The Evolution-Dominated Hydrodynamics and the Pseudorapidity Distributions in Nucleus-Nucleus Collisions at Low Energies at the BNL Relativistic Heavy Ion Collider

Zhi-Jin Jiang,1,∗ Hai-Li Zhang,1 Jie Wang,1 Ke Ma,1 and Liu-Mei Cai2

1College of Science, University of Shanghai for Science and Technology, Shanghai 200093, China
2Institute of Yanke Science and Instrument, Shanghai 200444, China
(Received June 5, 2014)

In the context of evolution-dominated hydrodynamics, we discuss the pseudorapidity distributions of the charged particles produced in Au-Au and Cu-Cu collisions at the corresponding energies of $\sqrt{s_{NN}} = 19.6$ and 22.4 GeV at BNL-RHIC. It is found that the evolution-dominated hydrodynamics alone can give a good description to the experimental measurements. This is different from that in collisions at the maximum energy of $\sqrt{s_{NN}} = 200$ GeV at BNL-RHIC or at $\sqrt{s_{NN}} = 2.76$ TeV at CERN-LHC. Where, in addition to evolution-dominated hydrodynamics, the leading particles have to be taken into account in order to adequately explain the experimental observations.

DOI: 10.6122/CJP.20140817 PACS numbers: 25.75.-q, 25.75.Ld, 24.10.Nz

I. INTRODUCTION

Because of the success in characterizing the elliptic flow and multiplicity production in particle or nucleus collisions [1–3], relativistic hydrodynamics has now been widely accepted as one of the most important tools for understanding the space-time evolution of the matter created in collisions [4–10].

Though at present there are powerful numerical methods to deal with certain hydrodynamics problems, however, this will require a very large scale of calculation and skillful sophisticated techniques to avoid instabilities. In contrast, analytical solutions, which are simple and transparent, and usually provide us with an invaluable insight into the features of matter created in collisions, are always our goal of pursuit. However, owing to the tremendous complexity of the hydrodynamics equations, the progress in finding exact hydrodynamical solutions is not going well. Up until now, the most part of this type of work only involves 1+1 dimensional flows with a simple equation of state [11–21]. The 3+1 dimensional hydrodynamics is less developed, and no general exact solutions are known so far.

An important application of 1+1 dimensional hydrodynamics is the analysis of the pseudorapidity distributions of the charged particles produced in particle or nucleus collisions. In our previous work [5], by taking into account the contributions from leading particles, we have successfully used the evolution-dominated hydrodynamic model [4] to

∗Electronic address: Jzj2650163.com
describe such distributions measured by the PHOBOS Collaboration in Au-Au and Cu-Cu collisions at the maximum energy of $\sqrt{s_{NN}} = 200$ GeV at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL), and by the ALICE Collaboration in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV at the Large Hadron Collider (LHC) at CERN. Now, what we are concerned with is that if this model can still work in nucleus-nucleus collisions at low energies. To clarify this query is just the subject of this paper. We can see that, unlike the cases at high energies, the evolution-dominated hydrodynamic model alone is enough in describing the experimental data measured at low energies at BNL-RHIC.

II. A BRIEF DESCRIPTION OF EVOLUTION-DOMINATED HYDRODYNAMICS

The key ingredients of evolution-dominated hydrodynamics are as follows:

1. The 1+1 expansion of a perfect fluid obeys the equation

\[
\frac{e^{2y} - 1}{2} \left( \frac{\partial (\varepsilon + p)}{\partial z^+} \right) + e^{2y} (\varepsilon + p) \left( \frac{\partial y}{\partial z^+} \right) + \frac{1 - e^{-2y}}{2} \left( \frac{\partial (\varepsilon + p)}{\partial z^-} \right) + e^{-2y} (\varepsilon + p) \left( \frac{\partial y}{\partial z^-} \right) + \frac{\partial p}{\partial z^+} - \frac{\partial p}{\partial z^-} = 0,
\]

\[
\frac{e^{2y} + 1}{2} \left( \frac{\partial (\varepsilon + p)}{\partial z^+} \right) + e^{2y} (\varepsilon + p) \left( \frac{\partial y}{\partial z^+} \right) + \frac{1 + e^{-2y}}{2} \left( \frac{\partial (\varepsilon + p)}{\partial z^-} \right) + e^{-2y} (\varepsilon + p) \left( \frac{\partial y}{\partial z^-} \right) + \frac{\partial p}{\partial z^+} - \frac{\partial p}{\partial z^-} = 0,
\]

where $\varepsilon$, $p$, and $y$ are, respectively, the energy density, pressure, and ordinary rapidity of the fluid. $z^\pm = t \pm z = x^0 \pm x^1 = \tau e^{\pm \eta}$ are the light-cone coordinates, $\tau = \sqrt{z^+ z^-}$ is the proper time, and $\eta = 1/2 \ln(z^+/z^-)$ is the space-time rapidity of the fluid.

