Ground State Properties of an Extended Hubbard Chain with Easy-Axis Magnetic Anisotropy

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We analytically study the ground state properties of a half-filled extended Hubbard chain for an arbitrary sign of the on-site and easy-axis spin interactions, using the weak-coupling theory combining bosonization with renormalization group (RG) approaches. The weak-coupling phase diagram consists of four different phases, characterized by the Luttinger-liquid with the TS orderings, the TS order, and the longitudinal SDW as well as the transverse SDW correlations. In the strong coupling regime and for on-site Coulomb repulsion, the system only shows a long-range ordered SDW ordering. The result exhibits that the dominant easy-axis anisotropy can change the topology of the structure of the conventional phase diagrams.

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I. INTRODUCTION

One-dimensional (1D) electron correlations in condensed matter physics have been a subject of intensive research in recent years, motivated not only by theoretical interest in studying novel concepts in 1D (e.g., charge-spin separation) and methods (e.g., exact diagonalization, quantum Monte Carlo, and density matrix renormalization group) but also by the discovery of quasi-1D conductors and high-Tc superconductivity. At half filling, correlation effects have the strongest impact on the low-energy physics due to the existence of an Umklapp scattering process. Especially the discovery of triplet superconductivity (TS) in the Bechgaard salts [1] and the ruthenate compounds [2], and the subsequent discovery of coexistence of the TS phase with ferromagnetism in some rare earth compounds [3–5], triggered interest in studies of correlated electron systems which exhibit close proximity of the magnetic and superconducting orderings [6–12].

A suitable model for investigating the competition, or even the coexistence, of magnetic order and superconductivity is the so-called t-U-J model [13–19]. It is an extended Hubbard model by explicitly including an intersite spin exchange interaction. Zhang used a 2D version of it to study correlated electrons in the Cu-O plane, showing a quantum transition from a Mott insulator to a gossamer superconductor at half filling with decreasing on-site repulsion, and finally to the RVB spin liquid phase away from half-filling [20]. Since in two dimensions it remains challenging to accurately solve this model, it may be instructive to analyze the 1D version. Among others, much effort has been devoted to investigating the resulting quantum phase diagrams for a half-filled band. Dai et al. studied the isotropic antiferromagnetic exchange by analytical and TMRG methods, and the phase...
diagram found consists of the critical spin-density-wave (SDW) phase for $J < 2U$ and a bond-charge-density-wave (BCDW) phase for $J > 2U$ in the weak-coupling regime; with increasing $J$, $U_c(J)$ deviates from the transition line $U_c = J/2$ and eventually tends to zero as $J \to +\infty$ [14]. Japaridze and Müller-Harmann studied the case for the anisotropic Heisenberg exchange and obtained rich phase diagrams which show the close proximity of the TS, SDW, and Peierls dimerized phases, revealing coexistence of the TS and SDW phases in a 1D itinerant electron system [15]. In particular, the experimental observation of the ferromagnetic easy-plane anisotropy in some ruthenate compounds [21] increased interest in the subject of the extreme anisotropy. Dziurzik et al. [22] studied the XY-type Heisenberg exchange case at half-filling analytically and numerically, and found that the phase diagram is composed of a sequence of transitions (with increasing ferro-exchange $J_{xy} < 0$) from a metallic phase with coexisting TS and longitudinal SDW (SDW$_z$) instabilities into an insulating Néel-type antiferromagnetic phase and, finally, for strong transverse ferro-exchange, into the insulating ferromagnetic XY phase. In contrast, in the case of antiferromagnetic exchange, the system shows a transverse SDW (SDW$_{x;y}$) phase, irrespectively of the magnitude of $J_{xy} > 0$.

Due to the crystal structure, spin-orbit coupling, or other factors, e.g., the theoretical interest, the exchange interaction is sometimes characterized by the easy-axis anisotropy (the Ising-type) instead of the usual XXZ-type anisotropy [14], the easy-plane anisotropy [22], or the fully anisotropic exchange (XYZ-type). In this paper, we discuss such an issue, further exploring the microscopic mechanisms for the competition between the insulating magnetic orderings and the superconducting correlations. The model Hamiltonian we consider is given by

$$H = -t \sum_{j, \alpha} \left( c_{j+1, \alpha}^\dagger c_{j, \alpha} + h.c. \right) + U \sum_j n_{j\uparrow} n_{j\downarrow} + J_z \sum_j S_j^z S_{j+1}^z,$$

