Temperature Profile for Ballistic and Diffusive Phonon Transport in a Suspended Membrane with a Radially Symmetric Heat Source

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Temperature profiles for phonon heat transport in a suspended membrane with a radially symmetric heat source are calculated for two extreme cases: (1) purely ballistic transport and (2) purely diffusive transport. Theoretical results confirm that it is possible to distinguish between these two transport mechanisms on the basis of the radial temperature profiles alone. Model results are also compared with the experimental data measured using 40-nm-thick free-standing silicon nitride membranes at temperatures below 1 K; superconductor-normal metal-superconductor tunnel junction (SINIS) thermometers are used for the measurements. The measured temperature profile is in quantitative agreement with the results obtained using the ballistic model only when the distance is below 50 μm.

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I. INTRODUCTION

Establishing the mechanism of thermal transport is a key challenge in the design of thermal devices for low-temperature applications. Suspended silicon nitride (SiN_x) membranes are widely used in many low-temperature applications because they have several advantages, including ease of fabrication and efficient thermal isolation from the environment. The aforementioned membranes are employed in bolometers used to detect radiation from sub-millimeter waves to gamma-rays [1]. Traditionally, the transport mechanisms in these membranes have been studied by measuring the temperature dependence of thermal conductance and the absolute value of heat transport [2–5], but the conclusions drawn are contradictory. Thus, it can be stated that the mechanism of phonon thermal transport in thin SiN_x membranes has not yet been fully understood.

Herein, we propose an alternative approach for elucidating the phonon transport mechanism; this approach involves the study of the temperature profile, $T(r)$, of the given SiN_x membrane. For this purpose, we determine the temperature increase at a given location on the membrane as a function of the distance from the heater. The obtained results can be used to obtain additional information about the nature of phonon transport in SiN_x membranes. We first calculate the temperature profiles for two extreme cases, i.e., purely ballistic heat transport and purely diffusive heat transport, in the simple geometry of a rcircular heater and a thermometer in the shape of a circular arc. The results clearly show that it is possible to distinguish between the transport mechanisms in the above mentioned

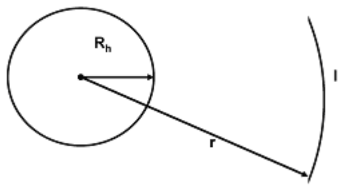

FIG. 1: Schematic of the studied sample geometry. A circular heater with radius R_h emits phonons, which can be detected by a thermometer of length l placed at a distance r from the center of the heater.

two cases solely on the basis of the shape of the $T(r)$ profile. Finally, we compare the results with experimental data obtained in the case of a 40-nm-thick free-standing SiN_x membrane for temperatures below 1 K. The measured data are in good quantitative agreement with the profile obtained for the ballistic model but deviate from the profile when the distance between the heating location and the heater increases.

II. THEORETICAL CONSIDERATIONS

The calculations can be greatly simplified if one considers a large highly symmetric membrane with a small circularly symmetric heater at the center and a narrow thermometer wire in the shape of a circular arc at a distance r from the center of the heater. The schematic of the geometry is presented in Fig. 1. The fact that heat can flow only along the membrane restricts the problem first to two dimensions, and the radial symmetry of the heater then reduces the problem to one dimension. Thus, temperature will depend only on the radial coordinate r. In principle, full radial symmetry would require the membrane edge (where it connects to the bulk substrate) to be circular also. However, if this membrane edge is far from the heater and the thermometer, the actual shape of the membrane edge plays no role, as the corrections to the temperature profile would appear only near the edge. This consideration is relevant in practice, because even though we can fabricate a circular heater using lithography, the membrane edge is square owing to the crystallographic etching process used to suspend the membrane [6].

