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Study of Fission and Evaporation Residue Cross Sections and Pre-Scission Neutron Multiplicities for ^{168}Yb

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A Kramers-modified statistical model was applied to the study of the fission properties of the compound nucleus ^{168}Yb formed in heavy ion-induced fusion reactions in an intermediate range of excitation energies. The evaporation residue cross section, fission cross section, and the average pre-fission neutron multiplicities were calculated for ^{168}Yb formed in ^{18}O + ^{150}Sm reactions and the results compared with the experimental data. To calculate these quantities, the effects of temperature and the spin K about the symmetry axis have been considered in the calculations of the potential energy surfaces and the fission widths. It was shown that the results of the calculations are in good agreement with the experimental data by using values of the temperature coefficient of the effective potential and scaling factor of the fission-barrier height equal to $k = 0.007 \pm 0.002$ MeV^{-2} and $r_s = 0.994 \pm 0.002$, respectively.

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I. INTRODUCTION

Fission is one of the most important decay channels of heavy excited nuclei. The first theoretical estimation of the transition-state fission decay width was given by Bohr and Wheeler [1]:

$$\Gamma_f^{\mathrm{BW}} = \frac{1}{2\pi}\frac{1}{\rho_{\mathrm{CN}}(E^*)}\int\limits_0^{E^*-B_f} \rho_{\mathrm{sad}}(E^* - B_f - \varepsilon)d\varepsilon, \tag{1}$$

where ρ_{CN} and ρ_{sad} are the level density of the compound nucleus at excitation energy E^* at the ground and saddle points, respectively. B_f is the height of the fission barrier, and ε represents the kinetic energy associated with the fission distortion. A number of modifications to the Bohr-Wheeler width have been proposed. By approximating the nuclear level density with a unique constant temperature formula $\rho \propto \exp(E_{\mathrm{int}}/T)$, Eq. (1) can be written as the expression $\Gamma_f^{\mathrm{BW}} = (T/2\pi)\exp(-B_f/T)$ [2, 3], where T and E_{int} are the temperature and the intrinsic energy, respectively. It should be noted that, if the above equation is recalculated correctly, taking into account the motion about the ground-state position, then the fission decay can be expressed as $\Gamma_f^{\mathrm{BW}} = (\hbar\omega_{\mathrm{eff}}/2\pi)\exp(-B_f/T)$. The slowing effects

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*Electronic address: `m_eslamizadeh@yahoo.com` of nuclear viscosity are included by using the Kramers-modified [4] Bohr-Wheeler model:

$$\Gamma_f(k) = \left(\sqrt{1+\gamma^2} - \gamma\right) \times \frac{\hbar\omega_{\text{eq}}}{2\pi} \exp\left(-\frac{B_{\text{eff}}}{T}\right), \tag{2}$$

where γ is the dimensionless nuclear viscosity given by $\gamma = \beta/2\omega_{\text{sp}}$, β is the reduced nuclear dissipation coefficient, and ω_{eq}, ω_{sp} are the curvatures of the potential energy surface at the equilibrium position and the fission saddle point, respectively. B_{eff}, ω_{eq}, and ω_{sp} are all assumed to be functions of K.

It should be stressed that Eq. (2) is the fission width for a system with fixed spin K about the symmetry axis. Therefore, by assuming axially symmetric shapes the full fission decay width can be obtained by summing over all possible K [5]:

$$\Gamma_f = \frac{\sum\limits_{K=-J}^{J} P(K)\Gamma_f(K)}{\sum\limits_{K=-J}^{J} P(K)}, \tag{3}$$

where $P(K) = (T/\hbar\omega_{\text{eq}})\exp(-V_{\text{eq}}/T)$ is the probability that the system is in a given K; V_{eq} is the sum of the nuclear, Coulomb, and rotational energies at the equilibrium position as a function of K.

In this paper we use a modified statistical model similar to Ref. [6] to reproduce the experimental data on the evaporation residue cross section, fission cross section, and the average pre-fission neutrons multiplicities for ^{168}Yb formed in ^{18}O + ^{150}Sm reactions. To reproduce these quantities, we consider the effects of temperature and the spin K about the symmetry axis on the calculations of the potential energy surfaces and the fission widths, as in Ref. [6].