where $t$ is the nearest-neighbor hopping integral, $c_{j\alpha}$ ($c_{j\alpha}^\dagger$) the annihilation (creation) operator of an electron with spin $\alpha$ at site $j$, $U$ the on-site interaction, and $J_z$ the intersite easy-axis exchange. In the limiting case $J_z = 0$, this is just the Hubbard model [23]. At half filling, the ground state exhibits critical SDW behavior for on-site repulsion, and is characterized by the coexistence of an insulating charge-density-wave (CDW) and singlet superconducting (SS) instabilities for on-site attraction. In the limit $U/t \gg 1$, it corresponds to the $t$-$J_z$ model [24], in which Batista and Ortiz, using an exact mapping to a spinless fermion model that can be exactly done by the Bethe ansatz, presented the quantum phase diagram for an arbitrary spin (integer or half-integer) and sign of the spin-spin interaction $J_z$.

In spite of its simple form, the Hamiltonian (1) is not a toy model. Because the problem studied is of close relevance for a special class of band magnetic materials, where the longitudinal exchange anisotropy might be dominated. The model considered breaks SU(2) and U(1) continuous symmetries in the spin space down into the $Z_2$-symmetry. It is anticipated that such a discrete symmetry leads to the occurrence of another phase transition, and eventually to a change of topology of the conventional phase diagrams. Compared to the $t$-$U$-$J_{xy}$ model [22], the Hamiltonian Eq. (1) contains more complex physics. Typically
the spin-flip terms are like the hopping terms in the fermion Hamiltonian and give rise to the motion of fermions, whereas the $S^z - S^z$ interaction leads to a four-fermion interaction between adjacent electrons. There exists a competition between the hopping term and the potential energy which costs $J_z$ if there are electrons present at adjacent sites. So, naively, for large $J_z$ (at least in the absence of $U$), one expects the potential to win and the electrons to be localized, and the model shows an insulating behavior. Furthermore, unlike the easy-plane exchange, the interaction $J_z$ can cause charge-spin coupling. Therefore, the longitude exchange is expected to have a non-trivial effect on the low-energy physics in place of the transverse exchange.

In this work, we focus mainly on the ground state properties of the model (1) in the regime $|U|, |J_z| < t$, where the weak-coupling theory including bosonization and renormalization group (RG) techniques may be applied. As a theoretical model, we considered both on-site repulsion ($U \geq 0$) and on-site attraction ($U < 0$). We will demonstrate that at half-filling the weak-coupling phase diagram consists of four distinct sectors, characterized by the Luttinger-liquid with $TS^\pm$ orderings (triplet correlations in the $S^z = \pm 1$ channels), the $TS^0$ ordering (triplet correlations in the $S^z = 0$ channel), the SDW $^z$ (longitudinal SDW) phase, and the SDW $^\pm$ (transverse SDW) phase. Besides, we also make a simple qualitative discussion of the strong coupling limit $J_z, |J_z| \gg t$, finding that the system only shows a long-range ordered SDW $^z$ ordering for physical on-site Coulomb repulsion.

II. THE WEAK-COUPLING THEORY: BOSONIZATION AND RG ANALYSIS

Depending upon the weak interactions, we construct the low-energy field theory for the Hamiltonian Eq. (1). In the Fourier developments of fermion operators we retain only the modes close to the Fermi points $\pm k_F$ in one dimension. In the continuum limit, the initial lattice fermion annihilation operator can be written in terms of fast and slow fields as

$$c_{j,\alpha} = \sqrt{a} \cdot [e^{-ik_F x} \psi_{-,\alpha}(x) + e^{ik_F x} \psi_{+,\alpha}(x)],$$

where $x = ja$, with $a$ being the lattice parameter. $\psi_{-,\alpha}$ and $\psi_{+,\alpha}$ denote left-moving and right-moving fermion fields, respectively.

