The Electron’s Anomalous Magnetic Moment Effects on the Laser Assisted Ionization of Atomic Hydrogen by Electronic Impact

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The electron-impact ionization of atomic hydrogen with the electron’s anomalous magnetic moment (AMM) effects are examined. The formulas for the laser-assisted relativistic triple differential cross section (TDCS) in the coplanar binary geometry developed earlier by Attaourti and Taj [Phys. Rev. A 69, 063411 (2004)] are used to check the consistency of our computations when the anomaly $\kappa$ is taken to be zero. We show that the terms containing the AMM effects even in the first Born approximation has an important contribution, so it must be included in any reliable analysis. A full analytical calculation for the TDCS is presented.

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I. INTRODUCTION

The hydrogen atom, because of its simplicity, has a central role in the understanding of chemistry and atomic physics. Apart from the fundamental interest, reactions involving atomic hydrogen have practical importance in controlled thermonuclear fusion and in the earth’s upper atmosphere. The ionization of atoms or ions in collisions by charged particles is important for diagnostics of high temperature plasmas as well as for the fundamental understanding of the atomic structure. In recent years, electron coincidence spectroscopy has become a powerful tool for testing the dynamic theories of final states with two outgoing electrons [1–4]. To the authors’ knowledge, no relativistic experimental data on the electron-impact ionization of atoms in laser assisted collisions has been given. A complete kinematic analysis can provide an overall symmetry of the impact ionization processes and facilitate the comparison between theory and experiment. The theoretical models developed through the years to uncover the details of these processes can be classified according to the impact energy of the projectile. In the low-to-intermediate energy regime, we found close-coupling methods based on a molecular approach [5, 6], in the intermediate regime, the classical
trajectory Monte Carlo has been widely used [7], while in the intermediate-to-high energy regime, the distorted wave method can be applied [8, 9]. Furthermore, there are methods that can be applied throughout the complete range of impact energies [10–12]. Though the distorted wave method offers several advantages, like including the correct asymptotic conditions of the wave functions due to the long-range behavior of the Coulomb interaction between the particles [13], it doesn’t take into account the electron’s anomalous magnetic moment effects. In this paper, we present a theoretical model for the relativistic electronic dressing in the laser-assisted ionization of atomic hydrogen by electron impact with the electron’s anomalous magnetic moment effects. For pedagogical purposes, in Section II we begin our study without AMM effects (the electron’s anomaly is taken to be zero). In Section III, we present our study with AMM effects. In Section IV, we discuss the results we have obtained. Throughout this work, we use atomic units $\bar{\hbar} = m = e = 1$ and work with the metric tensor $g^{\mu\nu} = g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. In many equations of this paper the Feynman ‘slash notation’ is used. For any 4-vector $A$, $A / = A /_\mu \gamma^\mu = A /_0 \gamma^0 - A /_\mu \gamma^\mu$ where the matrices $\gamma^\mu$ are the well known Dirac matrices.

II. THE TDCS IN LASER ASSISTED IONIZATION WITHOUT AMM

We now take into account the electronic relativistic dressing of all electrons which are described by Dirac-Volkov plane waves normalized to the volume $V$. This gives rise to a trace already given in detail in [14], but it will turn out that taking into account the relativistic electronic dressing of the ejected electron amounts simply to introducing a new sum on the $l_B$ photons that can be exchanged with the laser field. The transition amplitude is now given by

$$S_{fi} = -\frac{i}{c} \int_{-\infty}^{+\infty} dx^0 \langle \psi_i(x_1) \varphi_f(x_2) | V_d | \psi_i(x_1) \varphi_i(x_2) \rangle. \quad (1)$$