It should be stressed that many authors for studying different features of fission process used dynamical models based on the Langevin or Fokker-Planck equations [7–15].

The present paper has been arranged as follows. In Sec. II we describe the model and basic equations. The results of the calculations are presented in Sec. III. Finally concluding remarks are given in Sec. IV.

II. DETAILS OF THE MODEL AND BASIC EQUATIONS

In the present statistical model calculations, the effective potential can be obtained from the modified liquid-drop model (MLDM) as [5, 6, 16]

$$\begin{aligned} V_{\text{eff}}(c,A,Z,J,K) &= B_s(c)E_s^0(Z,A)(1-kT^2) + B_c(c)E_c^0(Z,A) \\ &\quad + \frac{(J(J+1)-K^2)\hbar^2}{I_\perp(c)\frac{4}{5}MR_0^2 + 8Ma^2} + \frac{K^2\hbar^2}{I_\parallel(c)\frac{4}{5}MR_0^2 + 8Ma^2}, \end{aligned} \tag{4}$$

where $B_s(c)$ and $B_c(c)$ are the surface and Coulomb energy terms, respectively. E_s^0 and E_c^0 are the surface and Coulomb energies of the corresponding spherical system as determined by Myers and Swiatecki [17, 18], M is the mass of the system, $R_0 = 1.2249A^{1/3}$ fm, and $a = 0.6$ fm. $I_\perp$ and $I_\parallel$ are the momenta of inertia with respect to the axes perpendicular and parallel to the symmetry axis of the fissioning nucleus.

It should be stressed that the magnitude of $k = c_S A^{2/3}/E_S^0$, where E_S^0 is the surface energy of the spherical system. Töke and Swiatecki [19] obtain $c_s \approx 0.27$ and other estimates of c_s give values of k that range from 0.007 to 0.022 MeV^{-2} [20–23]. In terms of $c_s \approx 0.27$, the value of $k \sim 0.016$ MeV^{-2}. It should be stressed that c_s is very sensitive to the assumed properties of the nuclear matter and to other approximations [24].

The total fusion cross section is usually calculated from

$$\sigma_{\rm Fus} = \sum_J \frac{d\sigma_{\rm Fus}(J)}{dJ}, \tag{5}$$

where, the angular momentum distribution of the compound nucleus can be described by the formula

$$\frac{d\sigma_{\rm Fus}(J)}{dJ} = \frac{2\pi}{k^2}\frac{2J+1}{1+\exp(\frac{J-J_c}{\delta J})}. \tag{6}$$

Here δJ is the diffuseness and J_c is the critical angular momentum. The parameters δJ and J_c can be approximated as follows [25]:

$$J_c = \sqrt{A_P A_T/A_{\rm CN}}\left(A_P^{1/3} + A_T^{1/3}\right)\left(0.33 + 0.205\sqrt{E_{\rm cm} - V_c}\right), \tag{7}$$

and

$$\delta J = \begin{cases} (A_P A_T)^{3/2} \times 10^{-5}\,[1.5 + 0.02(E_{\rm cm} - V_c - 10)] & \text{for } E_{\rm cm} > V_c + 10, \\ (A_P A_T)^{3/2} \times 10^{-5}\,[1.5 - 0.04(E_{\rm cm} - V_c - 10)] & \text{for } E_{\rm cm} < V_c + 10. \end{cases} \tag{8}$$

When $0 < E_{\rm cm} - V_c < 120$ MeV and when $E_{\rm cm} - V_c > 120$ MeV the term in the last brackets is put equal to 2.5.

Figure 1 shows the partial cross sections as a function of angular momentum for ^{168}Yb, for example, for projectile energy $E_{\rm cm} = 100$ and 150 MeV.

