
1.0	-0.11897918	0.00049973
1.2	-0.11903719	0.00052397
1.4	-0.11910458	0.00058386
1.6	-0.11915973	0.00069399
1.8	-0.11917460	0.00076763
2.0	-0.11922049	0.00075541
2.2	-0.11929920	0.00076692
2.4	-0.11939090	0.00081005
2.6	-0.11948893	0.00087977
2.8	-0.11959014	0.00097200
3.0	-0.11969245	0.00108331
3.2	-0.11979456	0.00120777
3.4	-0.11989759	0.00134807
3.6	-0.12000019	0.00150075
3.8	-0.12010201	0.00166434
4.0	-0.12020282	0.00183737
4.2	-0.12030257	0.00201850
4.4	-0.12040133	0.00220657
4.6	-0.12049926	0.00240070
4.8	-0.12059656	0.00260032
5.0	-0.12069340	0.00280509

and for most cases, they even start to decrease somewhat. We have discussed the physics behind such phenomenon earlier in the text. When the electric field is increased to a critical value, the autoionization width starts to increase and in a more rapid fashion. It is further noted that the critical value of the electric-field strengths differs from state to state. In general, the tighter bound (relative to the hydrogen N = 3 excited threshold) is for a doubly excited state, the stronger electric field would be required to break the two-electron bonding. Such critical field strengths for the $^1S^e(1)$, $^1D^e(1)$, $^1P^o(1)$, and $^1D^o(1)$ states are 7 x 10^{-4} Ry, 5 x 10^{-4} Ry, 2.8 x 10^{-4} Ry, and 1.5 x 10^{-4} Ry, respectively. Once the two-electron correlation bounding is suppressed by the strong electric field, the autoionization

of one of the two electrons is now similar to the autoionization for one-electron systems under the influence of external electric fields.

In summary, we have carried out a theoretical investigation of strong electric-field effects on the doubly excited intrashell Feshbach resonances of H^- below the $H(N=3)$ threshold. Our results are of accurate calculations that they are useful references for other theoretical and experimental works.

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REFERENCES

[1] Readers are referred to the following reviews: G. J. Schulz, Rev. Mod. Phys. 45, 378 (1973); J. S. Risley, Atomic Physics 6, 223 (1978); D. E. Golden, Advances in Atomic and Molecular Physics, 14, 1 (1978).

[2] H. C. Bryant ***et al.,*** Phys. Rev. Lett. 38, 228 (1977); D. W. MacArthur ***et al.,*** Phys. Rev. A32, 1921 (1985); P. G. Harris ***et al.,*** Phys. Rev. A42, 6443 (1990); M. Halka ***et al.,*** Phys. Rev. A46, 6942 (1992).

[3] P. A. M. Gram, J. C. Pratt, M. A. Yates-Williams, H. C. Bryant, J. B. Donahue, H. Sharifian, and H. Tooloonchi, Phys. Rev. Lett. 40, 107 (1978).

[4] H. C. Bryant, David A. Clark, Kenneth B. Butterfield, C. A. Frost, H. Sharifian, H. Tootoonchi, J. B. Donahue, P. A. M. Gram, M. E. Hamm, R. W. Hamm, J. C. Pratt, M. A. Yates, and W. W. Smith, Phys. Rev. **A27**, 2889 (1983); G. Comtet, C. J. Harvey, J. E. Stewart, H. C. Bryant, K. B. Butterfield, D. A. Clark, J. B. Donahue, P. A. M. Gram, D. W. MacArthur, V. Yuan, W. W. Smith, and Stanley Cohen, Phys. Rev. A35, 1547 (1987).

[5] S. Cohen, H. C. Bryant, C. J. Harvey, J. E. Steward, K. B. Butterfield, D. A. Clark, J. B. Donahue, D. W. MacArthur, G. Comtet, and W. W. Smith, Phys. Rev. A36, 4728 (1987).

[6] M. Halka, H. C. Bryant, C. Johnstone, B. Marchini, W. Miller, A. H. Mohagheghi, C. Y. Tang, K. B. Butterfield, D. A. Clark, S. Cohen, J. B. Donahue, P. A. M. Gram, R. W. Hamm, A. Hsu, D. W. MacArthur, E. P. MacKerrow, C. R. Quick, J. Tiee, and K. Rozsa, Phys. Rev. A46, 6942 (1992).

[7] E. Balslev and J. M. Combs, Commun. Math. Phys. 22, 296 (1971); B. Simon, Commun. Math. Phys. 27, 1 (1972).

[8] Y. K. Ho, Phys. Rep. 99, 1 (1983); W. P. Reinhardt, Ann. Rev. Phys. Chem. 33, 223 (1982); B. R. Junker, Adv. At. and Mol. Phys. 18, 208 (1982).

[9] Y. K. Ho and J. Callaway, Phys. Rev. A50, 4941 (1994).

[10] Y. K. Ho, Phys. Rev. A, (1995), accepted for publication.

[11] John J. Wendoloski and W. P. Reinhardt, Phys. Rev. A17, 195 (1978).

[12] N. Y. Du, I. I. Fabrikant, and A. F. Starace, Phys. Rev. A48, 2968 (1993).

[13] J. Callaway and A. R. P. Rau, J. Phys. B11, L289 (1978).

[14] A. Maquet, S. I. Chu, and W. P. Reinhardt, Phys. Rev. A27, 2946 (1983); C. Cerjan, R. Hedges, C. Holt, W. P. Reinhardt, K. Scheibner, and J. J. Wendoloski, Int. J. Quan. Chem. 14, 393 (1978).

[15] J. Callaway, Phys. Rev. A26, 199 (1982).

[16] A. Pathak, A. E. Kingston, and K. A. Berrington, J. Phys. B21, 2939 (1988).

[17] Y. K. Ho, J. Phys. B23, L71 (1990), and unpublished.

[18] Y. K. Ho, Phys. Rev. A45, 148 (1992).

[19] Y. K. Ho and A. K. Bhatia, Phys. Rev. A48, 3720 (1993).

[20] A. K. Bhatia and Y. K. Ho, Phys. Rev. A50, 4886 (1994).

[21] C. D. Lin, Phys, Rev. A28, 1876 (1983).