Nonzero θ_{13}, CP Violation, and Broken $\mu-\tau$ Symmetry

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Recently, evidence for a nonzero and relatively large θ_{13}, as measured in the latest experiment, has a strong implication on the well-known neutrino mixing matrix and the existence of CP violation in the neutrino sector ($J_{\rm CP} \neq 0$). One of the well-known mixing matricies is the tribimaximal (TBM) neutrino mixing matrix that predicts the mixing angle $\theta_{13} = 0$ and $J_{\rm CP} = 0$. In order to accommodate a nonzero θ_{13} and CP violation, we modified the TBM by introducing a simple perturbation matrix into the TBM matrix that can produce $\theta_{13} = 7.89°$, which is in agreement with the current experimental results, and the Dirac phase $\delta \approx \pi/2$ and $J_{\rm CP} \approx 0.0222$. The obtained neutrino mass matrix from the modified TBM with both nonzero θ_{13} and δ is a complex neutrino mass matrix. If we impose $\mu-\tau$ symmetry as a constraint into the neutrino mass matrix, one finds that the $J_{\rm CP} = 0$, which implies that CP violation cannot be accommodated in the $\mu-\tau$ symmetry scheme. We break the $\mu-\tau$ symmetry softly by introducing a small parameter x to perturb the neutrino mass matrix with the constraint that the trace of the perturbed neutrino mass matrix remains constant.

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I. INTRODUCTION

Recent experimental results show that the mixing angle $\theta_{13} \neq 0$, and its value is relatively large. Nonzero θ_{13} has some implications in our understanding about the neutrino sector. One of the implications when $\theta_{13} \neq 0$ is the possibility of CP violation in the neutrino sector, as well as in the quark sector. A nonzero and relatively large mixing angle θ_{13} have been reported by the following collaborations: MINOS [1], Double Chooz [2], T2K [3], Daya Bay [4], and RENO [5]. From the theoretical side, there are three types of the well-known neutrino mixing matrices: tribimaximal (TBM), bimaximal (BM), and democratic (DM), and all of these three types of neutrino mixing matrices predict the mixing angle $\theta_{13} = 0$. The evidence of nonzero θ_{13} is due to the achievement of experimental methods and tools, the assumption that the value of the mixing angle θ_{13} is zero must be amended or even ruled out. Concerning the well-known mixing matrix TBM, Ishimori and Ma [6] stated explicitly that the tribimaximal mixing matrix may be dead, due to the experimental fact that the mixing angle $\theta_{13} \neq 0$.

To explain the evidence of nonzero and relatively large θ_{13}, several authors have

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already proposed some methods and models. The simple way to accommodate a nonzero θ_{13} is to modify the neutrino mixing matrix by introducing a perturbation matrix into the known mixing matrix such that it can produces a nonzero θ_{13} [7–11], and the other is to build the model by using some discrete symmetries [12, 13]. A nonzero θ_{13} is also known to be related to the Dirac phase δ, as one can see in the standard parameterization of the neutrino mixing matrix. Thus, nonzero θ_{13} gives a clue to the possible determination of CP violation in the neutrino sector. Perturbations of the neutrino mixing matrix in order to accommodate both nonzero θ_{13} and CP violation have been reported [14–19].

Concerning the $\mu - \tau$ symmetry and the mixing angle θ_{13}, Mohapatra [20] stated explicitly that the neutrino mass matrix that obeys $\mu - \tau$ symmetry to be the reason for maximal $\mu - \tau$ mixing, one gets $\theta_{13} = 0$ and, conversely, if $\theta_{13} \neq 0$ it can provide the $\mu - \tau$ symmetry breaking manifest in the case of normal hierarchy. A nonzero θ_{13} and its implication for leptogenesis as an origin of matter is discussed in [21]. Aizawa and Yasue [22] analyzed the complex neutrino mass texture and $\mu - \tau$ symmetry, which can yield a small θ_{13} as a $\mu - \tau$ breaking effect. The $\mu - \tau$ symmetry breaking effect in relation with the small θ_{13} is also discussed in [23]. Another scenario that can produce a Dirac CP phase is by studying other mixing scenarios which deviate from tribimaximal mixing by leaving only one of the columns or one of the rows invariant, which is called "generalized trimaximal mixing" [24, 25]. An analysis of the correlation between CP violation and $\mu - \tau$ symmetry breaking can be read in [26, 27]. In [28, 29], the Dirac CP phase δ can be obtained by exploring the generalized $\mathbf{Z_2^s}$ symmetry without the assumption of $\mu - \tau$ symmetry.