The fission cross section can be obtained in terms of the fusion cross section as follows:

$$\sigma_{\rm Fiss} = \sum_J \sigma_{\rm Fus}(J)\frac{\Gamma_f}{\Gamma_{\rm tot}}. \tag{9}$$

At a given beam energy E, the total evaporation residue cross section can be obtained by summing of the cross sections from each J, these being the products of the capture cross sections and the probabilities of surviving fission $(1 - p_f(J, E^*))$:

$$\sigma_{\rm Er}(E) = \pi\lambda^2 \sum_{J=0} (2J+1)T_J(1 - P_f(J, E^*)). \tag{10}$$

We calculate the decay widths Γ_υ for particle emission and the fission width and use a standard Monte Carlo cascade procedure where the kind of decay is selected with the

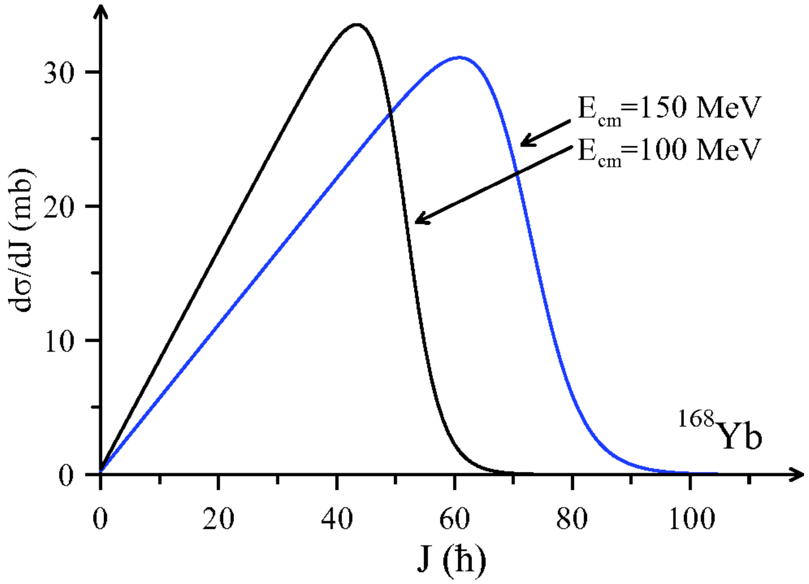

FIG. 1: The partial cross sections as a function of angular momentum for ^{18}O + ^{150}Sm.

weights $\Gamma_\nu/\Gamma_{\rm tot}$ with ($\nu = \text{fission}, n, p, \alpha, \gamma$). This procedure allows for multiple emissions of light particles and higher chance fission. After each emission act, we again recalculate the intrinsic energy and the angular momentum, and continue the cascade procedure until the intrinsic energy becomes smaller than either the fission barrier or the binding energy of a neutron. The loss of angular momentum is taken into account by assuming that each neutron, proton, or a γ quanta carries away $1\hbar$ while the α particle carries away $2\hbar$.

Decay widths for excited compound nuclei are calculated in the Hauser-Feschbach formalism [26], and the width of the gamma emission is calculated by the relation provided in [27, 28].

In many statistical model codes [29–34], authors have used the ratio of the level density, a_f/a_n, and a scaling of the barrier heights, f_B, which can be adjusted to reproduce experimental data at low and intermediate excitation energies. But fission in heavy-ion reactions cannot be accurately modeled as a function of the excitation energy using the J dependence of the $T = 0$ fission barriers and a fixed value of a_f/a_n. In the present paper, we want to consider other parameters as free parameters which perform similar roles as a_f/a_n and f_B [6]. We consider the temperature coefficient in the effective potential formula, k, and a scaling of the MLDM radii from their default values used to calculate the surface and Coulomb energies with the parameter r_s. The surface energy is proportional to the square of r_s, while the Coulomb energy is inversely proportional to r_s. A value $r_s = 1$ is the standard MLDM with fission-barrier heights in agreement with the FRLDM. Raising r_s above one decreases the Coulomb energy and increases the surface energy. This causes the fission barriers to increase. It should be stressed that the advantage of using r_s instead of f_B is that the curvature at the ground states and the fission transition points, the barrier locations, and the heights are all being determined in a self-consistent manner as a function of J, K, and T.

