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Temperature and Impurity Effects of the Magnetopolaron in an Asymmetric Quantum Dot

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Using the linear combination operator method, we investigate the properties of the strong-coupling impurity bound magnetopolaron in an asymmetric quantum dot (AQD). The relations between the vibrational frequency, the mean number of phonons, and the ground state binding energy with the temperature, the cyclotron frequency of the magnetic field, the electron-phonon coupling strength, and the Coulomb bound potential are derived. It is found that the mean number of phonons and the ground state binding energy are increasing functions of the temperature, the coupling strength, and the Coulomb bound potential. The vibrational frequency will increase with increasing Coulomb bound potential and coupling strength. The mean number of phonons is a decreasing function of the transverse and the longitudinal effective confinement lengths. The ground state binding energy is a decreasing function of the cyclotron frequency, whereas the vibrational frequency is an increasing function of it.

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I. INTRODUCTION

Recently, with the rapid development of material production technology, the physical characteristics of low dimensional materials have aroused great interest. Especially, there has been a great deal of interest in the investigation of quantum dots (QDs) both theoretically and experimentally [1–4]. Several investigators have studied the problem of a polaron and a magnetopolaron bound to a hydrogenic impurity, which is interacting with a longitudinal optical field resulting from an ionic crystal or a polar semiconductor. For example, with the Hasse variational approach, Arshak *et al.* [5] calculated the effect of electric and magnetic fields on the binding energy of the Coulomb impurity bound polaron in a QD. Within the effective mass approximation and finite barrier potential, the effect of an external magnetic field on the excited state energies in a spherical QD was investigated by Sadeghi and Rezaie [6], and using the variational method, the impurity energy and binding energy were calculated. Under the effective mass approximation, Cheng [7] presented theoretical results on the effects of particle-particle interaction and single spin $-\frac{5}{2}$ magnetic impurity (Mn^{2+}) on the paramagnetism of interacting spherical QDs. Employing the perturbation method and the compact density-matrix approach, Lu and Xie [8] made a detailed investigation of the impurity and exciton effects on the nonlinear optical properties of a disc-like QD under a magnetic field. Satyabrate *et al.* [9] studied a quantum-confined hydrogenic impurity in a spherical QD under the influence of parallel electric and magnetic fields by using a numerical technique and the complex absorbing potential method. Through a linear variational route, Kanchan *et al.* [10] investigated the frequency dependent linear and non-linear response properties of an electron impurity doped QDs influence on the impurity location. Using Kane's two-band approximation, Kazaryyan *et al.* [11] studied the impurity states of a narrow-gap semiconductor parabolic QD in the presence of an extremely strong magnetic field. The simultaneous effects of electric and magnetic fields in a GaAs/AlAs spherical QD with a hydrogenic impurity was investigated by Dane *et al.* [12], and a variational approach within the framework of the effective mass approximation was used in the calculations. Kanchan *et al.* [13] analyzed the interplay between the size and impurity position of a doped QD by using a number of diagnostic tools. However, few people have investigated the impurity properties of the magnetopolaron in an asymmetric QD so far by employing the linear combination operator method. Especially, the effects of the temperature and the impurity on the mean number of phonons and the ground state binding energy of the strong-coupling magnetopolaron in an AQD have never been studied yet.

In this paper, we investigated the temperature and impurity effects on the mean number of phonons and the ground state binding energy in an AQD by using the linear combination operator method.

II. THEORETICAL MODEL

A moving electron in the crystal QD is surrounded by the other medium. Due to the phonon field and the polar crystal boundary effect, the movment of the electron in every direction is quantized. In the presence of a parallel magnetic field along the z-direction with vector potential $A = B(-\frac{y}{2}, \frac{x}{2}, 0)$, the Hamiltonian of the electron-phonon interaction system with a hydrogenic impurity at the center can be written as

$$\begin{aligned} H \ = \ & \frac{1}{2m}\left(p_x - \frac{\overline{\beta}^2}{4}y\right)^2 + \frac{1}{2m}\left(p_y - \frac{\overline{\beta}^2}{4}x\right)^2 + \frac{p_z^2}{2m} + \frac{1}{2}m\omega_1^2\rho + \frac{1}{2}m\omega_2^2 z^2 \\ & + \sum_{\mathbf{q}} \hbar\omega_{\mathrm{LO}} a_{\mathbf{q}}^{+} a_{\mathbf{q}} + \sum_{\mathbf{q}} \left[V_q a_{\mathbf{q}} \exp\left(i\mathbf{q}\cdot\mathbf{r}\right) + h\cdot c\right] - \frac{e^2}{\varepsilon_0 r}, \end{aligned} \tag{1}$$