In this paper we derive a nonzero θ_{13} by modifying the TBM by introducing a simple perturbation matrix into the TBM, and we then calculate the mixing angle θ_{13} by using the mixing angles θ_{21} and θ_{32} from the experimental results. The modified TBM is used for obtaining the neutrino mass matrix M_ν in the flavor basis, and then we analyze the effect of $\mu - \tau$ symmetry on M_ν. This paper is organized as follows: in Section II, we modify the TBM by introducing a simple perturbation matrix. In Section III, we determine Dirac neutrino phase δ and the neutrino masses constrained by $\mu - \tau$ symmetry, and break the $\mu - \tau$ symmetry by introducing a small parameter x, which keeps the trace of the perturbed neutrino mass matrix constant. Finally, Section IV is devoted to our conclusion.

II. NONZERO θ_{13} FROM THE MODIFIED TBM

The neutrino mixing matrix existence is due to the experimental fact that mixing of flavors does exist in the leptonic sector, especially in the neutrino sector as well as in the quark sector. The neutrino eigenstates in the flavor basis $(\nu_e, \nu_\mu, \nu_\tau)$ relate to the neutrino eigenstates in the mass basis (ν_1, ν_2, ν_3) as follow:

$$\nu_i = V_{ij}\nu_j, \tag{1}$$

where $V_{ij}(i = e, \mu, \tau; j = 1, 2, 3)$ are the elements of the neutrino mixing matrix. The standard parameterization of the mixing matrix V reads:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2}$$

where c_{ij} is $\cos\theta_{ij}$, s_{ij} is $\sin\theta_{ij}$, and θ_{ij} are the mixing angles.

One of the well-known neutrino mixing matrices (V) is the TBM ($V_{\rm TBM}$), which is given by [30–35]

$$V_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \tag{3}$$

One can see from Eq. (3) that the entry $V_{e3} = 0$, which implies that the mixing angle θ_{13} must be zero in the TBM. However, the latest result from the long baseline neutrino oscillation experiment T2K indicates that θ_{13} is nonzero and relatively large. For a vanishing Dirac CP-violating phase ($\delta = 0$), the T2K collaboration reported that the values of θ_{13} for the neutrino mass in normal hierarchy (NH) are [3]

$$5.0^\circ \leq \theta_{13} \leq 16.0^\circ, \tag{4}$$

and for the neutrino mass in inverted hierarchy (IH)

$$5.8^\circ \leq \theta_{13} \leq 17.8^\circ. \tag{5}$$

The current combined data for the neutrino squared-mass difference are given by [36, 37]:

$$\Delta m^2_{21} = 7.59 \pm 0.20(^{+0.61}_{-0.69}) \times 10^{-5}\ {\rm eV}^2, \tag{6}$$

$$\Delta m^2_{32} = 2.46 \pm 0.12(\pm 0.37) \times 10^{-3}\ {\rm eV}^2,\ \text{(for NH)} \tag{7}$$

$$\Delta m^2_{32} = -2.36 \pm 0.11(\pm 0.37) \times 10^{-3}\ {\rm eV}^2,\ \text{(for IH)} \tag{8}$$

$$\theta_{12} = 34.5 \pm 1.0(^{+3.2}_{-2.8})^\circ, \quad \theta_{23} = 42.8^{+4.5}_{-2.9}(^{+10.7}_{-7.3})^\circ, \quad \theta_{13} = 5.1^{+3.0}_{-3.3}(\leq 12.0)^\circ, \tag{9}$$

at the 1σ (3σ) level. The latest experimental result for the value of θ_{13} is reported by the Daya Bay Collaboration which gives [4]

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat.}) \pm 0.005(\text{syst.}), \tag{10}$$

and the RENO Collaboration reported that [5]

$$\sin^2 2\theta_{13} = 0.113 \pm 0.013(\text{stat.}) \pm 0.014(\text{syst.}). \tag{11}$$