$V_d = 1/r_{12} - 1/r_1$ is the direct interaction potential where $r_1$ are the electron coordinates, $r_2$ are the atomic electron coordinates, and $r_{12} = |r_1 - r_2|$. The Dirac-Volkov wave function for the ejected electron reads as

$$\varphi_f(x_2) = \psi_{qB}(x_2) = \left[ 1 + \frac{k A_{(2)}}{2c(k+pB)} \right] \frac{u(p_B, s_B)}{\sqrt{2Q_BV}} \times \exp \left[ -i(q_B \cdot x_2) - i \int_0^{k.x_2} \frac{(A_{(2)}, p_B)}{c(k+pB)} d\phi_2 \right], \quad (2)$$

where $A_{(2)} = a_1 \cos(\phi_2) + a_2 \sin(\phi_2)$ is the four potential of the circularly polarized laser field felt by the ejected electron, $\phi_2 = k \cdot x_2 = k_0 x_2^0 - k \cdot x_2 = wt - k \cdot x_2$ is the phase of the laser field, and $w$ its frequency. The four-vector $q_\mu = (Q/c, q)$ is the four-momentum of the electron inside the laser field with wave four-vector $k$. For the atomic target, $\varphi_i(x_2) = e^{-i\epsilon_0 t/2} \varphi_i(r_2)$ is the relativistic wave function of atomic hydrogen in its ground state, and $\epsilon_b = c^2/(\sqrt{1 - \alpha^2} - 1)$ is the binding energy of the ground state of atomic hydrogen with $\alpha$.
being the fine structure constant. Proceeding along the lines of standard QED calculations, we obtain for the spin-unpolarized triple differential cross section evaluated for \( Q_f = Q_i + (n + l_B)\mathbf{w} + \varepsilon_b - Q_B \) the following formula:

\[
\frac{d\sigma}{dE_Bd\Omega_Bd\Omega_f} = \sum_{n,l_B=-\infty}^{+\infty} \frac{d\sigma^{(n,l_B)}}{dE_Bd\Omega_Bd\Omega_f},
\]

with

\[
\frac{d\sigma^{(n,l_B)}}{dE_Bd\Omega_Bd\Omega_f} = \frac{1}{2 |q_f|^2 |q_B|^2} \left( \sum_{s_l,s_f} |M_{fi}^{(n)}|^2 \right) \sum_{s_B} |\pi(p_B,s_B)\Gamma_{l_B}\gamma_0|^2 \times | \Phi_{1,1/2,1/2}(q = \Delta_{n+l_B} - q_B) - \Phi_{1,1/2,1/2}(q = -q_B + l_B\mathbf{k}) |^2.
\]

The functions \( \Phi_{1,1/2,1/2}(q) \) are the Fourier transforms of the relativistic atomic hydrogen wave functions, and the sum \( \left( \sum_{s_l,s_f} |M_{fi}^{(n)}|^2 / 2 \right) \) has already been evaluated in a previous work [15]. The quantity \( \Delta_{n+l_B} \) is simply given by \( \Delta_{n+l_B} = q_i - q_f + (n + l_B)\mathbf{k} \). Introducing the factor \( c(p_B) = 1/(2\epsilon(k,p_B)) \), the symbol \( \Gamma_{l_B} \) is defined as

\[
\Gamma_{l_B} = B_{1B}(z_B) + c(p_B)[\delta_{1\mathbf{k}}B_{1l_B}(z_B) + \delta_{2\mathbf{k}}B_{2l_B}(z_B)],
\]

where the three quantities \( B_{1B}(z_B) \), \( B_{1l_B}(z_B) \), and \( B_{2l_B}(z_B) \) are respectively given by

\[
\begin{align*}
B_{1B}(z_B) &= J_{l_B}(z_B)e^{il_B\phi_{0B}}, \\
B_{1l_B}(z_B) &= \{J_{l_B+1}(z_B)e^{i(l_B+1)\phi_{0B}} + J_{l_B-1}(z_B)e^{i(l_B-1)\phi_{0B}} \}/2, \\
B_{2l_B}(z_B) &= \{J_{l_B+1}(z_B)e^{i(l_B+1)\phi_{0B}} - J_{l_B-1}(z_B)e^{i(l_B-1)\phi_{0B}} \}/2i,
\end{align*}
\]

where \( z_B = \frac{|a|}{c(k,p_B)}\sqrt{\langle \hat{\mathbf{y}}.\mathbf{p}_B \rangle^2 + \langle \hat{\mathbf{x}}.\mathbf{p}_B \rangle^2} \) is the argument of the ordinary Bessel functions that will appear in the calculations, and the phase \( \phi_{0B} \) is defined by

\[
\phi_{0B} = \arctan(\langle \hat{\mathbf{y}}.\mathbf{p}_B \rangle/\langle \hat{\mathbf{x}}.\mathbf{p}_B \rangle).
\]