First, we consider the case of ballistic phonon transport, where no scattering takes place in the bulk of the membrane and all surface scattering is specular. This limit is the extreme case of a more general class of radiative transport models, where some or all of the surface scattering can be diffusive [7]. In these models, one uses the analogy between phonon radiation and photon radiation, where the radiation in vacuum is in dynamic equilibrium with the emitting and absorbing surfaces at a given temperature. In the case of ballistic phonon transport, the surfaces are the metallic heater and thermometer, which interact with the phonons because of electron-phonon coupling, and the analog of vacuum is the SiN membrane itself. One immediate important result is then obvious without the need for any explicit calculations: The geometry (size) of the heater and the thermometer will influence the temperature profile. In other words, the temperature of the thermometer in equilibrium with the phonon radiation can be defined and measured, but this temperature depends on the size of the thermometer and heater in the general case. Another corollary of this result is that if two or more thermometers are placed on the membrane, they influence each other so that the measured temperatures change. This has been observed experimentally [6].

In the following, we also simplify the calculations by assuming that the substrate is at 0 K. As our experiments are usually performed at very low substrate temperatures $T \sim 50$ mK, the error introduced is small if the measured phonon temperatures are high. Finally, if the membrane edge is fully absorbing (black body at $T = 0$ K), we can ignore it altogether in the calculation. Within these assumptions, in steady state, the net radiative powers emitted by the heater, P_h, and thermometer, P_t, are

$$\begin{aligned} P_h &= P_{heat} + \alpha P_t \\ P_t &= \beta P_h, \end{aligned} \tag{1}$$

where P_{heat} is the power input from the electrical circuit into the heater; $\alpha = R_h/2r$ and $\beta = l/2\pi r$, where R_h is the radius of the heater, r is the distance between the center of the heater and the thermometer, and l is the length of the thermometer arc. The factors α and β can be derived from the geometry: βP_h is the fraction of heater-emitted power intercepted by the thermometer, and αP_t is the fraction of thermometer-emitted power intercepted by the heater. In the above, we also assume that the thermometer does not intercept any of its own radiation, which is a good approximation if $l \ll r$. From Eq. (1), we can solve for P_t:

$$P_t = \frac{l}{2\pi r}\left(1 - \frac{lR_h}{4\pi r^2}\right)^{-1} P_{heat} \equiv P_{rad}, \tag{2}$$

which is equal to the power emitted by the thermometer in a phonon radiative transfer model, P_{rad}. When an effective model $P_{rad} = aT^n$ with a constant a is used, we finally obtain the distance dependence in the ballistic limit:

$$T = \left[\frac{Bl}{2\pi r}\left(1 - \frac{lR_h}{4\pi^2}\right)^{-1}\right]^{(1/n)}, \tag{3}$$

where $B = P_{heat}/a$. The dominant dependence at large distances $2\pi r > l$ is then simply $T \sim 1/r^{(1/n)}$, when the finite size correction (the term in parentheses) can be ignored. The constants a and n can be thought of as fitting parameters from a fit to T versus P_{heat} data at constant r. Theory does, however, predict for 3D phonons in the isotropic case that $n = 4$ and $a = 2ld\sigma$ where d is the membrane thickness, σ is the phononic Stefan-Boltzmann constant

$$\sigma = \frac{\pi^2 k_B^4}{120\hbar^3}\left(\frac{2}{c_t^2} + \frac{1}{c_l^2}\right), \tag{4}$$

and where c_t and c_l are the transverse and longitudinal speeds of sound, respectively. Thus, we obtain in the limit where the thermometer is small ($l \ll R_h$):

$$T(r) = \left(\frac{P_{heat}}{2\pi\sigma dr} \right)^{1/4} . \tag{5}$$

Interestingly, this temperature profile has a universal shape, and its magnitude is scaled only by the material parameters through σ and the thickness of the membrane d.

