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4x4 Matrix Algebra in the Theory of Optical Detection of Picosecond Acoustic Pulses in Anisotropic Media

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Matrix formalism is applied in the theory of optical detection of picosecond acoustic strain pulses in anisotropic multilayers, probed by oblique incident laser radiation. A 4 × 4 matrix formalism, which can be easily implemented in computer programs, is presented to calculate the 2 × 2 transient reflection matrix of the sample. Transient reflectometric and interferometric signals can be readily calculated from the transient reflection matrix. In addition, the rotation of the polarization state of the reflected probe induced by acoustic waves is also readily accessible. Therefore, the simulation of all the time-resolved picosecond ultrasonic experimental signals can be simulated in a single straightforward procedure. This matrix formalism is applied to treat the problem of shear wave detection within an isotropic sample.

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I. INTRODUCTION

Picosecond acoustics is a technique of characterization of physical properties of thin films. This technique involves the optical detection of picosecond acoustic pulses generated by femtosecond optical pulses through the optical pump-probe technique. The propagation of acoustic pulses within the sample modulates the optical reflection coefficient $\mathrm{r} = \rho e^{i\theta}$, where r and θ are its amplitude and phase, respectively. Most picosecond acoustics studies use the measurement of the transient reflectivity $\Delta\mathrm{r}/\mathrm{r} = (\Delta\rho/\rho) + \mathrm{i}\Delta\theta$. Here, Δr, $\Delta\rho$ and $\Delta\theta$ denote small acoustically induced variations in the reflection coefficient, its amplitude and phase, respectively. The theoretical predictions of the transient reflectivity Δr/r can be made by the application of the formulas that were derived for samples composed of a film on a substrate [1], and for multilayered samples [2, 3]. Formulas of Ref. [1–3] can be applied only to optically isotropic samples probed with an optical beam at normal incidence. When the sample is probed at oblique incidence, it becomes necessary to specify the incident polarization as transient reflectivities Δr/r are not the same for p- and s-polarized probes [4]. Nevertheless, the scalars $\Delta r_p/r_p$ and $\Delta r_s/r_s$ for p and s polarization, respectively, are not sufficient to characterize the transient reflectivity of an anisotropic sample [5]. Indeed, in this last case, an incident p- or s-polarized probe does not remain purely p- or s- polarized after reflection. A fraction of the p- (s-) polarized field may be converted into s (p) polarization. The complete characterization of the transient reflection properties of an anisotropic sample needs the additional scattering coefficients r_{sp} and r_{ps}. In consequence, the full description of the reflection properties of an anisotropic sample requires four coefficients: r_{pp}, r_{ss}, r_{sp}, and r_{ps}, which are equivalent to a description by a reflection Jones matrix $\mathbf{R}$, which is defined by the relationship $\mathbf{E}_r = \mathbf{R}\cdot\mathbf{E}_i$, where $\mathbf{E}_i$ and $\mathbf{E}_r$ are the Jones vectors of the incident and reflected fields, respectively. The Jones vectors $\mathbf{E}_i$, $\mathbf{E}_r$ and the Jones matrix $\mathbf{R}$ are explicitly expressed in terms of their p and s components as follows:

$$\mathbf{E}_i = \begin{pmatrix} \mathrm{E_{ip}} \\ \mathrm{E_{is}} \end{pmatrix} \; ; \mathbf{E}_r = \begin{pmatrix} \mathrm{E_{rp}} \\ \mathrm{E_{rs}} \end{pmatrix} \text{ and } \mathbf{R} = \begin{pmatrix} \mathrm{r_{pp}} & \mathrm{r_{ps}} \\ \mathrm{r_{sp}} & \mathrm{r_{ss}} \end{pmatrix}. \tag{1}$$