III. RESULTS OF THE CALCULATIONS AND DISCUSSION

In the present study, the evaporation residue cross section, fission cross section, and the average pre-fission neutron multiplicities have been calculated for ^{168}Yb formed in ^{18}O + ^{150}Sm reactions.

In the calculations, the parameters k and r_s are adjusted to reproduce the single residue and fission cross sections and single average pre-fission neutron multiplicities. Figures 1 and 2 show the results of the residue and fission cross sections for ^{168}Yb. It can be seen from Fig. 2 and 3 that the results of the calculations are in good agreement with the experimental data by using the values of $k = 0.007 \pm 0.002$ MeV^{-2} and $r_s = 0.994 \pm 0.002$.

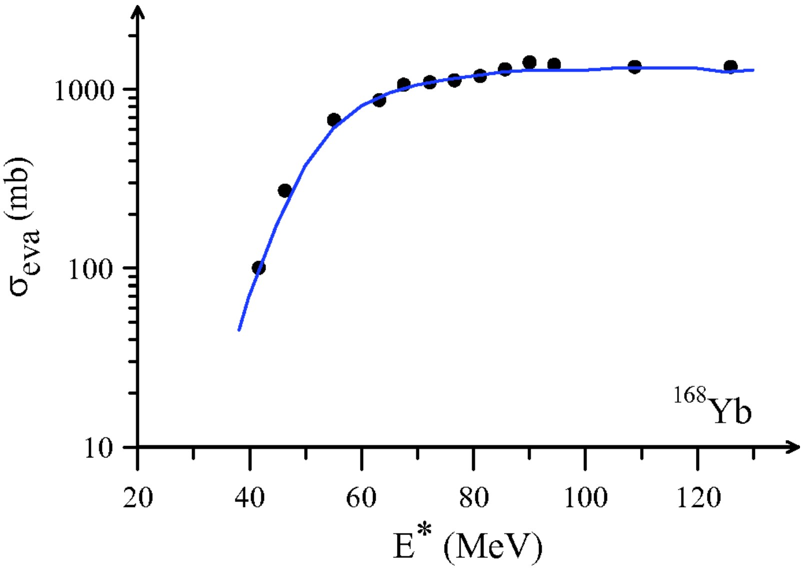

FIG. 2: The results of the evaporation residue cross section (solid line) as a function of excitation energy for ^{168}Yb calculated with considering $k = 0.007 \pm 0.002$ MeV^{-2} and $r_s = 0.994 \pm 0.002$. The experimental data for the evaporation cross section (closed circles) are taken from Refs. [35, 36].

Figure 4 shows the calculation results for the average pre-fission neutron multiplicities for ^{168}Yb.

It is clear from Fig. 4 that at lower and intermediate excitation energies the values of the average pre-fission neutron multiplicities calculated with $k = 0.007 \pm 0.002$ MeV^{-2} and $r_s = 0.994 \pm 0.002$ are close to the experimental data, but at higher excitation energies the calculated data are slightly lower than the experimental data. This is probably due to the compound nucleus at higher excitation energy being formed with a larger value of spin and temperature. Thus the fission barrier height will be reduced and therefore the neutron