The bosonization procedure is the most convenient way to analyze one-dimensional interacting electron systems, converting the fermionic mode to a quantum theory of bose fields [25–29]. The corresponding mapping formula reads

$$\psi_{r,\alpha}(x) = \frac{F_{r,\alpha}}{\sqrt{2\pi a}} \exp \left\{ \frac{i}{2} [r \phi_c(x) + r \alpha \phi_s(x) - \theta_c(x) - \alpha \theta_s(x)] \right\},$$

where $r = R, L$ and $\alpha = \uparrow, \downarrow$ refer to $+$ and $-$ in that order. The boson field $\phi_{\mu}(x)$ and the dual field $\theta_{\mu}(x)$ describe the collective charge ($\mu = c$) and the spin ($\mu = s$) fluctuations, satisfying the relation $[\phi_{\mu}(x), \theta_{\mu'}(x')] = i\pi \delta_{\mu,\mu'} \text{sgn}(x - x')$. The Hermitian operator $F_{r,\alpha}$ ensures the anticommutation relations of different fermion fields. Using the formalism, the
The bosonized form of the $g$-ology Hamiltonian is obtained as $H = \int dx \mathcal{H}$, with the Hamiltonian density being
\[
\mathcal{H} = \frac{v_p}{2\pi} \sum_{r=\pm} (\partial_x \phi_{r, c})^2 - g_{\pi} \frac{g_{\pi}}{2\pi^2} \cos 2\phi_c \sum_{r=\pm} (\partial_x \phi_{r, c})^2 - \frac{g_{\sigma}}{2\pi^2} (\partial_x \phi_{c, +}) (\partial_x \phi_{c, -}) + \frac{g_c}{2a^2\pi^2} \cos 2\phi_c \\
+ \frac{V}{2\pi} \sum_{r=\pm} (\partial_x \phi_{r, s})^2 - \frac{g_{\pi}}{2\pi^2} (\partial_x \phi_{s, +}) (\partial_x \phi_{s, -}) + \frac{g_s}{2a^2\pi^2} \cos 2\phi_s \\
- \frac{g_{\sigma s}}{2a^2\pi^2} \cos 2\phi_c \cos 2\phi_s + \frac{g_{\rho}}{2\pi^2} \cos 2\phi_c \cos 2\phi_s + \frac{g_{\rho s}}{2\pi^2} \cos 2\phi_c \cos 2\phi_s,
\]
where $\phi_{c/s} = \phi_{c/s, +} + \phi_{c/s, -}$ are the total phase fields.

Up to leading order in the expansion with respect to the lattice parameter, the coupling matrix elements are given by
\[
g_{\rho} = g_{1\parallel} - g_{2\parallel} - g_{2\perp} = - \left( U + \frac{J_z}{2} \right) a, \quad (5)
g_{\sigma} = g_{1\parallel} - g_{2\parallel} + g_{2\perp} = \left( U - \frac{3J_z}{2} \right) a, \quad (6)
g_{c} = g_{3\perp} = - \left( U + \frac{J_z}{2} \right) a, \quad (7)
g_{s} = g_{1\perp} = \left( U + \frac{J_z}{2} \right) a, \quad (8)
g_{cs} = g_{\rho s} = -J_z a/2, \quad g_{c\sigma} = g_{\rho s} = J_z a/2. \quad (9)
\]
Here the indices $\parallel$ and $\perp$ denote the scattering of electrons with the same and opposite spins, respectively. The renormalized velocities are $v_{p/\sigma} = 2\pi a + (g_4) / 2\pi$, respectively, for the charge and spin degrees of freedom. In the following they will not be considered due to a secondary effect. The marginal coupling constants $g_{\rho}$ and $g_{c}$ ($g_{\sigma}$ and $g_{s}$) determine the low-energy behavior of the charge (spin) mode. The terms $g_{1\parallel}$ and $g_{1\perp}$ ($g_{3\parallel}$ and $g_{3\perp}$) represent the backward (Umklapp) scattering processes. $g_{2\parallel}$ and $g_{2\perp}$ ($g_{4\parallel}$ and $g_{4\perp}$) denote the forward scattering with the different (same) branch. The charge-spin coupling term $g_{cs}$ ($g_{\rho s}$) is generated from the Umklapp scattering of electrons with opposite (parallel) spins, while $g_{\rho s}$ ($g_{\rho s}$) stems from the backward scattering of electrons with parallel (opposite) spins.

Although the Hamiltonian Eq. (4) cannot be solved exactly, the renormalization-group (RG) analysis allows us to investigate the relative importance of various couplings. In the RG procedure, these couplings are thought of as a function of some scaling parameter $l$, e.g., the logarithm of the effective bandwidth. Note that the SU(2) symmetry of the charge sector ensures $g_{\rho} = g_{c}$, $g_{c\sigma} = g_{\rho s}$, and $g_{cs} = g_{\rho s}$. The one-loop RG equations for five
The above RG equations with sine-Gordon models, and we can analyze properties of the charge and spin modes separately. With this approximation in mind, the model Hamiltonian reduces to two decoupled scaling dimensionality 4. Therefore, as in the case of other studies \cite{31–35}, we can neglect the charge-spin coupling terms, assuming charge-spin separation in the weak-coupling regime. ∆e(\(\Delta_{c(s)}\)) denotes the charge (spin) gap.