If we consider that an acoustic pulse propagates within the sample, then it will induce transient perturbations of sample reflectivity, which may depend on the time t. Perturbations of the $\mathbf{R}$ matrix is $\Delta\mathbf{R}(\mathrm{t}) = \mathbf{R}'(\mathrm{t}) - \mathbf{R}$, where $\mathbf{R}'$ and $\mathbf{R}$ are the perturbed and unperturbed reflection matrices, respectively. However, the description of transient perturbations by the normalized matrix $\Delta\mathbf{R}\cdot\mathbf{R}^{-1}$, where $\mathbf{R}^{-1}$ is the inverse of the $\mathbf{R}$-matrix, provides important advantages since this matrix can be expressed in function of parameters characterizing the amplitude and polarization of the reflected field $\mathbf{E}_r$ as follows [6]:

$$\Delta\mathbf{R}\cdot\mathbf{R}^{-1} = \begin{pmatrix} \left(\frac{\Delta A_0}{A_0} + i\Delta\alpha\right)_p & -(\Delta\psi + i\Delta\chi)_s \\ (\Delta\psi + i\Delta\chi)_p & \left(\frac{\Delta A_0}{A_0} + i\Delta\alpha\right)_s \end{pmatrix}. \tag{2}$$

The subscripts p and s in Eq. (2) specify the polarization states of the reflected probe. The diagonal components depend only on the variations of the complex amplitude $A = A_0.\exp(i\alpha)$ of the reflected field and the off-diagonal components depend only on the changes of the two polarization parameters: ψ, the orientation angle, and χ, the ellipticity angle of the polarization ellipse. The real part $\Delta\psi$ of the complex rotation $\Delta\psi + i\Delta\chi$ is the rotation of the polarization ellipse, and the imaginary part $\Delta\chi$ is the variation of ellipticity. All the quantities of the $\Delta\mathbf{R}\cdot\mathbf{R}^{-1}$ matrix can be measured. The quantity $\Delta A_0/A_0$ can be obtained experimentally by transient reflectivity measurements [7]. The variations of the phase $\Delta\alpha$ can be measured by transient interferometry [8–10]. Finally, the angular quantities $\Delta\psi$ and $\Delta\chi$ can be measured by transient polarimetry [11, 12]. The $\Delta\mathbf{R}\cdot\mathbf{R}^{-1}$ matrix provides a complete description of the sample transient reflectivity; so it is possible to predict the transient optical response of the sample for any arbitrary polarization state of the reflected probe.

The 2×2 matrix formalism pioneered by Abelès [5] could be used to calculate the $\Delta\mathbf{R}\cdot\mathbf{R}^{-1}$ matrix, provided that there is no coupling between p and s polarization states.

This is in particular the case for longitudinal strain pulses that propagate within a stratified isotropic structure. Conversely, the Abelès formalism is not sufficient for anisotropic samples because of the coupling that may arise between p and s optical modes. The Abelès formalism also cannot be applied when an isotropic medium sustains shear strain [13]. A general method of calculation was presented by Matsuda et al. [14] to handle the problem of anisotropic multilayered samples. The general theory of Ref. [14], based on the resolution of the general 2^{nd} order wave equation for a monochromatic electric field, can be used to calculate the transient reflectivity, i.e., the transient $\mathbf{\Delta R{\cdot}R}^{-1}$ matrix, of a planar stratified anisotropic nanostructure perturbed by acoustic plane waves, for an arbitrary angle of incidence and polarization of the optical probe. We propose here an alternative method of calculation of the $\mathbf{\Delta R{\cdot}R}^{-1}$ matrix, which is based on a 4×4 matrix formalism [5, 15]. This matrix method does not require the explicit determination of the electric field **E** or magnetic field **H** within the sample. Moreover, the matrix formalism can be easily implemented in computer programs to calculate the $\mathbf{\Delta R{\cdot}R}^{-1}$ matrix, and it gives in a single procedure, all the quantities that are needed for the simulations of time-resolved measurements. In the next section, we will present the application of this matrix method to picosecond acoustics. As an example, the formalism will be applied to calculate the transient optical response of a sample perturbed by shear and longitudinal acoustic pulses that propagate within a substrate. Then, the most favourable experimental conditions (incidence angle and polarisation state of the incident probe) to detect shear waves in an isotropic sample will be deduced.