where $\overline{\beta}^2 = \frac{2e}{c}B$, and m is the band mass, ω_1 and ω_2 are the transverse and longitudinal confinement strengths of the three-dimensional anisotropic harmonic potential in the $x-y$ plane and the z direction, respectively. $a_{\mathbf{q}}^{+}(a_{\mathbf{q}})$ denotes the creation(annihilation) operator of the LO phonons with wave vector $\mathbf{q}(\mathbf{q}_{\|}, q_z)$, and $\mathbf{r}(\boldsymbol{\rho}, z)$ is the position vector of an electron.

$$V_q = i\left(\frac{\hbar\omega_{\mathrm{LO}}}{q}\right)\left(\frac{\hbar}{2m\omega_{\mathrm{LO}}}\right)^{1/4}\left(\frac{4\pi\alpha}{V}\right)^{1/2}, \tag{2}$$

$$\alpha = \left(\frac{e^2}{2\hbar\omega_{\rm LO}}\right)\left(\frac{2m\omega_{\rm LO}}{\hbar}\right)^{1/2}\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right). \tag{3}$$

We use the Fourier expansion of the Coulomb bound potential,

$$-\frac{e^2}{\varepsilon_0 r} = -\frac{4\pi e^2}{\varepsilon_0 V}\sum_{\mathbf{q}}\frac{1}{q^2}\exp(-i\mathbf{q}\cdot\mathbf{r}). \tag{4}$$

We apply the following Lee-Low-Pines transformation to Eq. (1):

$$U = \exp\left[\sum_{\mathbf{q}}\left(a_{\mathbf{q}}^{+} f_q - a_{\mathbf{q}} f_q^{*}\right)\right], \tag{5}$$

then we introduce the linear combination operators

$$P_j = \left[\frac{m\hbar\lambda}{2}\right]^{\frac{1}{2}}(b_j + b_j^{+}), \tag{6}$$

$$r_j = i\left[\frac{\hbar}{2m\lambda}\right]^{\frac{1}{2}}(b_j - b_j^{+}), \quad j = x, y, z,$$

where $f_q(f_q^*)$ and λ are the variational parameters, and λ expresses the vibrational frequency of the magnetopolaron. We choose the following ground state wave function of the system:

$$|\psi_0\rangle = |0\rangle_a\,|0\rangle_b\,, \tag{7}$$

where $|0\rangle_b$ is the vacuum state of the b operator and $|0\rangle_a$ is the unperturbed zero phonon state, which satisfies $b_j\,|0\rangle_b = a_q\,|0\rangle_a = 0$. We calculate

$$F(\lambda, f_q) = \left\langle \Psi_0\right| U^{-1}HU\left|\,\Psi_0\right\rangle. \tag{8}$$

We can obtain f_q by using the variational method for f_q^*. Inserting f_q into F and replacing the summation by the integration, we have

$$F(\lambda) = \frac{3\hbar\lambda}{4} + \frac{\hbar\omega_1^2}{2\lambda} + \frac{\hbar\omega_2^2}{4\lambda} + \frac{\hbar\beta^4}{32m^2\lambda} - \frac{\alpha\hbar\omega_{\rm LO}}{\sqrt{\pi}}\left(\frac{\lambda}{\omega_{lo}}\right)^{\frac{1}{2}} - \frac{2e^2}{\varepsilon_0}\sqrt{\frac{m\lambda}{\pi\hbar}}. \tag{9}$$

Performing the variation of $F(\lambda)$ with respect to λ, we obtain

$$\lambda^2 - \left(\frac{2\alpha}{3}\sqrt{\frac{\omega_{\rm LO}}{\pi}} + \frac{4}{3}\beta\right)\lambda^{3/2} - \left(\frac{2}{3}\omega_1^2 + \frac{1}{3}\omega_2^2 + \frac{\omega_c^2}{6}\right) = 0, \tag{10}$$

where $\beta = \frac{e^2}{\varepsilon_0}\sqrt{\frac{m}{\pi\hbar}}$ and $\omega_c = \frac{eB}{mc}$ are the Coulomb bound potential and the cyclotron frequency of the magnetic field. Solving Eq. (10), we get the vibrational frequency of the polaron:

$$\lambda = \lambda_0. \tag{11}$$

By substituting λ_0 into Eq. (9), we obtain the impurity bound magnetopolaron ground state energy E_0.