These reduced equations determine the RG flow diagrams, shown in Fig. 1.

\[ \frac{d\tilde{g}_c}{dl} = -2\tilde{g}_c^2 - \tilde{g}_s\tilde{g}_{cs} - \tilde{g}_{cs}\tilde{g}_{c\sigma}, \]  
\[ \frac{d\tilde{g}_\sigma}{dl} = -2\tilde{g}_\sigma^2 - \tilde{g}_c\tilde{g}_{c\sigma} - \tilde{g}_{c\sigma}^2, \]  
\[ \frac{d\tilde{g}_s}{dl} = -2\tilde{g}_s\tilde{g}_s - \tilde{g}_c\tilde{g}_{cs} - \tilde{g}_{cs}^2, \]  
\[ \frac{d\tilde{g}_{c\sigma}}{dl} = -4\tilde{g}_c\tilde{g}_{c\sigma} - 2\tilde{g}_{c\sigma} - 2\tilde{g}_\sigma\tilde{g}_c - 4(\tilde{g}_s + \tilde{g}_{cs})\tilde{g}_{cs}, \]  
\[ \frac{d\tilde{g}_{cs}}{dl} = -4(\tilde{g}_c + \tilde{g}_{c\sigma})\tilde{g}_{cs} - 2(1 + \tilde{g}_\sigma)\tilde{g}_{cs} - 2(\tilde{g}_c + \tilde{g}_{c\sigma})\tilde{g}_s, \]

with initial values \(\tilde{g}_\nu(0) = g_\nu/(4\pi ta)\).

From these scaling equations, one finds that the \(\tilde{g}_c, \tilde{g}_\sigma, \) and \(\tilde{g}_s\) terms are marginal with the scaling dimensionality 2, while the \(\tilde{g}_{c\sigma}\) and \(\tilde{g}_{cs}\) are irrelevant operators with higher scaling dimensionality 4. Therefore, as in the case of other studies \cite{31–35}, we can neglect the charge-spin coupling terms, assuming charge-spin separation in the weak-coupling regime. With this approximation in mind, the model Hamiltonian reduces to two decoupled sine-Gordon models, and we can analyze properties of the charge and spin modes separately. The above RG equations with \(\tilde{g}_{c\sigma} = \tilde{g}_{cs} = 0\) become modified as follows. For the charge sector

\[ \frac{d\tilde{g}_c(l)}{dl} = -2\tilde{g}_c^2(l), \]  

for the spin sector

\[ \frac{d\tilde{g}_\sigma(l)}{dl} = -2\tilde{g}_\sigma^2(l), \quad \frac{d\tilde{g}_s(l)}{dl} = -2\tilde{g}_s(l)\tilde{g}_s(l). \]  

These reduced equations determine the RG flow diagrams, shown in Fig. 1.
The low-energy physics in the charge sector belongs to the universality class of the level-1 $SU_1(2)$ WZNW model [36]. Eq. (15) can be integrated immediately, giving
\[ \tilde{g}_c(l) = \frac{g_c(0)}{4\pi ta + 2g_c(0)l}. \] (17)
The flux is exactly fixed on the separatrix $\tilde{g}_c(l) = \tilde{g}_c(0)$ [see Fig. 1(a)]. For $g_c(0) > 0$, $\tilde{g}_c(l)$ flows to a weak coupling fixed point $\tilde{g}_c^*(l \to +\infty) = 0$, indicating that the charge excitation is gapless ($\Delta_c = 0$). For $g_c(0) < 0$, $\tilde{g}_c(l)$ grows at low energies with increasing length scale, and at a distance scale identified with a correlation length $\xi_c = a \exp[2\pi ta/g_c(0)]$, goes to a strong coupling fixed point $\tilde{g}_c^*(l \to \ln \xi_c) = -\infty$. In this case, a charge gap opens ($\Delta_c \neq 0$) accompanied by the expectation value of the charge field $\langle \phi_c \rangle = 0$. Consequently, the charge-gap transition occurs at the critical point $g_c = g_c = 0$, corresponding to the line
\[ U + \frac{J_z}{2} = 0. \] (18)

In the spin sector, the usual SU(2) symmetry preserved in the pure Hubbard model is broken by a finite $J_z$. We cannot directly solve a pair of coupled equations (16) as in the charge sector, but can turn to the RG diagram Fig. 1(b), which is divided into two qualitatively different sections by the separatrices $g_s = \pm g_\sigma$.