II. THE 4 × 4 MATRIX METHOD

An analytical solution of Maxwell equations for an inhomogeneous anisotropic medium cannot be presented, in general, in explicit form. However, the problem is considerably simplified if the medium is composed of a stack of planar layers that are homogeneous in the plane of the layers. Let the xy-plane be parallel to the plane of the layers. We suppose that these layers may be inhomogeneous in the z-direction, which is the direction perpendicular to the layers (Fig. 1). We consider that the stack is embedded between two semi-infinite homogeneous media: an ambient isotropic medium characterized by a refractive index n_0 and occupying the $z < z_0$ half-space, and a substrate medium ($z > z_N$), which may be anisotropic. We suppose that the stack is composed of N layers, numbered from 1 to N and that the interface lying between layers number (i - 1) and (i) is located at coordinate $z = z_i$. The multilayered sample is probed by an optical beam, characterized by an incident wave-vector $\mathbf{k}_0$, which arrives from the ambient medium at an incidence angle ϕ_0. The vacuum incident probe wave-vector is $k = \omega/c$, where ω is the angular frequency of the monochromatic optical probe and c the vacuum velocity of light. The incidence plane is the zx-plane; so the x-component $k_{0x} = k\ n_0\ \sin\phi_0 = k\ \eta$ is positive and the y-component k_{0y} is zero. The x- and y-components of the wave-vector $\mathbf{k}_0$ are conserved in the multilayered sample.

We suppose that the monochromatic electric field **E** and magnetic field **H** of the

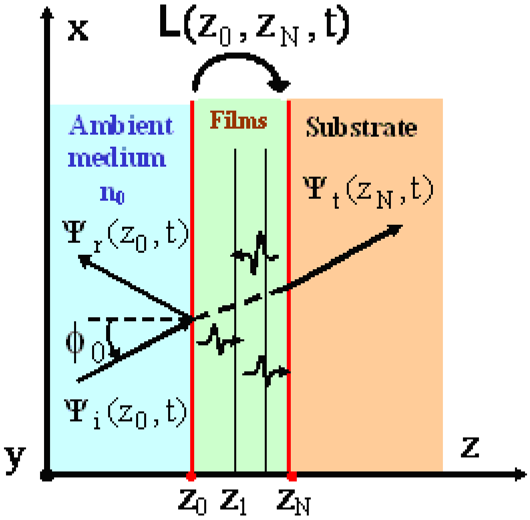

FIG. 1: Stratified planar structure. Acoustic waves propagate within a stack of thin anisotropic films.

optical probe have the form: $\mathbf{E}$(x,z,t) = $\mathbf{E}_0$(z) exp[i(ωt - kηx)] and $\mathbf{H}$(x,z,t) = $\mathbf{H}_0$(z) exp[i(ωt - kηx)]. Then, the set of Maxwell equations are equivalent to the following 1st order linear differential equation:

$$\frac{\partial \mathbf{\Psi}(z,t)}{\partial z} = -i\,k\,H(z,t)\cdot\mathbf{\Psi}(z,t), \tag{3}$$

where

$$\mathbf{\Psi}(z,t) = \begin{pmatrix} E_x \\ H_y \\ E_y \\ -H_x \end{pmatrix} \text{ and } H(z,t) = \begin{pmatrix} -\frac{\eta\,\varepsilon_5}{\varepsilon_3} & 1-\frac{\eta^2}{\varepsilon_3} & -\frac{\eta\,\varepsilon_4}{\varepsilon_3} & 0 \\ \varepsilon_1-\frac{\varepsilon_5^2}{\varepsilon_3} & -\frac{\eta\,\varepsilon_5}{\varepsilon_3} & \varepsilon_6-\frac{\varepsilon_4\,\varepsilon_5}{\varepsilon_3} & 0 \\ 0 & 0 & 0 & 1 \\ \varepsilon_6-\frac{\varepsilon_4\,\varepsilon_5}{\varepsilon_3} & -\frac{\eta\,\varepsilon_4}{\varepsilon_3} & \varepsilon_2-\frac{\varepsilon_4^2}{\varepsilon_3}-\eta^2 & 0 \end{pmatrix} \tag{4}$$