In the diffusive case, one can define, as usual, a local phonon temperature without explicit consideration of the thermometer geometry. The radial heat flow is written as

$$P = -S(r)\kappa(T)\frac{dT}{dr}, \tag{6}$$

where $S(r) = 2\pi rd$ is the cross-sectional area the heat transport in the case of the studied geometry, at a distance r from the center, and d is the thickness of the membrane. Now we assume for the thermal conductivity $\kappa(T) = bT^m$, where b is a constant, and from Eq. (6), we obtain

$$-\int_{r_0}^{r} \frac{P}{2\pi rd} dr = \int_{T_0}^{T} bT^m dT \tag{7}$$

After noting that the total power cannot depend on r, and integrating, Eq. (7) gives

$$\frac{P}{2\pi d} \ln\left(\frac{r_0}{r}\right) = \frac{b}{m+1}\left(T^{m+1} - T_0^{m+1}\right), \tag{8}$$

where a point (r_0, T_0) is known. From Eq. (8), we can solve the temperature profile in the diffusive case:

$$T = \left[T_0^{m+1} + \frac{(m+1)P}{2\pi bd} \ln\left(\frac{r_0}{r}\right) \right]^{1/(m+1)}, \tag{9}$$

where the power P must be equal to the power input from the electrical circuit, $P = P_{heat}$ and is therefore known. In our experimental situation, the only fixed boundary condition is at the membrane edge, where the temperature must equal the substrate temperature. Thus, in Eq. (9), r_0 is the membrane edge, and with the same approximation as before, namely, the substrate temperature equals zero, $T_0 = 0$ K. The temperature of the heater is not fixed but is dependent on the input power. We thus note a significant difference between the diffusive and ballistic cases: The diffusive model requires the presence of the edge of the membrane, whereas the ballistic conductance can be considered for an infinite membrane also.

Figure 2 compares the temperature profiles calculated from either the ballistic or the diffusive model, Eqs. (5) and (9), respectively. In the diffusive case, we have chosen the constant in thermal conductivity, b, in such a way that the temperatures at the heater

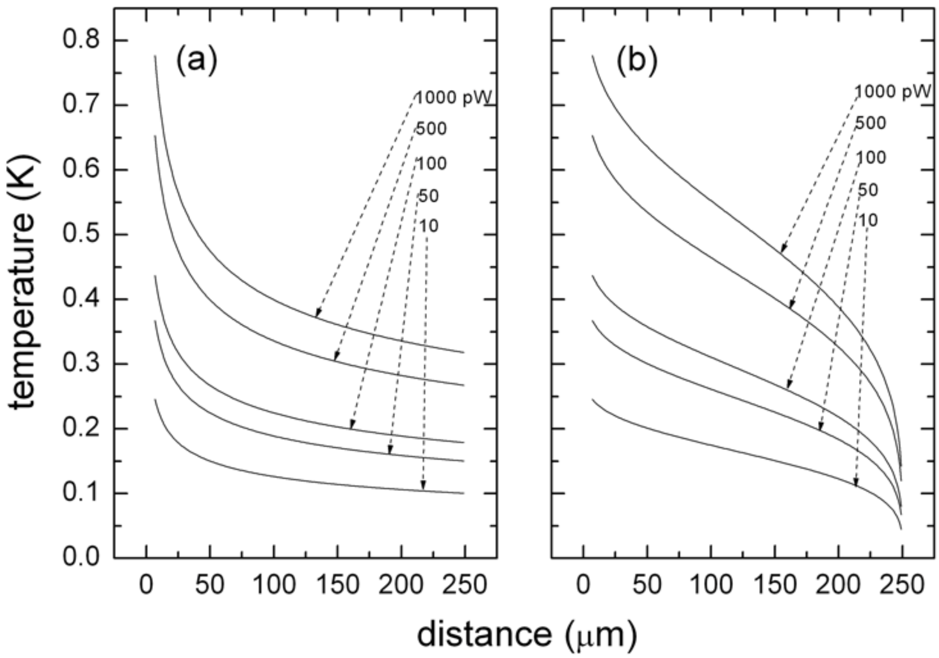

FIG. 2: Theoretical temperature profiles for (a) ballistic [Eq. (5)] and (b) diffusive [Eq. (9)] phonon transport in membranes for radially symmetric geometry. Different curves correspond to varying input powers in sequence $P = 10, 50, 100, 500, 1000$ pW.

location are equal for both models for the same input power P_{heat}. The plots are calculated using the speeds of sound for SiN $c_t = 6200$ m/s and $c_l = 10\ 300$ m/s and a membrane thickness $d = 200$ nm. The heater is located at $r = 7$ μm and the membrane edge at $r = 250$ μm. Furthermore, we select $n = m + 1 = 4$, so that in both cases $P \sim T^4$. Physically, $m = 3$ corresponds to diffuse surface scattering, which is expected to be the dominant scattering mechanism at low temperatures [8].