(i) For $g_\sigma \geq |g_s|$, the system is dominated in the weak-coupling (WC) sector. With increasing length scale, the perturbation interaction $\tilde{g}_c(l)$ associated with the cosine term flows to zero, and the spin excitation spectrum is massless ($\Delta_s = 0$). Depending on the bare coupling constants $g_\sigma$ and $g_s$ given by Eqs. (6) and (8), the WC regime corresponds to
\[ J_z \leq \min\{0; 2U\}. \] (19)

(ii) For $g_\sigma < |g_s|$, with increasing length scale the scaling trajectories eventually scale to the strong-coupling (SC) sector, $\tilde{g}_s^*(l) \to \pm \infty$. This leads to the dynamical generation of a finite energy gap in the spin excitation. In spin-gapped state, the spin field $\phi_s(x)$ is not free but ordered with the vacuum expectation values $\langle \phi_s \rangle = \pi/2$ for $g_s > 0$ and zero for $g_s < 0$. So there are two different SC sectors in the spin channel. For
\[ J_z > \max\{0; -2U\} \] (20)
the spin excitation is massive and the field $\phi_s$ gets ordered with the vacuum expectation value $\langle \phi_s \rangle = \pi/2$; while for
\[ 2U < J_z < -2U \] (21)
the spin channel is gapped with the vacuum expectation value $\langle \phi_s \rangle = 0$.

With the results of the spin excitation spectrum, we obtain two spin-gap transition lines, which correspond to
\[ J_z = 0 \] (22)
for $U > 0$, and to
\[ U - \frac{J_z}{2} = 0 \] (23)
for $U < 0$. 
III. THE WEAK-COUPLING PHASE DIAGRAM

In this section, we discuss the character of each phase which appears in the ground state phase diagram. If the charge and spin excitation spectra are both massless, the corresponding phase is a Luttinger liquid (LL) with the TS\(^\pm\) correlations. If the charge excitation remains gapless but the spin gap opens, we identify such a phase as either a Luther-Emery (LE) metal with the SS ordering or a TS\(^0\) phase, depending on the sign of \(\Delta_g\). In addition to these metallic phases, there exist the charge-gapped insulating phases, identified as the CDW, BSDW, SDW, and BCDW correlations. At half filling the correspondence between the fixed points and physical phases are summarized in Table I.

In order to characterize the dominant phases, we introduce a set of order parameters \(O_{TS}, O_{SS}, O_{CDW}, O_{BSDW}, O_{SDW}, \) and \(O_{BCDW}\), which depend on the field operators \(O_1 = \cos \phi, O_2 = \sin \phi, \) and \(O_3 = \exp(\pm i\theta)\). For the charge part \((\mu = c)\), \(O_{c1, c2, c3}\) represent the “dimer”, “Néel”, and “doublet” states, respectively; for the spin part \((\mu = s)\), \(O_{s1, s2, s3}\) denote the “singlet”, “triplet\(^0\)”, and “triplet\(^\pm\)” states, respectively.

The order parameters corresponding to the superconducting phases are given by \[O_{TS\pm} = \frac{1}{\sqrt{2}} \left( c_{j,+,j+1,+,\pm} + c_{j,+,j+1,+,\pm} \right) \propto \exp(i\theta_c) \exp(\pm i\theta_s), \tag{24}\]
\[O_{TS^0} = \frac{1}{\sqrt{2}} \sum_{\alpha} c_{j,\alpha,\alpha}^\dagger c_{j+1,\alpha,\alpha} \propto \exp(i\theta_c) \sin \phi_s, \tag{25}\]
\[O_{SS} = c_{j,+,j+1,+,\pm} \propto \exp(i\theta_c) \cos \phi_s. \tag{26}\]