Equation (3) was introduced by Berreman to calculate the reflection matrix $\mathbf{R}$ of inhomogeneous anisotropic samples [5, 15]. The electromagnetic field at each point is completely determined by a state vector $\mathbf{\Psi}$, which contains only the tangential components of the electromagnetic field; so the conditions of continuity of the tangential components of $\mathbf{E}$ and $\mathbf{H}$ at each interface can be translated into the condition of continuity of the state vector $\mathbf{\Psi}$ at each interface. The integration of the 1st order differential Eq. (3) determines the vector field $\mathbf{\Psi}$ at any point z of the sample. The differential propagation operator (or matrix) H is completely determined by the permittivity tensor, expressed in Eq. (4) in the Voigt notation, and by the constant η = n$_0$ sinϕ_0. The transfer matrix $L_i(z_{i-1}, z_i, t)$ of the ith layer is defined by the relationship $\mathbf{\Psi}(z_i,t) = L_i(z_{i-1}, z_i, t)\cdot\mathbf{\Psi}(z_{i-1},t)$. The $L_i(z_{i-1}, z_i, t)$ matrix is calculated by integration of Eq. (3). So, the transfer matrix of the whole stack from z_0 to z_N is

$$L(z_0, z_N, t) = L_N(z_{N-1}, z_N, t) \cdot \ldots \cdot L_2(z_1, z_2, t) \cdot L_1(z_0, z_1, t). \tag{5}$$

The transfer matrix from z to z′ inside a homogeneous layer, i.e. characterized by a constant H-matrix, is $L(z, z') = \exp[-ik(z' - z)H]$.

The incident and reflected fields of the probe at point z = z_0 are represented by state vectors $\mathbf{\Psi}_i = E_{ip} \cdot \mathbf{\Phi}_p^+ + E_{is} \cdot \mathbf{\Phi}_s^+$ and $\mathbf{\Psi}_r = E_{rp} \cdot \mathbf{\Phi}_p^- + E_{rs} \cdot \mathbf{\Phi}_s^-$, respectively, where $\mathbf{\Phi}_{p,s}^{+;-}$ are four eigenvectors representing the optical eigenmodes of the ambient medium and corresponding to the eigenpolarization of the incident or the reflected optical probe. The four eigenvalues associated to eigenvectors $\mathbf{\Phi}_{p,s}^{+;-}$ are $\nu_p^+ = \nu_s^+ = n_0 \cos\phi_0$ and $\nu_p^- = \nu_s^- = -n_0 \cos\phi_0$. The subscripts p or s refer to polarization state, and the + or − signs in the superscripts specify the direction of propagation of the mode. The z-components of the wave-vectors of eigenmodes $\mathbf{\Phi}_{p,s}^{+;-}$ are $k\nu_{p,s}^{+;-}$. The amplitudes E_{ip}, E_{is}, E_{rp}, and E_{rs} are the components of Jones vectors of Eq. (1). The optical eigenvectors of the ambient medium are expressed explicitly as follows:

$$\mathbf{\Phi}_p^+ = \begin{pmatrix} \cos\phi_0 \\ n_0 \\ 0 \\ 0 \end{pmatrix}, \mathbf{\Phi}_p^- = \begin{pmatrix} \cos\phi_0 \\ -n_0 \\ 0 \\ 0 \end{pmatrix}, \mathbf{\Phi}_s^+ = \begin{pmatrix} 0 \\ 0 \\ 1 \\ n_0 \cos\phi_0 \end{pmatrix}$$
$$, \text{and } \mathbf{\Phi}_s^- = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -n_0 \cos\phi_0 \end{pmatrix}, \tag{6}$$