In Fig. 2 it can be clearly observed that the temperature profiles of the two cases have differing shapes and that the shape does not vary with input power. The ballistic case [Fig. 2(a)] has a steep drop close to the heater and then a slow decay at large distances, whereas the diffusive profile [Fig. 2(b)] is almost linear, except for a steeper initial drop near the heater, and then a strong drop near the membrane edge. At the membrane edge the two cases display completely different behavior, as in the ballistic case the edge plays no role. The profiles should also be compared to the well-known 1D case [7], in which the temperature profile is constant or linear in the ballistic and diffusive cases respectively.

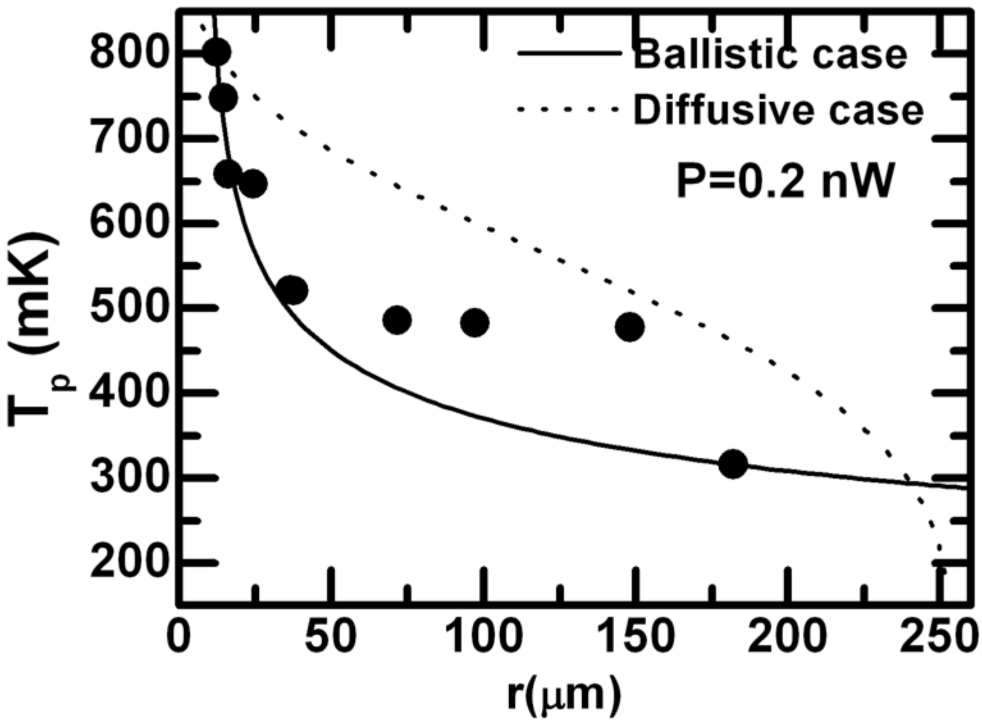

FIG. 3: Plot of local phonon temperature T_p versus distance from the center r at constant heating power $P = 0.2nW$. Black dots: measured data. Black solid line: data obtained for the ballistic model from Eq. (3). Black dotted line: data obtained for the diffusive model from Eq. (9).