If E_e and E_p denote the energies of the uncoupled electron and phonon, respectively, then the ground state binding energy of the magnetopolaron is given by

$$E_b = E_e + E_p - E_0 = \frac{2\alpha\hbar\omega_{\text{LO}}}{\sqrt{\pi}}\sqrt{\frac{\lambda_0}{\omega_{\text{LO}}}} - \frac{\hbar}{4\lambda_0}\left(2\omega_1^2 + \omega_2 + \frac{\omega_c^2}{2}\right) + 2\beta\sqrt{\lambda_0}. \tag{12}$$

Choosing the usual polaron units ($\hbar = 2m = \omega_{\text{LO}} = 1$), the vibrational frequency and the ground state binding energy of the strong-coupling impurity bound magnetopolaron in an AQD are

$$\lambda^2 - \left(\frac{2\alpha}{3\sqrt{\pi}} + \frac{4\beta}{3}\right)\lambda^{\frac{3}{2}} - \left(\frac{8}{3l_1^4} + \frac{4}{3l_2^4} + \frac{\omega_c^2}{6}\right) = 0 \tag{13}$$

and

$$E_b = 2\alpha\frac{\sqrt{\lambda_0}}{\sqrt{\pi}} + 2\sqrt{\lambda_0}\beta - \frac{1}{\lambda_0}\left(\frac{2}{l_1^4} + \frac{1}{l_2^4} + \frac{\omega_c^2}{8}\right), \tag{14}$$

where $l_1 = \sqrt{\frac{\hbar}{m\omega_1}}$, $l_2 = \sqrt{\frac{\hbar}{m\omega_2}}$ are the transverse and longitudinal effective confinement lengths, respectively. The mean number of optical phonons of the ground state around the electron is

$$N = \langle\psi_0|\, U^{-1}\sum_q a_q^+ a_q U\,|\psi_0\rangle = \frac{\alpha}{\sqrt{\pi}}\sqrt{\lambda_0}. \tag{15}$$

III. TEMPERATURE EFFECTS

At finite temperature, the electron-phonon system is no longer entirely in the ground state. The lattice vibrations excite not only the real phonon but also the electron in a parabolic potential. The properties of the polaron are a statistical average over various states. According to the quantum statistics theory, the statistical average number of the optical phonons is

$$\bar{N} = \left[\exp\left(\frac{\hbar\omega_{\text{LO}}}{k_B T}\right) - 1\right]^{-1}. \tag{16}$$

where k_B is the Boltzmann constant. However, the value of λ_0 in Eq. (15) relates not only to the value of N but also to the value of $\bar{N}$, which should be self-consistently calculated with Eq. (15), and then we can obtain the relation between λ, E_b, N, and T.

IV. NUMERICAL RESULT AND DISCUSSION

To show more obviously the influence of the temperature and impurity on the properties of the magnetopolaron in an AQD, we perform a numerical calculation. The results are presented in Figs. 1–9.

Figure 1 shows the relationship between the vibrational frequency λ and the electron-phonon coupling strength α for fixed $l_1 = 1.2$, $l_2 = 1.6$, and $\omega_c = 8.0$. The dotted and the solid lines correspond to the cases of Coulomb bound potential $\beta = 0.5$ and $\beta = 0.2$, respectively. Figure 2 illustrates the relationship between the vibrational frequency λ and the electron-phonon coupling strength α for fixed $l_1 = 1.2$, $l_2 = 1.6$, and $\beta = 0.5$. The dotted and the solid lines correspond to the cases of cyclotron frequency of the magnetic field $\omega_c = 8.0$ and $\omega_c = 5.0$, respectively. The relational curves of the mean number of phonons N with the electron-phonon coupling strength α for fixed $l_1 = 0.8$, $l_2 = 1.2$, and $\omega_c = 5.0$ are shown in Fig. 3. The dotted and the solid lines correspond to the cases of Coulomb bound potential $\beta = 8.0$ and $\beta = 2.0$, respectively. From these figures, we can see that the vibrational frequency and the mean number of phonons increase rapidly with increasing the electron-phonon coupling strength. This is because the larger the electron-phonon coupling strength is, the stronger the electron-phonon interaction is. Therefore, it leads to the increment of the electron energy and makes the electron to interact with more phonons. As a result, the vibrational frequency and the mean number of phonons are all increased. From them we can also find that the vibrational frequency and the mean number of phonons are increasing functions of the Coulomb bound potential.