The order parameters corresponding to the insulating phases are given by \[O_{CDW} = (-1)^j \sum_{\alpha} c_{j,\alpha,\alpha}^\dagger c_{j,\alpha,\alpha} \propto \sin \phi_c \cos \phi_s, \tag{27}\]
\[O_{SDW^+} = (-1)^j \sum_{\alpha, \alpha'} c_{j,\alpha,\alpha}^\dagger \sigma_{\alpha,\alpha'}^+ c_{j,\alpha',\alpha} \propto \cos \phi_c \sin \phi_s, \tag{28}\]
\[O_{SDW^-} = (-1)^j \sum_{\alpha, \alpha'} c_{j,\alpha,\alpha}^\dagger \sigma_{\alpha,\alpha'}^- c_{j,\alpha',\alpha} \propto \cos \phi_c \exp(\pm i\theta_s), \tag{29}\]
\[O_{BCDW} = (-1)^j \sum_{\alpha} \left( c_{j,\alpha,\alpha}^\dagger c_{j+1,\alpha,\alpha} + h.c. \right) \propto \cos \phi_c \cos \phi_s, \tag{30}\]
\[O_{BSDW^+} = (-1)^j \sum_{\alpha, \alpha'} \left( c_{j,\alpha,\alpha}^\dagger \sigma_{\alpha,\alpha'}^+ c_{j+1,\alpha',\alpha} + h.c. \right) \propto \sin \phi_c \sin \phi_s, \tag{31}\]
\[O_{BSDW^-} = (-1)^j \sum_{\alpha, \alpha'} \left( c_{j,\alpha,\alpha}^\dagger \sigma_{\alpha,\alpha'}^- c_{j+1,\alpha',\alpha} + h.c. \right) \propto \sin \phi_c \exp(\pm i\theta_s). \tag{32}\]

Let us now consider the quantum phase diagram of the model Eq. (1) in the weak-coupling regime. The dominating phase can be determined by looking at which of these order parameters takes a maximum amplitude. From Eqs. (18)–(32) one obtains that the \(U-J_z\) plane consists of four different sectors [see Fig. 2].
TABLE I: Correspondence between the dominant phases and fixed points at the half-filled band and the fixed points in the RG analysis.

<table>
<thead>
<tr>
<th>$\tilde{g}_c^*(l) = 0$</th>
<th>$\tilde{g}_c^*(l) = +\infty$</th>
<th>$\tilde{g}_c^*(l) = -\infty$</th>
</tr>
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<tbody>
<tr>
<td>TS$^\pm$</td>
<td>BSDW$^\pm$</td>
<td>SDW$^\pm$</td>
</tr>
<tr>
<td>TS$^0$</td>
<td>BSDW$^z$</td>
<td>SDW$^z$</td>
</tr>
<tr>
<td>SS</td>
<td>CDW</td>
<td>BCDW</td>
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FIG. 2: The field-theoretical prediction for the ground state phase diagram of the half-filled 1D $t$-$U$-$J_z$ model at weak coupling ($t = 1$).

In the sector A: $U + J_z/2 > 0$ with $J_z > 0$. Upon increasing the length scale, the system scales to the SG regime, where $\tilde{g}_c^*(l) = -\infty$ and $\tilde{g}_s^*(l) = +\infty$. The charge and spin excitations are both gapped, and the phase fields get ordered with the expectation values $\langle \phi_c \rangle = 0$ and $\langle \phi_s \rangle = \pi/2$, respectively. It is easily obtained that the order parameter $O_{SDW^z}$ is maximal. Therefore, all the fluctuations except the longitudinal SDW$^z$ phase are completely suppressed. This corresponds to a true long-range ordered phase with the Ising ordering in the ground state. Really, one gets the same result according to the fixed-point physics (see Table I).

In the sector B: $U + J_z/2 > 0$ with $J_z < 0$. Under the RG flows, the charge mode scales towards a SC fixed-point $\tilde{g}_c^*(l) = -\infty$, but the spin mode flows to a WC fixed-point $\tilde{g}_s^*(l) = 0$. The charge excitation remains gapped, while the spin excitation is gapless. The charge field is locked at $\langle \phi_c \rangle = 0$, whereas the spin field $\phi_s$ is free. The order parameter $O_{SDW^z}$ is maximal. The transverse $SDW^\pm$ phases are realized while the other instabilities disappear.