Concerning the TBM status in the context of nonzero θ_{13}, Ishimori and Ma [6] concluded that TBM may be dead, but A_4 is alive and even getting healthier. Modification of the neutrino mixing matrix, by introducing a perturbation matrix into the neutrino mixing matrix in Eq. (3), is a simple way to obtain a nonzero θ_{13}. The value of θ_{13} can be obtained in some parameters that can be fitted from the experimental results. A nonzero of V_{e3} for the TBM can be obtained by charge lepton corrections and renormalization group running [7]. In this paper, the modified TBM neutrino mixing matricies to be considered are given by

$$V'_{\rm TBM} = V_{\rm TBM} V_y, \tag{12}$$

where V_y is the perturbation matrix. We take the form of the perturbation matrix as follows:

$$V_y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_y & s_y e^{-i\delta} \\ 0 & -s_y e^{i\delta} & c_y \end{pmatrix}, \tag{13}$$

where c_y is $\cos y$, s_y is $\sin y$, and δ is the Dirac CP phase.

By inserting Eqs. (3) and (13) into Eqs. (12), we then have the modified neutrino mixing matrix as follows:

$$V'_{\rm TBM} = \begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3}c_y & \frac{\sqrt{3}}{3}s_y e^{-i\delta} \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3}c_y - \frac{\sqrt{2}}{2}s_y e^{i\delta} & \frac{\sqrt{3}}{3}s_y e^{-i\delta} + \frac{\sqrt{2}}{2}c_y \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3}c_y + \frac{\sqrt{2}}{2}s_y e^{i\delta} & \frac{\sqrt{3}}{3}s_y e^{-i\delta} - \frac{\sqrt{2}}{2}c_y \end{pmatrix}. \tag{14}$$

By comparing Eqs. (14) with the neutrino mixing in the standard parameterization form as shown in Eq. (2), we have:

$$\tan\theta_{12} = \left|\frac{\sqrt{2}c_y}{2}\right|, \quad \tan\theta_{23} = \left|\frac{\frac{\sqrt{3}}{3}s_y e^{-i\delta} + \frac{\sqrt{2}}{2}c_y}{\frac{\sqrt{3}}{3}s_y e^{-i\delta} - \frac{\sqrt{2}}{2}c_y}\right|, \quad \sin\theta_{13}e^{-i\delta} = \left|\frac{\sqrt{3}}{3}s_y\right| e^{-i\delta}, \tag{15}$$

and for $\delta = 0$ [10]:

$$\tan\theta_{12} = \left|\frac{\sqrt{2}c_y}{2}\right|, \quad \tan\theta_{23} = \left|\frac{\frac{\sqrt{3}}{3}s_y + \frac{\sqrt{2}}{2}c_y}{\frac{\sqrt{3}}{3}s_y - \frac{\sqrt{2}}{2}c_y}\right|, \quad \sin\theta_{13} = \left|\frac{\sqrt{3}}{3}s_y\right|. \tag{16}$$

From Eqs. (15) and (16) it is apparent that for $y \to 0$, the value of $\tan\theta_{12} \to \sqrt{2}/2$ and $\tan\theta_{23} \to 1$ which imply that $\theta_{12} \to 35.264^\circ$ and $\theta_{23} \to 45^\circ$. From Eq. (15), one can see that it is possible to determine the value y and therefore the value of θ_{13} by using the experimental values of θ_{12} of Eq. (9).

By inserting the experimental value of θ_{12} of Eq. (9) into Eq. (15), we obtain the value of c_y for $1\sigma(3\sigma)$:

$$c_y = 0.97133^{+0.03742}_{-0.04528}\left(^{+0.08279}_{-0.09789}\right), \tag{17}$$

which gives: $0^\circ < y < 22^\circ$ for 1σ and $0^\circ < y < 29^\circ$ for 3σ and the central value of $y = 13.7537^\circ$. From Eqs. (15) and (17), we have for the central value of y:

$$0 < \sin\theta_{13} < 0.21628, \tag{18}$$

for 1σ, and

$$0 < \sin\theta_{13} < 0.27991, \tag{19}$$

for 3σ, and the central value of the mixing angle $\theta_{13} = 7.89^\circ$, which is in agreement with the T2K [3] and Daya Bay experimental results [4].