Figure 4 presents the mean number of phonons N as a function of the transverse effective confinement length l_1 for fixed $l_2 = 1.2$, $\alpha = 6.0$, and $\omega_c = 5.0$. The dotted and the solid lines correspond to the cases of the Coulomb bound potential $\beta = 1.0$ and $\beta = 0.2$, respectively. Figure 5 plots the relation between the mean number N as a function of the longitudinal effective confinement length l_2 for fixed $l_1 = 0.8$, $\alpha = 6.0$, and $\omega_c = 5.0$. The dotted and the solid lines correspond to the cases of the Coulomb bound potential $\beta = 0.5$ and $\beta = 0.2$, respectively. Figure 6 demonstrates the dependence of the mean number N on the temperature T. From Figs. 4 and 5, we find that the mean number of phonons will increase rapidly with decreasing the transverse and the longitudinal effective confinement lengths l_1 and l_2. These can be attributed to the interesting quantum size confining effects. The reason is that the motion of the electron is confined by the confining potential. With the increase of the confining potential (that is, with decreasing ρ and z), the interaction between the electron and the phonons is enhanced, because of a smaller range of particle motion. As a result of this, the mean number of phonons is increased. Figure 6 shows that the mean number of phonons is an increasing function of the temperature.

Figs. 7, 8, and 9 express the ground state binding energy E_b changes with the temperature T at different coupling strength α, Coulomb bound potential β, and cyclotron frequency ω_c of the magnetic field, respectively. From Figs. 6, 7, 8, and 9, we can see that the mean number of phonons and the ground state binding energy are increasing functions of the temperature T. The reason is that higher temperature increases the speed of the thermal activity of the electron and the phonon, so that the electron will interact with more

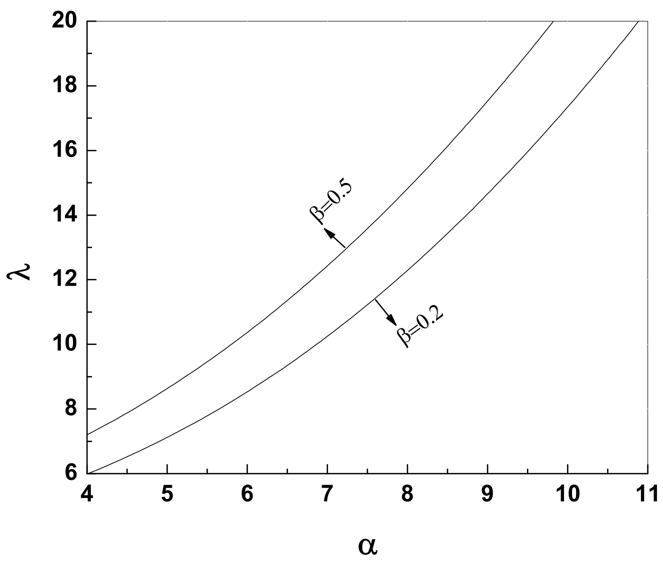

FIG. 1: Relational curves of the vibrational frequency λ with the electron-phonon coupling strength α for fixed $l_1 = 1.2$, $l_2 = 1.6$, and $\omega_c = 8.0$.

phonons. Hence, they will be enhanced with increasing temperature.

From Figs. 1, 3, 4, 5, and 7, one finds that the vibrational frequency, the mean number of phonons, and the ground state binding energy will increase with increasing Coulomb bound potential. There is a Coulomb potential between the electron and the hydrogen-like impurity because of the existence of a hydrogen-like impurity in the center. Since the presence of the Coulomb potential is equivalent to introducing another new confinement on the electrons, which leads to greater electron wavefunction overlapping with each other, the electron-phonon interaction will be enhanced. Therefore, the vibrational frequency, the mean number phonons, and the ground state binding energy of the impurity bound magnetopolaron in an AQD are all increased. Figs. 2 and 9 also show that the vibrational frequency is an increasing function of the cyclotron frequency, whereas the ground state binding energy is a decreasing function. From the expressions for $\omega_c = eB/mc$ one can see that the vibrational frequency will increase with the increase of the magnetic field B. With the increase of B, the electron energy and the electron-phonon coupling energy are enhanced, because of the existence of the magnetic field. Therefore, the vibrational frequency is increased. However, the last term in Eq. (14) is a contribution from the magnetic field term to the ground state binding energy, which is a negative value, and the ground state binding energy is decreased. From Fig. 7, it can be seen that the ground state binding energy is an increasing function of the coupling strength. The reasons are