The reflection matrix is determined by the condition of continuity for the vector field $\mathbf{\Psi}$ at a conveniently chosen interface. At the interface z = z_0, the sum $\mathbf{\Psi}_i(z_0^-) + \mathbf{\Psi}_r(z_0^-)$ must be equal to the field vector $\mathbf{\Psi}(z_0^+)$. In consequence, we must consider also the four optical eigenmodes of the semi-infinite substrate (z>z_N) to express the transmitted vector $\mathbf{\Psi}_t(z_N)$ in the substrate. Since we suppose that no light arrives directly from the substrate, the two independent eigenvectors $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ of the substrate H-matrix, with optical propagation towards the +z-direction, form a basis for the decomposition of the vector $\mathbf{\Psi}_t(z_N)$. Thus, eigenvalues ν_1 and ν_2 associated to eigenvectors $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ both have a positive real part. Then, the transmitted vector at z = z_N is $\mathbf{\Psi}_t = C_1 \cdot \mathbf{\Phi}_1 + C_2 \cdot \mathbf{\Phi}_2$, where C_1 and C_2 are constants. Finally, the condition of continuity of the state vector $\mathbf{\Psi}$ at the interface z = z_0 is expressed as follows:

$$\mathbf{\Psi}_i + \mathbf{\Psi}_r = L^{-1}\left(C_1 \cdot \mathbf{\Phi}_1 + C_2 \cdot \mathbf{\Phi}_2\right), \tag{7}$$

where L^{-1} is the inverse matrix of the L(z_0,z_N,t) transfer matrix.

By setting:

$$\mathbf{\Psi}_1 = L^{-1}\mathbf{\Phi}_1^= \begin{pmatrix} \Psi_{11} \\ \Psi_{21} \\ \Psi_{31} \\ \Psi_{41} \end{pmatrix} \text{ and } \mathbf{\Psi}_2 = L^{-1}\mathbf{\Phi}_2^= \begin{pmatrix} \Psi_{12} \\ \Psi_{22} \\ \Psi_{32} \\ \Psi_{42} \end{pmatrix}, \tag{8}$$

Eq.(7) can be written as follows:

$$\begin{pmatrix} E_{ip}+E_{rp} \\ E_{ip}-E_{rp} \\ E_{is}+E_{rs} \\ E_{is}-E_{rs} \end{pmatrix} = C_1^{\cdot} \begin{pmatrix} \Psi_{11}/\cos\phi_0 \\ \Psi_{21}/n_0 \\ \Psi_{31} \\ \Psi_{41}/n_0\ \cos\phi_0 \end{pmatrix} + C_2^{\cdot} \begin{pmatrix} \Psi_{12}/\cos\phi_0 \\ \Psi_{22}/n_0 \\ \Psi_{32} \\ \Psi_{42}/n_0\ \cos\phi_0 \end{pmatrix}. \tag{9}$$

Equation (9) splits into a system of two linear equations:

$$\begin{cases} \mathbf{E}_i + \mathbf{E}_r = \mathbf{A}_0 \cdot \mathbf{C} \\ \mathbf{E}_i - \mathbf{E}_r = \mathbf{B}_0 \cdot \mathbf{C} \end{cases}, \tag{10}$$

where

$$\mathbf{C} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}, \mathbf{A}_0 = \begin{pmatrix} \Psi_{11}/\cos\phi_0 & \Psi_{12}/\cos\phi_0 \\ \Psi_{31} & \Psi_{32} \end{pmatrix}$$
$$\text{and } \mathbf{B}_0 = \begin{pmatrix} \Psi_{21}/n_0 & \Psi_{22}/n_0 \\ \Psi_{41}/n_0\ \cos\phi_0 & \Psi_{42}/n_0\ \cos\phi_0 \end{pmatrix}. \tag{11}$$