In the sector C: $|J_z| < -2U$. The properties of the model in this case are determined...
by the dominating on-site attraction $U < 0$. Upon increasing the parameter of the RG procedure, the initially positive charge coupling decreases and finally tends to zero $\tilde{g}_c^*(l) = 0$. Contrary to that in the sectors A and B, the charge mode is gapless and free. This leads to the complete suppression of the insulating phases in favor of metallic phases. Simultaneously, the initially positive spin coupling grows with increasing length scale, tending to positive infinity $\tilde{g}_s^*(l) = +\infty$, and then the spin gap opens. The behavior of the system is characteristic of a spin-gapped TS$^0$ phase.

In the sector D: $|U| < -J_z/2$. This sector is dominated by the ferromagnetic longitudinal exchange $J_z < 0$. The whole system is in the weak-coupling regime, where $\tilde{g}_c^*(l) = \tilde{g}_s^*(l) = 0$, and the charge and spin excitations are gapless. The ground state is a LL phase with TS$^\pm$ orderings.

We finally consider a particular case $U = 0$, where the weak-coupling analysis indicates the existence of two phases. For $J_z > 0$, both the charge and spin gaps open, accompanied by the vacuum expectation values $\langle \phi_c \rangle = 0$ and $\langle \phi_s \rangle = \pi/2$. This leads exclusively to the occurrence of the SDW$^z$ phase and to the suppression of all the other correlations. Whereas for $J_z < 0$ the charge and spin excitation spectra are both massless. As is common in the half-filled band case, the gapless charge excitation leads to the disappearance of the insulating states, and provides a possibility for the realization of the superconducting instabilities. Furthermore, a gapless spin gap suppresses the TS$^0$ and SS$^\pm$ correlations. In this situation, the LL phase with TS$^\pm$ orderings is dominate in the ground state. The result is in good agreement with that given by Fig. 2, where the SDW$^z$ and TS$^\pm$ phases are exactly fixed on the $J_z$ axis.

IV. THE STRONG-COUPLING REGIME

In this sector, we present a simple description of the strong coupling case, and restrict our considerations to the physical area $U \geq 0$. As we have known above, in the weak-coupling regime and for on-site Coulomb repulsion, the line $J_z = 0$ marks the spin-gap transition from the spin-gapped sector for $J_z > 0$ into the spin-gapless sector for $J_z < 0$. This is in obvious contrast with the properties of the $t$-$U$-$J_{xy}$ model, where the spin gap opens at $J_{xy} < 0$ and closes at $J_{xy} > 0$, indicating that the easy-axis anisotropy and the easy-plane anisotropy have an distinct effect on the low-energy spin excitations of the 1D $t$-$U$-$J$ model. Simultaneously, in the limit of strong ferromagnetic transverse exchange $|J_{xy}| \gg t$, the $t$-$U$-$J_{xy}$ model with $U = 0$ is equivalent to the XY spin chain, undergoing a transition from the regime with gapped spin and gapless charge excitation into an insulating magnetic phase with gapless spin excitations, while in the case of antiferromagnetic exchange $J_{xy} > 0$, a phase with gapless spin and gapped charge easy-plane correlations, predicted by the weak-coupling theory, evolves smoothly to the strong coupling limit $J_{xy} \gg t$ [22]. This hints at the $J_{xy} \leftrightarrow -J_{xy}$ asymmetry in the strong exchange interaction.

However, it is not clear what happens to the present model in the strong coupling regime. Taking into account the failure of the weak-coupling approach in this regime, we
briefly make a qualitative analysis. Usually, it is interesting to consider the unconstrained $t$-$J_z$ chain ($U = 0$). In the strong-coupling limit $J_z, |J_z| \gg t$, one can bosonize the Ising-type Heisenberg chain, taking the hopping term as a perturbation. Note that in the pure Heisenberg model the charge excitations are completely suppressed, so that the phase field $\phi_c$ is pinned to zero and the spin excitation spectrum is always massive (if only $J_z > J_{xy}$). In the presence of a small $t$-term the charge field is strictly frozen to be at its vacuum expectation and $\phi_c$ approaches to zero. This will weakly reduce the charge gap, not destroying the insulating properties of the system, while effectively inducing the backward scattering related to the spin channel. If one bosonizes the $t$-$J_z$ chain, it should clearly be found that the effective Hamiltonian becomes of a sine-Gordon form, only describing the spin degree of freedom, where the feedback of $\phi_c$ (induced by the $t$-term) on the spin excitation is encoded in the cosine term. It is due to this relevant backward scattering that a spin gap will open everywhere in the strong coupling limit. That is, there is no indication of further transitions in the antiferromagnetic case, and this regime is stable against any $J_z > 0$, governed by a long-range ordered antiferromagnetic (Néel) phase ($AF_z$). Instead, in the strong ferromagnetic exchange regime, above some critical value $|J_c|$ the system shows a uniaxial ferromagnetic ordering ($F_z$) with

$$\chi_{F_z} = \frac{1}{N} \sum_j \langle S^z_j \rangle \neq 0, \quad (33)$$

in complete suppression of the TS phase.