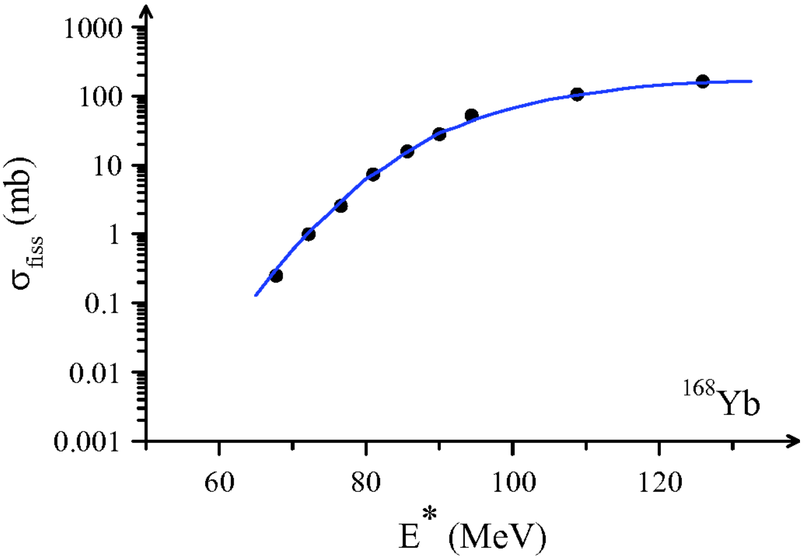

FIG. 3: The results of the fission cross section (solid line) as a function of the excitation energy for ^{168}Yb calculated with considering $k = 0.007 \pm 0.002$ MeV^{-2} and $r_s = 0.994 \pm 0.002$. The experimental data for the fission cross section (closed circles) are taken from Refs. [35, 36].

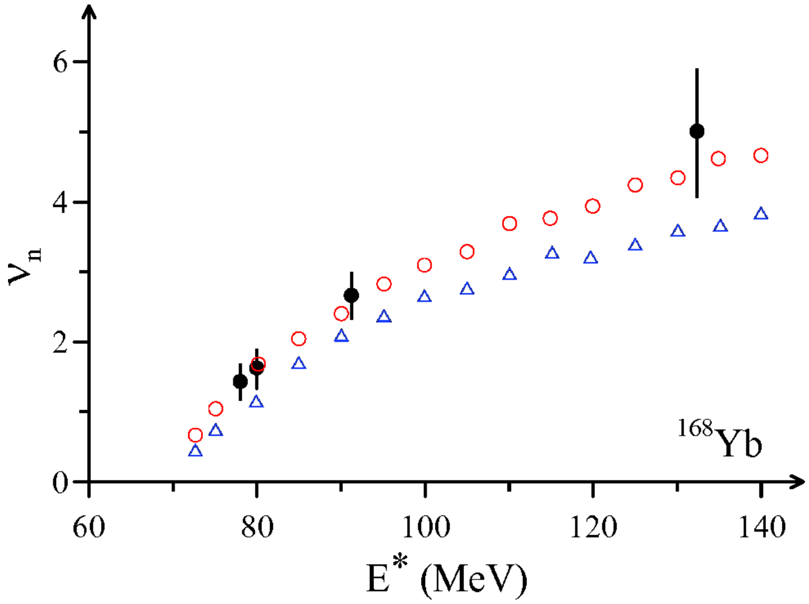

FIG. 4: The results of the pre-fission neutron multiplicities (open circles) as a function of the excitation energy for ^{168}Yb calculated with considering $k = 0.007 \pm 0.002$ MeV^{-2} and $r_s = 0.994 \pm 0.002$. The open triangles show the results of the pre-fission neutron multiplicities calculated with considering $k = 0$ and $r_s = 1$. The experimental data for pre-fission multiplicities of neutrons (closed circles) are taken from Ref. [37].

particles widths are comparable to the fission widths, and so the calculations data for the pre-scission neutron multiplicities are slightly lower than the experimental data.

It should be mentioned that in the present calculations, we use the magnitude of the reduced nuclear dissipation coefficient equal to 3×10^{21} s^{-1}.

Finally, it would be useful to see how the extracted ^{168}Yb parameters from this work fit into the systematic of Leston's fits [6]. In Figs. 5 and 6 the extracted ^{168}Yb parameters from this work's fit are compared with the systematic of Leston's fits [6].