Eq. (1) is complicated, non-linear, and coupled. In order to solve it, one introduces the Khalatnikov potential $\chi$ [13], which relates to $z^\pm$, $\tau$, and $\eta$ by the equations

\[
z^\pm(\theta, y) = \frac{1}{2T_0} e^{\theta z y} \left( \frac{\partial x}{\partial \theta} \pm \frac{\partial x}{\partial y} \right),
\]

\[
\tau(\theta, y) = \frac{e^\theta}{2T_0} \sqrt{\left( \frac{\partial x}{\partial \theta} \right)^2 - \left( \frac{\partial x}{\partial y} \right)^2},
\]

\[
\eta(\theta, y) = y + \frac{1}{2} \ln \left( \frac{\partial x/\partial \theta + \partial x/\partial y}{\partial x/\partial \theta - \partial x/\partial y} \right),
\]

where

\[
\theta = \ln \left( \frac{T_0}{T} \right),
\]

\[
\eta = \frac{1}{2} \ln \left( \frac{z^+/z^-}{} \right).
\]
\( T \) is the temperature of the fluid, and \( T_0 \) is its initial scale. In terms of \( \chi \), Eq. (1) can be reduced to
\[
\frac{\partial^2 \chi(\theta, y)}{\partial \theta^2} - \frac{[g(\theta) - 1]}{g(\theta)} \frac{\partial \chi(\theta, y)}{\partial \theta} - g(\theta) \frac{\partial^2 \chi(\theta, y)}{\partial y^2} = 0,
\]
where \( 1/\sqrt{g(\theta)} = c_s(\theta) \) is the speed of sound. The above equation is now a linear second-order partial differential equation, which works for any form of \( g(\theta) \).

(2) Experimental investigations have shown that the speed of sound is a constant of about \( c_s = 0.35 \) or \( g = 8.16 \) \([22]\), which is almost independent of the interaction energy and system \([23–25]\). In this case, Eq. (5) has the solution
\[
\chi(\theta, y) = \frac{e^{\frac{\theta - 1}{2} \theta}}{4\sqrt{g}} \int_0^\theta \int_0^{y-(y-y')/\sqrt{g}} d\theta' F(\theta', y') I_0 \left( \frac{g - 1}{2} \sqrt{(\theta - \theta')^2 - \frac{(y - y')^2}{g}} \right),
\]
where \( F(\theta', y') \) stands for the initial distribution of the source of fluid.

(3) In collisions at high energy, owing to the violent compression and Lorentz contraction of the interaction system along the beam direction, the initial pressure gradient of the created matter in this direction is very large. In contrast, the effect of the initial motion of the source is negligible. The flow of fluid is mainly dominated by the following evolution. In this evolution-dominated picture, the initial distribution of the source takes the form \([4, 5, 12]\)
\[
F(\theta', y') = Ce^{-\frac{\theta - 1}{2} \theta} \Theta(\theta') \delta(y'),
\]
where \( C \) is a constant. Inserting it into Eq. (6), it reads
\[
\chi(\theta, y) = Ce^{-\theta} \int_{y/\sqrt{g}}^{\theta} d\theta' e^{\frac{\theta - 1}{2} \theta'} I_0 \left( \frac{g - 1}{2} \sqrt{\theta'^2 - \frac{y^2}{g}} \right).
\]

III. THE RAPIDITY DISTRIBUTIONS OF THE CHARGED PARTICLES IN NUCLEUS-NUCLEUS COLLISIONS

From the Khalatnikov potential of Eq. (8), we can obtain the rapidity distributions of the charged particles produced in nucleus-nucleus collisions. To this end, we first evaluate the entropy distributions at the freeze-out temperature \( T_{FO} = T_0 e^{-\theta_{FO}} \) as a function of the rapidity \( y \). The entropy distributions at the freeze-out temperature are defined as the amount of entropy flowing through a space-like hypersurface with a fixed temperature \( T_{FO} \) in a unit rapidity interval. They take the form \([4, 5]\)
\[
\frac{dS}{dy} = s_{FO} u^\mu \frac{d\lambda\mu}{dy} = s_{FO} u^\mu n^\nu \frac{d\lambda}{dy},
\]
where $n^\mu$ is the 4-dimensional unit vector of the hypersurface,

$$n^\mu n_\mu = n^+ n^- = 1. \quad (10)$$

d\lambda is the space-like slab element along hypersurfaces defined as $d\lambda^\mu = d\lambda n^\mu$ meeting

$$d\lambda^2 = d\lambda^\mu d\lambda_\mu = -dz_{FO}^2 - dz_{FO}^2 + \cdots \quad (11)$$

where the minus sign accounts for the space-like characteristic of $d\lambda$.