The sum over the spins of the ejected electron can be transformed to traces of gamma matrices. Using REDUCE [16], we find

\[
\sum_{s_B} |\pi(p_B,s_B)\Gamma_{l_B}\gamma_0|^2 = 4\{E_BJ_{l_B}^2(z_B) + wc(p_B)(\cos(\phi_{0B})(a_1.p_B) + \sin(\phi_{0B})(a_2.p_B)) + J_{l_B}(z_B)(J_{l_B+1}(z_B) + J_{l_B-1}(z_B)) - a^2w(k,p_B)e^2(p_B)(J_{l_B}^2(z_B) + J_{l_B}^2(z_B))\}.
\]

As expected, in the absence of the laser field only the term 4\( E_BJ_{l_B}^2(z_B = 0)\delta_{l_B,0} = 4E_B \) contributes to the TDCS.
III. THE TDCS IN LASER ASSISTED IONIZATION WITH AMM

We now take into account the AMM effects of all electrons (incident, scattered, and ejected) which are described in the weak-field approximation (WFA) [17] by

\[ \psi(x) = \left[ 1 - (\alpha kA + \beta \hat{k} + \delta \hat{p}kA) \right] \frac{u(p,s)}{\sqrt{2VQ_0}} \times \exp \left[ -i(qx) - i \int_0^{kx} \frac{(Ap)}{c(kp)} d\phi \right] \]

with

\[ \alpha = \frac{1}{2(kp)} \left( \frac{\kappa c}{2} - \frac{1}{c} \right), \quad \beta = \frac{\kappa A^2}{4c(kp)}, \quad \delta = \frac{\kappa}{4(kp)}. \]

The transition amplitude with AMM effects is given by

\[ S_{AMM}^{fi} = \frac{i}{c} \int_{-\infty}^{+\infty} dx \langle \psi_f(x_1) \varphi_f(x_2) | V_d | \psi_i(x_1) \varphi_i(x_2) \rangle. \]

Proceeding along the same lines as before, the spin-unpolarized triple differential cross section with the AMM effects evaluated for \( Q_f = Q_i + (n + l_B)w + \varepsilon_B - Q_B \) is given by

\[ \frac{d\sigma_{AMM}}{dE_B d\Omega_B d\Omega_f} = \sum_{n,l_B=-\infty}^{+\infty} \frac{d\sigma_{AMM}(n,l_B)}{dE_B d\Omega_B d\Omega_f}, \]

with

\[ \frac{d\sigma_{AMM}(n,l_B)}{dE_B d\Omega_B d\Omega_f} = \frac{1}{2} \frac{|q_f||q_B|^2}{|q_f - q_i - n k|^2} \sum_{s_B} |\vec{u}(p_B, s_B)\Delta_{l_B} \gamma^0|^2 \times |\Phi_{1,1/2,1/2}(q = \Delta_{n+l_B} - q_B) - \Phi_{1,1/2,1/2}(q = -q_B + l_B k)|^2. \]

The spinorial part \((\sum_{s_i,s_f} | M_{fi}^{(n)} |^2 / 2)\) is the factor in which electron’s AMM effects are reflected [18]. However, the novelty in the various stages of the calculations when including the AMM effects of the ejected electron is contained in the symbol \( \Delta_{l_B} \), which is given as

\[ \Delta_{l_B} = (1 - \frac{k}{\beta_B}) B_{l_B}(z_B) - [\delta_B \phi_1 k \hat{p}B + \alpha_B \phi_1 \hat{k}] B_{l_B}(z_B) \\
- [\delta_B \phi_2 k \hat{p}B + \alpha_B \phi_2 \hat{k}] B_{2l_B}(z_B), \]

where the three quantities \( B_{l_B}(z_B), B_{l_B}(z_B), \) and \( B_{2l_B}(z_B) \) are given in Eq. (7).