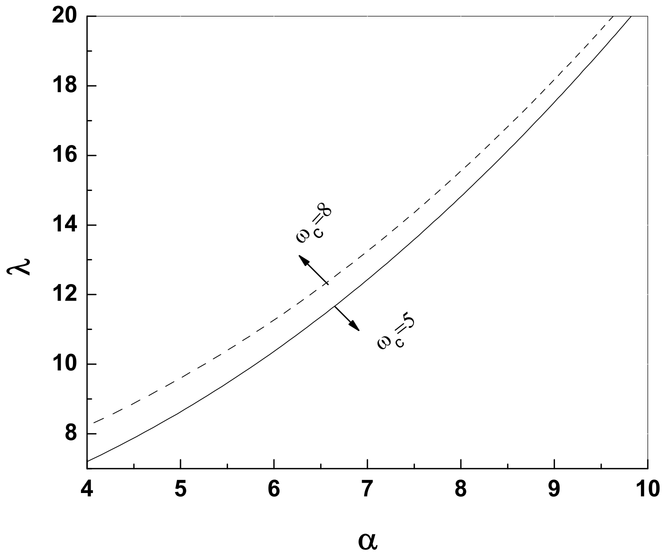

FIG. 2: Relational curves of the vibrational frequency λ with the electron-phonon coupling strength α for fixed $l_1 = 1.2$, $l_2 = 1.6$, and $\beta = 8.0$.

essentially the same as with the explanation of the variation relationships of the vibrational frequency and the mean number of phonons in Figs. 1–2, respectively. So, we will not repeat them here.

V. CONCLUSION

In conclusion, based on the linear combination operator method, we have investigated the temperature and impurity effects of the vibrational frequency, the mean number of phonons, and the ground state binding energy of the strong-coupling impurity bound magnetopolaron in an AQD. It is found that the mean number of phonons and the ground state binding energy are increasing functions of the temperature, the coupling strength, and the Coulomb bound potential.

The vibrational frequency will increase with increasing Coulomb bound potential and coupling strength. The mean number of phonons is a decreasing function of the transverse and the longitudinal effective confinement lengths. The ground state binding energy is a decreasing function of the cyclotron frequency, whereas the vibrational frequency is an increasing function of it.

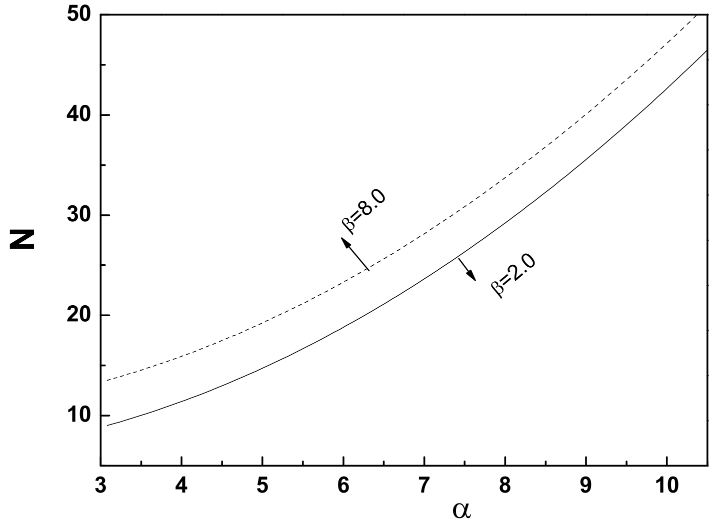

FIG. 3: Relational curves of the mean number of phonons N with the electron-phonon coupling strength α for fixed $l_1 = 0.8$, $l_2 = 1.2$, and $\omega_c = 5.0$.