From Eq. (15) we can have the relation all mixing angles and the Dirac phase δ in general as follows:

$$\tan\theta_{23} = \left|\frac{\sin\theta_{13}e^{-i\delta} + \tan\theta_{12}}{\sin\theta_{13}e^{-i\delta} - \tan\theta_{12}}\right|. \tag{20}$$

After performing some algebraic effort on Eq. (20), we finally have:

$$\delta = \arccos\left[\frac{(\tan^2\theta_{23} - 1)(\tan^2\theta_{12} + \sin^2\theta_{13})}{2(\tan^2\theta_{23} + 1)\sin\theta_{31}\tan\theta_{12}}\right]. \tag{21}$$

It is also apparent from Eq. (21) that the Dirac phase δ relates to the mixing angles θ_{23}, θ_{12}, and θ_{13}. Thus, it is possible to detect the Dirac phase δ by measuring the precise values of the mixing angles θ_{23}, θ_{12}, and θ_{13} in a future experiment.

If we insert the values of $\theta_{23} = 42.8^\circ$ and $\theta_{12} = 34.5^\circ$, as shown in Eq. (9), and $\sin\theta_{13} = 0.137265$ into Eq. (21), then we have

$$\delta \approx \pi/2. \tag{22}$$

The Jarlskog invariant $J_{\rm CP}$, which is very useful for quantifying CP violation, is given by [38]:

$$J_{\rm CP} = {\rm Im}\left[(V'_{\rm TBM})_{11}(V'_{\rm TBM})_{22}(V'_{\rm TBM})^*_{12}(V'_{\rm TBM})^*_{21}\right]. \tag{23}$$

If we insert the corresponding values of $V'_{\rm TBM}$ of Eq. (14) into Eq. (23) with the values of c_y in Eq. (17) and δ in Eq. (22), then we have

$$J_{\rm CP} \approx 0.0222. \tag{24}$$

III. BROKEN $\mu - \tau$ SYMMETRY AND THE DIRAC PHASE δ

Concerning the $\mu - \tau$ symmetry, most of the authors discuss the $\mu - \tau$ symmetry in relation with the mixing angle θ_{13} [20] and its implication for the origin of matter via leptogenesis [21]. That the effect of $\mu - \tau$ symmetry breaking in the neutrino mass matrix can give rise to the leptonic CP violation was proposed by Mohaptara and Rodejohann [26]. In this section, we analyze the $\mu-\tau$ symmetry in relation with the Dirac phase δ. We begin with the assumption that the charged lepton mass matrix is diagonal, then we evaluate the neutrino mass matrix M_ν in the flavor eigenstate basis. In the basis where the charged lepton mass matrix is diagonal, the neutrino mass matrix can be diagonalized by a unitary matrix V as follows:

$$M_\nu = VMV^T, \tag{25}$$

where the diagonal neutrino mass matrix M is given by:

$$M = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \tag{26}$$

If the unitary matrix V is the modified TBM mixing matrix ($V'_{\rm TBM}$) as shown in Eq. (14), then from Eq. (25) the neutrino mass matrix in the flavor basis is given by

$$M_\nu = \begin{pmatrix} A & B & C \\ B & D & E \\ C & E & F \end{pmatrix}, \tag{27}$$

where

$$A = \frac{2m_1}{3} + \frac{m_2}{3}c_y^2 + \frac{m_3}{3}s_y^2 e^{-2i\delta}, \tag{28}$$

$$B = -\frac{m_1}{3} + m_2\left(\frac{1}{3}c_y^2 - \frac{\sqrt{6}}{6}c_y s_y e^{i\delta}\right) + m_3\left(\frac{1}{3}s_y^2 e^{-2i\delta} + \frac{\sqrt{6}}{6}s_y c_y e^{-i\delta}\right), \tag{29}$$

$$C = -\frac{m_1}{3} + m_2\left(\frac{1}{3}c_y^2 + \frac{\sqrt{6}}{6}c_y s_y e^{i\delta}\right) + m_3\left(\frac{1}{3}s_y^2 e^{-2i\delta} - \frac{\sqrt{6}}{6}s_y c_y e^{-i\delta}\right), \tag{30}$$

$$D = \frac{m_1}{6} + m_2\left(\frac{\sqrt{3}}{3}c_y - \frac{\sqrt{2}}{2}s_y e^{i\delta}\right)^2 + m_3\left(\frac{\sqrt{3}}{3}s_y e^{-i\delta} + \frac{\sqrt{2}}{2}c_y\right)^2, \tag{31}$$