Besides, it is widely accepted that the on-site repulsion $U > 0$ enlarges the charge-gapped sector at the cost of the spin-gapped one. Thus, in the case of ferromagnetic exchange and for large $U$ the critical value $|J_c|$ will decrease with increasing $U$. As a result, one expects that, for $U \gg t, |J_z|$, a direct transition from antiferromagnetism ($AF_z$) into ferromagnetism ($F_z$) has to occur. However, the antiferromagnetic SDW$^z$ phase is expected not to change qualitatively. Furthermore, it is difficult to determine the location of the phase boundaries.

V. SUMMARY

To further understand the microscopic mechanisms for the competition or even coexistence of superconductivity and insulating magnetism, we have analytically studied the ground state properties of a 1D half-filled $t$-$U$-$J_z$ model in the weak-coupling regime. The model describes the interactions of correlated electrons including the on-site and intersite spin exchange restricted in the easy-axis direction. With the weak-coupling theory based on the charge-spin separation hypothesis, the phase boundaries are determined by solving the reduced RG equations. The charge-gap transition occurs at $U_c = -J_z/2$. The spin-gap transition takes place at $J_z = 0$ for $U > 0$ and $U_c = J_z/2$ for $U < 0$. The ground state shows four distinct phases: a gapped charge and spin phase dominated by the LRO SDW$^z$ correlation, a gapped charge and gapless spin phase dominated by the SDW$^\pm$ correlations, a gapless charge and gapped spin phase characterized by TS$^0$ correlation, and a gapless
charge and spin phase with the TS± correlations. Particularly, in the case of vanishing on-site interaction $U = 0$, the SDW± and TS0 instabilities disappear. This indicates that the presence of correlated energy $U$ has an important effect on the ground state properties. The suggested weak-coupling phase diagram is plotted in Fig. 2. Secondarily, we simply made a qualitative discussion of the ground state properties in the strong coupling regime $J_z, |J_z| \gg t$, finding that the system is governed only by an insulating SDWz phase in the physical parameter range $U \geq 0$.

Our work extends a previous study of the t-U-J model by Dziurzik et al. [22], pushing an anisotropy issue to another limiting case, so it is necessary and interesting to compare them with each other. In both models, the charge channel obeys an SU(2) symmetry because of the half filling. The difference is the spin channel. In Ref. [22], the exchange interaction involves exclusively easy-plane magnetic anisotropy, and the spin channel has a global U(1) invariance, generated by the operators $S^x$ and $S^y$. Our model discussed the easy-axis anisotropy, and the spin space satisfies the $Z_2$ symmetry with respect to the $S^z \rightarrow -S^z$. It is due to different symmetries that lead to the opposite spin excitation behavior in two extremely anisotropic exchange interactions. For instance, in the case of easy-axis anisotropy, a spin gap opens in the antiferromagnetic regime and closes in the ferromagnetic regime. This is in complete contrast with the easy-plane anisotropy. Consequently, we will get a remarkable result that, if one makes the replacements: $J_z \rightarrow J_{xy}$, SDWz $\rightarrow$ SDWz,y, SDW± $\rightarrow$ SDWz, and TS± $\rightarrow$ TS0 in the regime $U \geq 0$, the phase diagram Fig. 2 has exactly the same form as that given by Ref. [22] at weak coupling, except for a different charge-gap transition position. This indicates that the XY spin exchange ($J_{xy}$) can stabilize planar ferro- (antiferro) magnetic orderings and TS0 correlations, and that the longitudinal exchange ($J_z$) favors uniaxial ferro- (antiferro) magnetic orderings and TS± correlations. However, we did not find coexistence of the SDW and TS phases. Besides, as a phenomenological model, we also considered the case for on-site attraction in the weak coupling regime.

In summary, the result demonstrates that the longitudinal exchange and transverse exchange have a different effect on the ground state properties of a 1D correlated electron system. Our work is further expected to provides a significant sight into the competing mechanisms for the insulating and superconducting phases.

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