The sum over the spin of the ejected electron can be transformed to traces of gamma matrices. Using REDUCE [16], and after tedious calculations, we obtain the trace of the
ejected electron in its final form:

\[
\sum_{s_B} \left| \mathfrak{u}(p_B, s_B) \Delta l_B \gamma^0 \right|^2 = \frac{1}{2c^2(k.p_B)} \left[ -4c^2 \kappa a^2 \omega + 8c^2(k.p_B) \right]
\]

\[
\times E_B + \kappa^2 a^2 \omega] J^2_{l_B}(z_B) + \frac{\omega}{2c(k.p_B)} \left[ - \cos(\phi_0)(a_1.p_B) \kappa^2 a^2 + 4 \sin(\phi_0) \right]
\]

\[
\times (a_2.p_B)] J_{l_B}(z_B)(J_{l_B+1}(z_B) + J_{l_B-1}(z_B)) + \frac{a^2}{2c^2(k.p_B)}
\]

\[
\times [-c^2 \kappa^2(k.p_B)E_B + 2c^2 \kappa \omega - 2\omega](J^2_{l_B+1}(z_B) + J^2_{l_B-1}(z_B)). \tag{15}
\]

The first check to be done is to take \( \kappa = 0 \) in order to recover all the results in the absence of the anomalous magnetic moment effect. When this is done, one recovers the simple trace result given in Eq. (8). Once again, when no radiation field is present, this trace reduces to \( 4E_B J^2_{l_B}(z_B = 0) \delta_{l_B,0} = 4E_B \).

**IV. RESULTS AND DISCUSSIONS**

In this section, the results of the applications of the foregoing equations are presented by numerically evaluating the TDCSs for the value of the electron’s anomaly \( \kappa = 115952188.4 \times 10^{-12} \) [19]. We have chosen the angular frequency \( \omega = 0.043 \) a.u. of a Nd:YAG laser. We have also discussed the laser-assisted TDCSs under three kinds of conditions: (a) without taking into account the AMM effect of all electrons (incident, scattered, and ejected), (b) taking into account only the AMM effect for the incident and scattered electrons, (c) finally, taking into account the AMM effects for all electrons. We choose a geometry where \( p_i \) is along the \( Oz \) axis (\( \theta_i = \phi_i = 0^\circ \)). For the scattered electron (\( \theta_f = 45^\circ, \phi_f = 0^\circ \)), and for the ejected electron \( \phi_B = 180^\circ \) and the angle \( \theta_B \) varies approximately from \( 30^\circ \) to \( 60^\circ \). This is an angular situation where we have a coplanar geometry.

In Figure 1, we give the relation between the TDCSs and the angle of the ejected electron corresponding to three cases: the solid-line indicates results obtained by neglecting the AMM effects of all electrons in the formalism, the long dash-line indicates results obtained by considering the AMM effects in the formalism but with the electron’s anomaly \( \kappa = 0 \) and the electrical field strength \( E = 0 \) a.u. The dashed-line is the results obtained by using the plane waves. The results show that the three approaches give identical curves. In Figure 2, we show the TDCS with and without AMM effects for \( n = 1 \) and \( l_B = -1 \). We have obtained the same curve for the case \( n = -1 \) and \( l_B = 1 \). Once again this figure justifies clearly the accuracy and the consistency of our new formalism, even if it contains a very long analytical formula which is not prone to calculation by hand.

Figure 3 illustrates the variation of TDCSs with AMM effects of the incident and scattered electrons versus the angle of the ejected electron. It follows from this figure that three times magnitude between the two approaches has been recognized in the vicinity of
FIG. 1: The TDCSs as a function of the angle $\theta_B$. The incident electron kinetic energy is $T_i = 2700$ eV and the ejected electron kinetic energy is $T_B = 1349.5$ eV.