$$E = \frac{m_1}{6} + m_2\left(\frac{1}{3}c_y^2 - \frac{1}{2}s_y^2 e^{2i\delta}\right) + m_3\left(\frac{1}{3}s_y^2 e^{-2i\delta} - \frac{1}{2}c_y^2\right), \tag{32}$$

$$F = \frac{m_1}{6} + m_2\left(\frac{\sqrt{3}}{3}c_y + \frac{\sqrt{2}}{2}s_y e^{i\delta}\right)^2 + m_3\left(\frac{\sqrt{3}}{3}s_y e^{-i\delta} - \frac{\sqrt{2}}{2}c_y\right)^2. \tag{33}$$

As one knows that the TBM neutrino mixing matrix leads to the neutrino mass matrix with $\mu - \tau$ symmetry as the underlying symmetry, then in the rest of the remaining parts of this paper we also impose the $\mu - \tau$ symmetry as a constraint on the obtained neutrino mass matrix M_ν from the modified TBM. By imposing $\mu - \tau$ symmetry into the neutrino mass matrix in Eq. (27) it then implies $B = C$ and $D = F$.

For the $B = C$ and $D = F$ constraints, we have the relation

$$\frac{m_2}{m_3} = e^{-2i\delta}. \tag{34}$$

By inserting Eq. (34) into Eq. (27), we then have the neutrino mass matrix in the flavor basis as follows:

$$M_\nu = \begin{pmatrix} P & Q & Q \\ Q & R & S \\ Q & S & R \end{pmatrix}, \tag{35}$$

where:

$$P = \frac{1}{3}\left(2m_1 + m_2\right), \tag{36}$$

$$Q = \frac{1}{3}(m_2 - m_1), \tag{37}$$

$$R = \frac{1}{6}\left(m_1 + m_2(2 + 3e^{2i\delta})\right), \tag{38}$$

$$S = \frac{1}{6}\left(m_1 + m_2(2 - 3e^{2i\delta})\right). \tag{39}$$

The effect of $\mu - \tau$ symmetry on the neutrino mass matrix of Eq. (27) is to reduce the number of parameters including the parameters that were introduced in the perturbation mixing matrix. From the three parameters (c_y, s_y, δ) in the perturbation mixing matrix of Eq. (13), only the Dirac phase parameter δ appears when the neutrino mass matrix in Eq. (35) is constrained by $\mu - \tau$ symmetry. The eigenvalues of the neutrino mass matrix of Eq. (35) are

$$\lambda_1 = R - S, \tag{40}$$

$$\lambda_2 = \frac{1}{2}\left(P + R + S - \sqrt{P^2 - 2RP - 2PS + R^2 + 2RS + S^2 + 8Q^2}\right), \tag{41}$$

$$\lambda_3 = \frac{1}{2}\left(P + R + S + \sqrt{P^2 - 2RP - 2PS + R^2 + 2RS + S^2 + 8Q^2}\right). \tag{42}$$

From Eqs. (40), (41), and (42), we have

$$\lambda_1 + \lambda_2 + \lambda_3 = 2R + P. \tag{43}$$

Alternatively, the Jarlskog rephasing invariant $J_{\rm CP}$ can also be determined from the relation [38]:

$$J_{\rm CP} = -\frac{{\rm Im}\left[(M_\nu^{'})_{e\mu}(M_\nu^{'})_{\mu\tau}(M_\nu^{'})_{\tau e}\right]}{\Delta m_{21}^2 \Delta m_{32}^2 \Delta m_{31}^2}, \tag{44}$$

where $(M_\nu^{'})_{ij} = (M_\nu M_\nu^\dagger)_{ij}$ and $i, j = e, \nu, \tau$. For the neutrino mass matrix constrained by $\mu - \tau$ symmetry, the value of $J_{\rm CP}$ can be obtained by inserting the values of Q and S in Eqs. (37) and (39) into Eq. (44) which gives

$$J_{\rm CP} = 0. \tag{45}$$

From Eq. (45), one can see that the exact $\mu - \tau$ symmetry leads to $J_{\rm CP} = 0$, as claimed by many authors. If one still wants to get the value of $J_{\rm CP} \neq 0$ in the scheme of $\mu - \tau$ symmetry, one must break the $\mu - \tau$ symmetry softly by introducing some parameters to break the neutrino mass matrix.