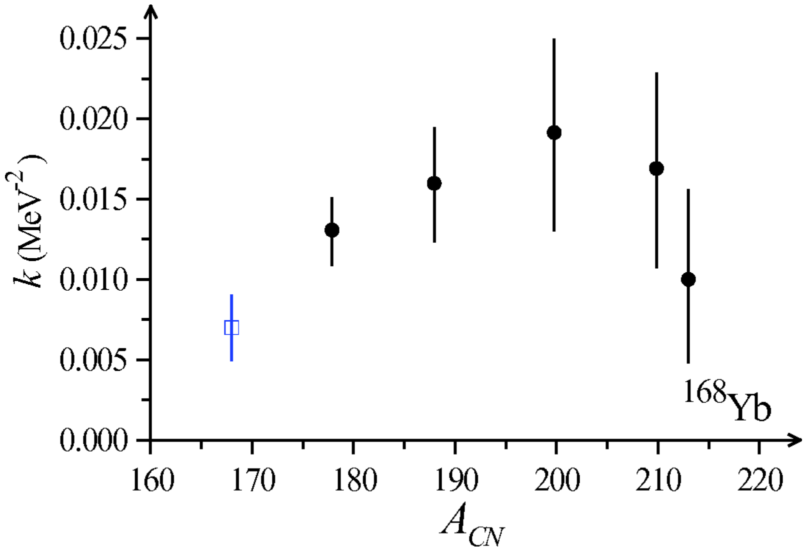

FIG. 5: Fit parameter k for ^{168}Yb from this work (open square) compared with the systematic of Leston's fits [6] (closed circles).

IV. CONCLUSIONS

In the framework of a Kramers-modified statistical model the average pre-fission neutron multiplicities and the fission and evaporation residue cross sections for ^{168}Yb formed in ^{18}O + ^{150}Sm reactions were calculated, and the results compared with the experimental data. To calculate these quantities, the effects of temperature and the spin K about the symmetry axis have been considered in the calculations of the potential energy surfaces and the fission widths.

It should be stressed that in many statistical model codes, authors have used the ratio of the level density, a_f/a_n and a scaling of the FRLDM barrier heights, f_B, as free parameters to reproduce the experimental data. But fission in heavy-ion reactions cannot be accurately modeled as a function of the excitation energy using a fixed value of a_f/a_n and the J dependence of the $T = 0$ fission barriers. But in the present research, we considered other parameters as free parameters which perform similar roles to those of a_f/a_n and f_B.

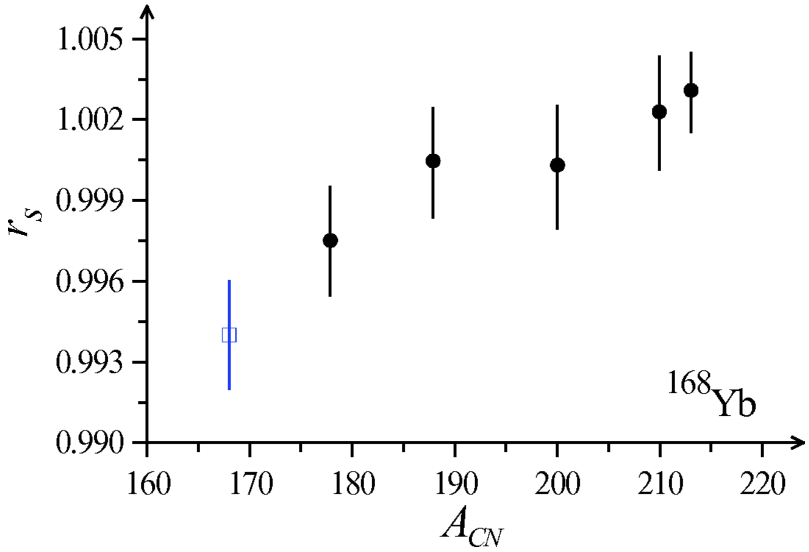

FIG. 6: Fit parameter r_s for ^{168}Yb from this work (open square) compared with the systematic of Leston's fits [6] (closed circles).

We considered the temperature coefficient in the effective potential formula, k, and a scaling of the MLDM radii from their default values used to calculate the surface and Coulomb energies with the parameter r_s.

In the present work, the parameters k and r_s were adjusted to reproduce a single average pre-fission neutron multiplicity and a single residue and fission cross sections for ^{168}Yb. It was shown that the results of the above mentioned experimental data are in good agreement with the experimental data by using the values of $k = 0.007 \pm 0.002$ MeV^{-2} and $r_s = 0.994 \pm 0.002$. It should be stressed that our result for k is consistent with the other investigations [19–24].

Acknowledgements

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