FIG. 2: The TDCSs as a function of the angle $\theta_B$ for $n = 1$ and $l_B = -1$ (we obtain the same figure for $n = -1$ and $l_B = 1$). The incident electron kinetic energy is $T_i = 2700$ eV and the ejected electron kinetic energy is $T_B = 1349.5$ eV. The geometric parameters are $\theta_i = 0^\circ$, $\phi_i = \phi_f = 0^\circ$, $\theta_f = 45^\circ$, and $\phi_B = 180^\circ$.

$\theta_f = 45^\circ$. In our previous paper published earlier [20], the TDCS with the electron’s AMM effects always overestimates the TDCS without the electron’s AMM effects. This result is not justified if one considers only the AMM effects of the incident and scattered electrons. Figure 4 shows the same dependence of the TDCS with the AMM effects of all electrons (incident, scattered, and ejected) and an emergent picture which is completely different. Indeed, the value of the TDCS with AMM effects at its maximum overestimates the TDCS.
without AMM. This means that, by introducing the AMM effects of the ejected electron, we have obtained a qualitative result similar to that obtained in our previous paper [20]. For all energies, even if the process is very different \((e, 2e)\), we have reached the same conclusion in which the TDCS with AMM always overestimates the TDCS without AMM.

**FIG. 3:** The TDCSs (with AMM effects for incident and scattered electrons) as a function of the angle \(\theta_B\). The incident electron kinetic energy is \(T_i = 5109\) eV and the ejected electron kinetic energy is \(T_B = 2554.5\) eV. The electrical strength field is \(E = 0.2\) a.u. and the number of photons exchanged are \(n = \pm 10\) and \(l_B = \pm 10\).

**FIG. 4:** The TDCSs (with AMM effects for incident, scattered and ejected electrons) as a function of the angle \(\theta_B\). The incident electron kinetic energy is \(T_i = 5109\) eV and the ejected electron kinetic energy is \(T_B = 2554.5\) eV. The electrical strength field is \(E = 0.2\) a.u. and the number of photons exchanged are \(n = \pm 10\) and \(l_B = \pm 10\).
FIG. 5: The TDCS with AMM as a function both of the angle $\theta_B$ and the electrical field strength ($\mathcal{E}$ scaled in $10^{-2}$). The incident electron kinetic energy is $T_i = 5109$ eV and the ejected electron kinetic energy is $T_B = 2554.5$ eV. The geometric parameters are the same and the number of photons exchanged are $n = \pm 5$ and $l_B = \pm 5$.

Figure 5 shows a three-dimensional plot of the calculated triple differential cross section with the electron’s anomalous magnetic moment effects. Two characteristic features of this landscape are obtained: first, the abrupt fall in the triple differential cross section at small and large angles; second, for the angles, particular in the vicinity of ($\theta_B = 45^\circ$) which represents the binary coplanar geometry, the ejected electron loses its Coulombian behavior and the TDCS decreases with the intensity. We would like also to mention that the plane wave results should not be too reliable for the slower electrons, in this case the long-range Coulomb interaction should in no way be neglected. We are not in a position to compare our semirelativistic results with the existing theoretical works, since the present theory is particularly meant for a binary coplanar geometry and takes into account the electron’s AMM effects, whereas the other theoretical results refer to the non-relativistic case. Thus for a proper comparison we have to await the experimental data.

V. CONCLUSION

In this paper, we have extended our treatment of the ionization of atomic hydrogen by electronic impact in the presence of a circularly polarized laser field to the case of the ionization with the introduction of the electron’s anomalous magnetic moment effects. The calculations have been performed in the framework of the first Born approximation and in the binary coplanar geometry. These results show, notably in comparison to more simplified approaches (TDCS without the electron’s AMM effects), the importance of the full Dirac approach, especially in the case of intense laser fields and high energies. Important
differences have been found when the formalism of the triple differential cross sections with and without AMM is used.

References