To break the $\mu - \tau$ symmetry softly, we introduce a small parameter x ($x << R$) that perturbs the neutrino mass matrix in Eq. (35) but keeps the trace of the perturbed neutrino mass matrix constant. This perturbation technique have been successfully applied by Damanik [39] to a perturbed neutrino mass matrix which is constrained by invariance under a cyclic permutation. In this perturbation scenario, the neutrino mass matrix in Eq. (35) reads

$$M_\nu = \begin{pmatrix} P & Q & Q \\ Q & R - ix & S \\ Q & S & R + ix \end{pmatrix}, \tag{46}$$

for which the trace of the perturbed M_ν remains constant: $2R+P$. The perturbed neutrino mass matrix in Eq. (46) produces

$$M_\nu^{'} = \begin{pmatrix} a & b & c \\ b^* & d & e \\ c^* & e^* & d \end{pmatrix}, \tag{47}$$

where

$$a = \left(M_\nu M_\nu^\dagger\right)_{ee} = \frac{2m_1^2 + m_2^2}{3}, \tag{48}$$

$$b = \left(M_\nu M_\nu^\dagger\right)_{e\mu} = \frac{m_2^2 - m_1^2 + ix(m_2 - m_1)}{3}, \tag{49}$$

$$c=\left(M_\nu M_\nu^\dagger\right)_{e\tau}=\frac{m_2^2-m_1^2-ix(m_2-m_1)}{3}, \tag{50}$$

$$d=\left(M_\nu M_\nu^\dagger\right)_{\mu\mu}=\frac{m_1^2+5m_2^2}{6}+x^2, \tag{51}$$

$$e=\left(M_\nu M_\nu^\dagger\right)_{\mu\tau}=\frac{m_1^2+m_2^2}{6}+ix\left(m_2(\cos(2\delta)-\frac{2}{3})-\frac{m_1}{3}\right). \tag{52}$$

Substituting Eqs. (49), (52), and the complex conjugate of Eq. (50) into Eq. (44), we have the Jarlskog invariant:

$$J_{\rm CP}=\frac{1}{9}\frac{\left[m_2(\Delta m_{21}^2)^2(1-\cos(2\delta))x+(m_2(m_2-m_1)^2\cos(2\delta)-\frac{1}{3}m_1^3-\frac{2}{3}m_2^3+m_1m_2^2)x^3\right]}{\Delta m_{21}^2\Delta m_{32}^2\Delta m_{31}^2}. \tag{53}$$

Because the value of the parameter x is very small, the terms that are related to x^3 in Eq. (53) can be neglected, and the Jarlskog invariant in this scheme is as follows:

$$J_{\rm CP}\approx\frac{1}{9}\frac{xm_2\Delta m_{21}^2(1-\cos(2\delta))}{\Delta m_{32}^2\Delta m_{31}^2}. \tag{54}$$

IV. CONCLUSION

The nonzero and relatively large θ_{13} from the latest experimental results have a serious implication on the well-known neutrino mixing matrix. One of the well-known mixing matrices is the tribimaximal (TBM) neutrino mixing matrix which predicts $\theta_{13}=0$. We modified the TBM by introducing a simple perturbation matrix into the TBM matrix and taking advantage of the experimental results of the mixing angles, we then can obtain $\theta_{13}=7.89°$, which is in agreement with the present experimental results. The Dirac phase $\delta\approx\pi/2$ and the Jarlskog rephasing invariant $J_{\rm CP}\approx 0.0222$ are also obtained. The obtained neutrino mass matrix from the modified TBM with both nonzero θ_{13} and δ is a complex neutrino mass matrix. If we impose $\mu-\tau$ symmetry, as a constraint into the neutrino mass matrix, one finds that the Jarlskog rephasing invariant $J_{\rm CP}=0$, which implies that CP violation cannot be accommodated in the $\mu-\tau$ symmetry scheme. If one still wants to use $\mu-\tau$ symmetry as the underlying symmetry for the neutrino mass matrix, one must break the $\mu-\tau$ symmetry softly. A simple way to break the $\mu-\tau$ symmetry, which can produce $J_{\rm CP}\neq 0$, is to introduce the small parameter x into the neutrino mass matrix obeying $\mu-\tau$ symmetry such that the trace of the broken $\mu-\tau$ symmetry of neutrino mass matrix remains constant.

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