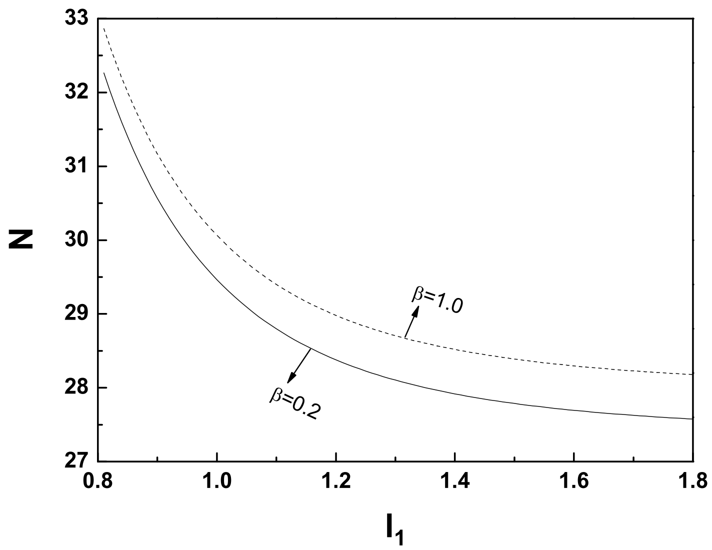

FIG. 4: Relational curves of the mean number of phonons N with the transverse confinement length l_1 for fixed $l_2 = 1.2$, $\alpha = 6.0$, and $\omega_c = 5.0$.

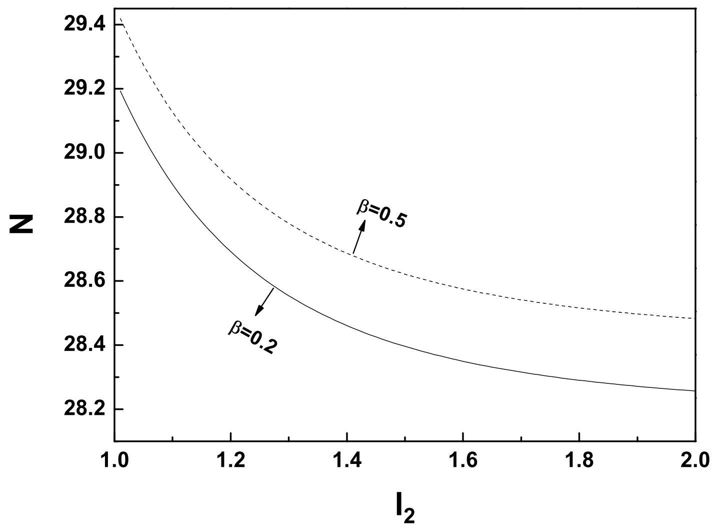

FIG. 5: Relational curves of the mean number of phonons N with the longitudinal confinement length l_2 for fixed $l_1 = 0.8$, $\alpha = 6.0$, and $\omega_c = 5.0$.

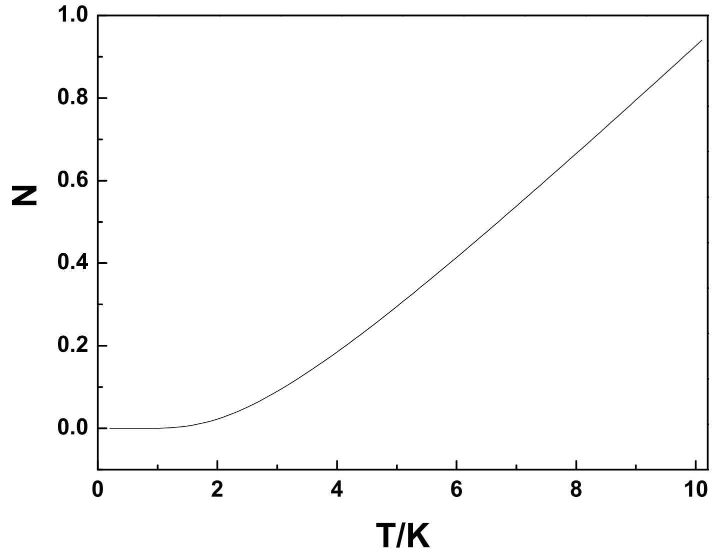

FIG. 6: Relational curves of the mean number of phonons N with the temperature T.