The set of equations (10) is solved by eliminating vector $\mathbf{C}$ to obtain the relationship $\mathbf{E}_r = [(\mathbf{A}_0 - \mathbf{B}_0)\cdot(\mathbf{A}_0 + \mathbf{B}_0)^{-1}]\cdot\mathbf{E}_i = \mathbf{R}\cdot\mathbf{E}_i$. It is preferable to multiply both $\mathbf{A}_0$ and $\mathbf{B}_0$ matrices by $\mathrm{n}_0 \cos\ \phi_0$ in order to avoid divisions by zero for reflection matrix is

$$\mathbf{R} = (\mathbf{A} - \mathbf{B})\cdot(\mathbf{A} + \mathbf{B})^{-1}, \tag{12}$$

where

$$\mathbf{A} = \begin{pmatrix} n_0\,\Psi_{11} & n_0\Psi_{12} \\ n_0\ \cos\phi_0\Psi_{31} & n_0\ \cos\phi_0\,\Psi_{32} \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} \cos\phi_0\,\Psi_{21} & \cos\phi_0\,\Psi_{22} \\ \Psi_{41} & \Psi_{42} \end{pmatrix}. \tag{13}$$

The procedure of calculation of the reflection matrix $\mathbf{R}$ is summarized as follows: (i) the eigenvectors $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ of the substrate H-matrix are calculated, (ii) Eq. (5) is used to calculate the L^{-1} matrix and then vectors $\mathbf{\Psi}_1$ and $\mathbf{\Psi}_2$, (iii) Eqs. (12) and (13) are used to calculate the $\mathbf{R}$ matrix. The calculation of the $\mathbf{R}$ matrix is rather easy, since it requires simple matrix operations that are available in most computing softwares.

When acoustic waves propagate within the sample, the reflection matrix is perturbed. The same method can be applied to calculate the perturbed reflection matrix $\mathbf{R}'$. Then the perturbation of the reflection matrix can be expressed as follows:

$$\Delta\mathbf{R} = [\Delta(\mathbf{A} - \mathbf{B}) - \mathbf{R}\cdot\Delta(\mathbf{A} + \mathbf{B})]\cdot(\mathbf{A} + \mathbf{B})^{-1}. \tag{14}$$

To calculate $\Delta\mathbf{R}$ for an arbitrary distributed strain field within the sample, it is advantageous to calculate first the perturbation $\delta\mathbf{R}(\mathrm{z})$ of the reflection matrix induced by delta-like acoustic strain of unit magnitude located at each point z within the sample. Then, the overall variation of the reflection matrix is

$$\Delta\mathbf{R}(t) = \int_{z_0}^{z_N} \left[\sum_{A=1,\ldots,3} \delta\mathbf{R}_A(z)\cdot S_A(z,t) \right] dz, \tag{15}$$

where S_A represents the magnitude of the strain at point z associated to the acoustic mode specified by subscript A. The sum in Eq. (15) includes only three terms since only three independent plane acoustic waves may propagate in an elastic medium. If acoustic waves propagate in the ambient medium, it is possible to define the first layer as a perturbed buffer layer of ambient medium. The thickness of this buffer layer should be about the absorption length of the acoustic pulses. For picosecond pulses, this attenuation length is generally less than 1 μm. A similar buffer layer (the last layer number N) can be defined in the substrate to take account for the attenuation of acoustic waves propagating in the substrate.

In the next section, we will show an application of the matrix method to calculate the reflection matrix of a sample perturbed by short longitudinal and shear acoustic pulses.