III. EXPERIMENT

The measurement samples comprised of a 40-nm-thick suspended low-stress SiN_x membrane (dimensions: $\sim 550 \times 550\ \mu m^2$), which was anisotropically etched from double-sided, low-pressure chemical vapor deposition (LPCVD) nitridized (100) silicon wafers in aqueous KOH solution. At the center of the membrane, we fabricated a circular Cu wire heater, whose radius, width, and thickness were 7 μm, $\sim$ 200 nm, and 30 nm, respectively. The wire was directly connected to superconducting Nb leads at both ends of the heater wire to form superconductor-normal metal (SN) junctions. A symmetric normal-metal-insulator-superconductor (NIS) tunnel junction pair made of Cu/AlO_x/Al was fabricated at a distance r from the center to measure the local phonon temperature of the membrane. Several samples were fabricated to study the temperature profiles, by varying the distance between the heater and the thermometer and maintaining the heater and thermometer sizes constant. Further details of the sample fabrication process and geometry are presented in Ref. [6].

In the measurement, we applied a slowly ramping DC voltage into the heater wire and measured the input Joule heating power $P = IV$ by a four-probe configuration. Owing to Andreev reflection in the SN contacts, all the input power is dissipated uniformly only in the Cu wire, thereby causing radially symmetric phonon power emission (via electron-phonon coupling) to the membrane. Simultaneously, the current-biased SINIS tunnel junction thermometer measures the response in the local phonon temperature T_p at distance r. A more detailed description of the measurement technique and tunnel junction thermometry is presented in Refs. [6, 9].

In Fig. 3, we present the measured data (black dots) for nine distances when the input heating power P is 0.2 nW. The measured temperature profiles look very similar for other values of P, as expected. At this level of heating, the measured temperatures are sufficiently high so that the membrane phonons are expected to be in the 3D limit [9]. For comparison, we have also plotted the theoretical models in the ballistic and diffusive cases from Eqs. (3) and (9). For the diffusive model, we use the temperature exponent $m = 3$ and fix the fitting parameter b such that the measured data agree with the model data for two points $(r_0, T_0) = (12.2\ \mu\text{m}, 802\ \text{mK})$ and $(r_1, T_1) = (250\ \mu\text{m}, 50\ \text{mK})$ (the membrane edge). For the ballistic model, we use $n = 4$ and the fitting constant $B = 7.8 \times 10^{11}\ \text{K}^4$.

Clearly, the measured data agree qualitatively better with the shape of the ballistic model, particularly at short distances $r < 50\ \mu$m. The diffusive model would predict a temperature profile that is almost linear and much higher in absolute value than the measured data. After the strong decrease in temperature, the measured temperatures form a plateau between $r \sim 75\ \mu$m and $r \sim 150\ \mu$m, , where the temperature hardly decreases with distance. Although the ballistic model predicts a flat temperature profile at large distances, the absolute values of the measured temperatures are much higher than that expected from the simplest ballistic theory, Eq. (3). This is not fully understood at the moment, but it may be caused by a fraction of the surface scattering events being diffusive, increasing the thermal resistance. Moreover, at the final point, $r = 180\ \mu$m, the measured temperature drops again quite significantly compared to the previous points. This is most likely because of the effect of the membrane edge, but an understanding of this behavior is also currently lacking.

IV. CONCLUSIONS

We carried out theoretical and experimental investigations of the temperature profiles for thermal transport in thin suspended SiN_x membranes at sub-Kelvin temperatures. We also calculated the radial temperature profile for circular symmetry, under the following two conditions: (1) purely ballistic transport and (2) purely diffusive transport. The temperature profiles for these two transport mechanisms were markedly different even in the case of localized heating in the membranes with 2D heat flow, similar to the case of the better-known 1D heat flow [7]. The results obtained with theoretical models were compared with the measured data, which agree qualitatively with the results of the ballistic model. However, complete quantitative agreement between the results of the simple ballistic transport model and the experimental results was not observed. This was expected, in hindsight, because we considered only a simplified limit where surface scattering was specular. Better agreement between the theoretical and experimental results can be obtained if a more refined model that takes into account a certain degree of diffusive surface scattering is used.

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