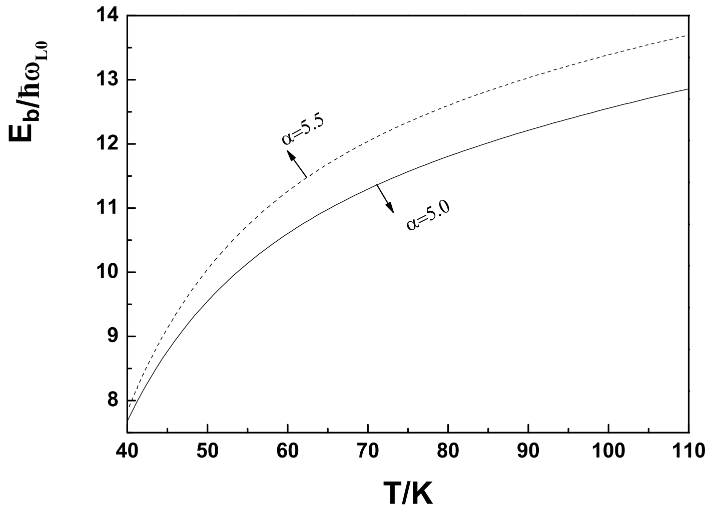

FIG. 7: Relational curves of the ground state binding energy E_b with the temperature T for fixed $l_1 = 1.2$, $l_2 = 1.6$, $\beta = 0.2$, and $\omega_c = 10.0$.

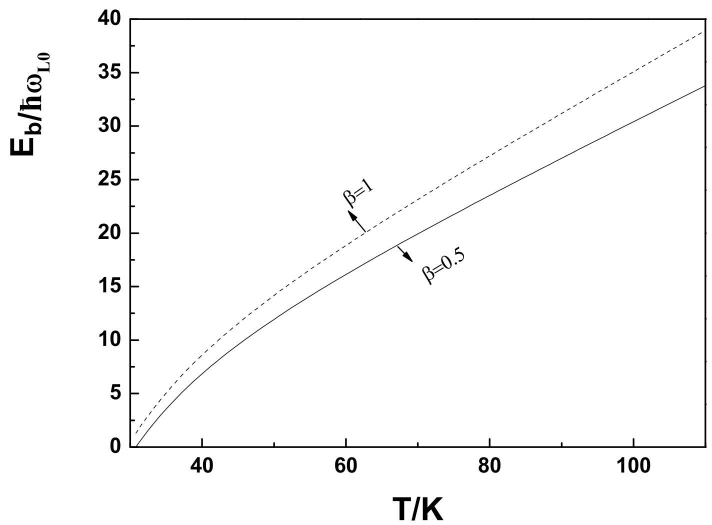

FIG. 8: Relational curves of the ground state binding energy E_b with the temperature T for fixed $l_1 = 1.2$, $l_2 = 1.6$, $\alpha = 0.2$, and $\omega_c = 8.0$.

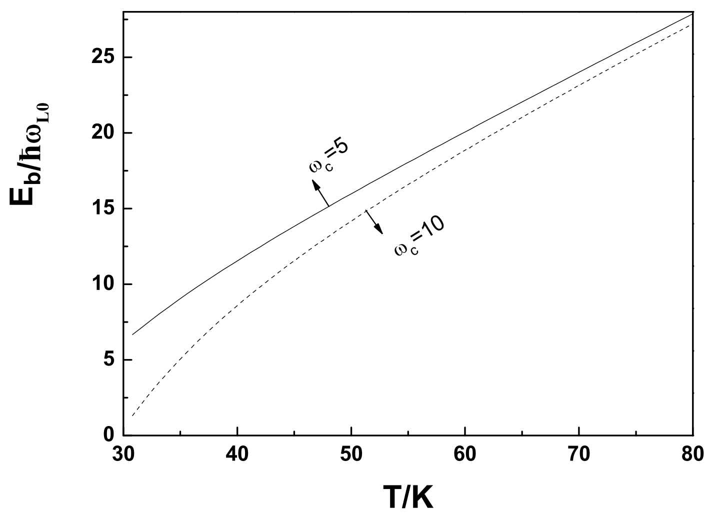

FIG. 9: Relational curves of the ground state binding energy E_b with the temperature T for fixed $l_1 = 1.2$, $l_2 = 1.6$, $\beta = 0.2$, and $\alpha = 6.0$.

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