III. SHORT ACOUSTIC PULSE IN AN ISOTROPIC HOMOGENEOUS MEDIUM

We consider an optically isotropic and homogeneous medium perturbed by acoustic waves. This strain field, expressed by the strain tensor field S_{kl}, induces a perturbation of the permittivity tensor $\Delta\varepsilon_{ij} = -n^4 p_{ijkl} \cdot S_{kl}$, where n and p_{ijkl} are the refractive index and the photoelastic tensor of the medium, respectively. In the Voigt notation, for an isotropic medium, the permittivity-strain relationship is written as follows:

$$\begin{pmatrix} \Delta\varepsilon_1 \\ \Delta\varepsilon_2 \\ \Delta\varepsilon_3 \\ \Delta\varepsilon_4 \\ \Delta\varepsilon_5 \\ \Delta\varepsilon_6 \end{pmatrix} = -n^4 \begin{pmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ S_3 \\ S_4 \\ S_5 \\ 0 \end{pmatrix}$$

$$= -n^4 \begin{pmatrix} p_{12} S_3 \\ p_{12} S_3 \\ p_{11} S_3 \\ 0 \\ 0 \\ 0 \end{pmatrix} - n^4 \begin{pmatrix} 0 \\ 0 \\ 0 \\ p_{44} S_4 \\ p_{44} S_5 \\ 0 \end{pmatrix} = \Delta\varepsilon_L + \Delta\varepsilon_T, \tag{16}$$

where $S_3 = \partial u_z/\partial z$, $S_4 = \partial u_y/\partial z$, $S_5 = \partial u_x/\partial z$, and u_x, u_y, u_z are the components of the displacement vector $\mathbf{u}$ of the acoustic wave. The longitudinal and transverse displacement vectors are $\mathbf{u}_L = (0,\ 0,\ u_L)^T$ and $\mathbf{u}_T = (u_T \cos\varphi,\ u_T \sin\varphi,\ 0)^T$, respectively, where the superscript T denotes the transposition of the row vector and the angle φ represents the direction of the transverse displacement vector in the xy-plane. Equation (16) shows that the perturbation of the permittivity tensor can be considered as a superposition of longitudinal and transverse parts.

The perturbation of the permittivity tensor induces a perturbation of the differential propagation matrix *H*. In most picosecond acoustics experiments, the magnitude of acoustic strain S_A is below 10^{-3}. So, the first order expansion in S_A of the perturbed

H-matrix, $H' = H + (\partial H/\partial \mathrm{S}_A)\cdot\mathrm{S}_A = H + D_A\cdot\mathrm{S}_A$, is sufficient. We call the D_A matrix the "sensitivity matrix" associated to acoustic mode A. Equations (4) and (16) can be used to calculate sensitivity matrices for longitudinal (L) and transverse (T) acoustic modes, which are expressed as follows:

$$D_L = \begin{pmatrix} 0 & -\mathrm{n}^2\sin^2\phi\ \mathrm{p}_{11} & 0 & 0 \\ -\mathrm{n}^4\mathrm{p}_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\mathrm{n}^4\mathrm{p}_{12} & 0 \end{pmatrix} \text{ and } D_T$$

$$= \mathrm{n}^3\sin\phi\ \mathrm{p}_{44} \begin{pmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & \cos\phi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \sin\phi & 0 & 0 \end{pmatrix} \tag{17}$$

The D_L matrix is composed of two diagonal blocks of 2 × 2 matrices. The upper and the lower blocks are concerned respectively with the p and s polarizations. The form of the D_L matrix means that the p and s optical modes are not coupled by longitudinal acoustic waves; so p and s optical modes can then be treated as independent optical modes. On the contrary, the D_T matrix contains non-zero off-diagonal blocks of 2 × 2 matrices, which are responsible for the coupling between p and s optical modes. This coupling vanishes for $\varphi = 0$, i.e., when the transverse displacement lies in the plane of incidence. In contrast, maximum coupling is achieved when the transverse displacement is perpendicular to the plane of incidence. Moreover, at normal incidence ($\phi = 0^\circ$), the D_T matrix vanishes. Thus, non-vanishing sensitivity to plane shear waves that propagate along the z-axis is achieved only if the optical probe is reflected on the sample at oblique incidence.

We consider a delta-like acoustic strain pulse within an isotropic substrate medium. This strain pulse can be considered as the limit case of a thin birefringent homogeneously strained slab of thickness e, when e tends to zero. The transfer matrix **L** of the slab, with homogeneous strain S_A, is $L = \lim_{e\to 0}\exp(-ikeH') = 1 - iku_AD_A$, where $u_A = \lim_{e\to 0} eS_A$. The symbol 1 is the 4 × 4 identity matrix and u_A is the displacement associated to the delta-strain. Then $L^{-1} = 1 + iku_AD_A$, as we suppose that $\mathrm{ku}_A <<1$.

The two optical eigenmodes of the unperturbed medium are p- and s-polarized modes. Then, $\mathbf{\Phi}_1 = \mathbf{\Phi}_p+$ and $\mathbf{\Phi}_2 = \mathbf{\Phi}_s+$; so $\mathbf{\Psi}_1 = L^{-1}\mathbf{\Phi}_1 = \mathbf{\Phi}_p + +\mathrm{iku}_A D_A\cdot\mathbf{\Phi}_p+$ and $\mathbf{\Psi}_2 = L^{-1}\mathbf{\Phi}_2 = \mathbf{\Phi}_s + +\mathrm{iku}_A D_A\cdot\mathbf{\Phi}_s+$. Finally, the reflection matrices for delta-like longitudinal and shear strains are

$$\delta\mathbf{R}_L = \begin{pmatrix} \mathrm{a}_p & 0 \\ 0 & \mathrm{a}_s \end{pmatrix}\cdot\mathrm{u}_L \text{ and } \delta\mathbf{R}_T = \begin{pmatrix} 0 & \mathrm{a}_T \\ -\mathrm{a}_T & 0 \end{pmatrix}\cdot\mathrm{u}_T, \tag{18}$$

respectively, where

$$\mathrm{a}_p = \mathrm{ikn}^3(\mathrm{p}_{12}\cos^2\phi - \mathrm{p}_{11}\sin^2\phi)/(2\cos\phi), \tag{19}$$

$$\mathrm{a}_s = \mathrm{ikn}^3\mathrm{p}_{12}/(2\cos\phi), \tag{20}$$

$$a_T = ikn^3 p_{44} \sin\phi \sin\varphi/(2\cos\phi). \tag{21}$$

Equations (18) to (21) were used to calculate the $\mathbf{\Delta R{\cdot}R}^{-1}$ matrix of a sample perturbed by a delta-like acoustic pulse within the substrate [6]. The $(\mathbf{\Delta R{\cdot}R}^{-1})_L$ matrix has only diagonal components; so according to Eq. (2), longitudinal waves does not induce rotation of polarization for p- or s-polarized probe. On the contrary, the $(\mathbf{\Delta R{\cdot}R}^{-1})_T$ matrix for transverse acoustic waves have only non-diagonal components, which means that p- or s-polarized probe will exhibit a rotation of polarization but no change in amplitude. This result proves the relevance of polarimetric measurements to detect shear acoustic waves in isotropic media.

IV. CONCLUSIONS

A matrix method of calculation of the acousto-optic response of a stratified planar anisotropic inhomogeneous sample was presented. A transfer matrix characterizes completely the reflection properties of the multilayered sample and determines its reflection Jones matrix **R**. So the optical field within the sample has not to be known explicitly. The method was applied to calculate the perturbation $\mathbf{\Delta R} = \mathbf{R'} - \mathbf{R}$ and the transient reflection $\mathbf{\Delta R{\cdot}R}^{-1}$ matrix, whose components are directly related to measured time-resolved acousto-optic signals. The method can also be applied to calculate the effects of displacements of the interfaces, which have a non-negligible impact on the acousto-optic response of a multilayer.

This matrix method can be also extended to calculate the effect of other kinds of physical perturbations on the **R**-matrix, such as the influence of the transient electric field through the electro-optic effect, or the transient magnetic field through the magneto-optic effect.

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