In the $(\theta, y)$-base, the fixed-temperature hypersurface can be defined by

$$\tau_{FO}(y) = \tau(\theta_{FO}, y),$$

$$\eta_{FO}(y) = \eta(\theta_{FO}, y).$$

The tangential vector of the hypersurface is

$$t^+(y) \equiv z_{FO}'(y) = (\tau_{FO}' + \eta_{FO}' \tau_{FO}) e^{\eta_{FO}},$$

$$t^-(y) \equiv z_{FO}'(y) = (\tau_{FO}' - \eta_{FO}' \tau_{FO}) e^{-\eta_{FO}}, \quad (12)$$

where the primes represent the derivatives with respect to $y$. According to the definitions,

$$n^\mu(y) t_\mu(y) = \frac{1}{2} [n^+(y) t^-(y) + n^-(y) t^+(y)] = 0.$$

Owing to Eq. (12), the above equation turns into

$$n^+(y) (\eta_{FO}' \tau_{FO} - \tau_{FO}') e^{-\eta_{FO}} = n^-(y) (\eta_{FO}' \tau_{FO} + \tau_{FO}') e^{\eta_{FO}}.$$

This equation together with Eq. (10) gives

$$n^+(y) = \sqrt{\frac{\eta_{FO}' \tau_{FO} + \tau_{FO}' e^{\eta_{FO}}}{\eta_{FO}' \tau_{FO} - \tau_{FO}' e^{-\eta_{FO}}}},$$

$$n^-(y) = \sqrt{\frac{\eta_{FO}' \tau_{FO} - \tau_{FO}' e^{-\eta_{FO}}}{\eta_{FO}' \tau_{FO} + \tau_{FO}' e^{\eta_{FO}}}}. \quad (13)$$

Eq. (12) translates Eq. (11) into

$$d\lambda = \sqrt{\frac{\tau_{FO}^2 + \tau_{FO}' e^{\eta_{FO}}}{\eta_{FO}' \tau_{FO} - \tau_{FO}' e^{-\eta_{FO}}}} dy. \quad (14)$$

Making use of Eq. (13), we obtain

$$u^\mu n_\mu = \frac{1}{\sqrt{\frac{\tau_{FO}^2 + \tau_{FO}' e^{\eta_{FO}}}{\eta_{FO}' \tau_{FO} - \tau_{FO}' e^{-\eta_{FO}}}}} \left[\eta_{FO}' \tau_{FO} \cosh (\eta_{FO} - y) + \tau_{FO}' \sinh (\eta_{FO} - y)\right]. \quad (15)$$
Using Eqs. (14) and (15), Eq. (9) reads
\[
\frac{dS}{dy} = s_{\text{FO}} \left[ \eta'_{\text{FO}} \tau'_{\text{FO}} \cosh (\eta_{\text{FO}} - y) + \tau'_{\text{FO}} \sinh (\eta_{\text{FO}} - y) \right].
\] 
(16)

Furthermore, from Eq. (2) it is known that
\[
\cosh(\eta - y) = \frac{e^{\theta}}{2 \tau T_0} \frac{\partial \chi}{\partial \theta} \bigg|_{\theta = \theta_{\text{FO}}},
\]
\[
\sinh(\eta - y) = \frac{e^{\theta}}{2 \tau T_0} \frac{\partial \chi}{\partial y} \bigg|_{\theta = \theta_{\text{FO}}},
\] 
(17)

From Eq. (3) one can deduce
\[
\tau'_{\text{FO}} = \frac{e^{\theta}}{2 T_0} \left( \frac{\partial \chi}{\partial \theta} \right) \left( \frac{\partial^2 \chi}{\partial \theta \partial y} - (\partial \chi/\partial y) \left( \frac{\partial^2 \chi}{\partial y^2} \right) \right) \bigg|_{\theta = \theta_{\text{FO}}},
\]
\[
\eta'_{\text{FO}} = \frac{(\partial \chi/\partial \theta)^2 - (\partial \chi/\partial y)^2 - (\partial \chi/\partial y) (\partial^2 \chi/\partial \theta \partial y) + (\partial \chi/\partial \theta) (\partial^2 \chi/\partial y^2)}{(\partial \chi/\partial \theta)^2 - (\partial \chi/\partial y)^2} \bigg|_{\theta = \theta_{\text{FO}}}. 
\] 
(18)

These two equations together with Eq. (5) make Eq. (16) become
\[
\frac{dS}{dy} = s_{\text{FO}} \frac{2 T_{\text{FO}} T_0}{g} \left[ \frac{\partial^2 \chi(\theta, y)}{\partial \theta^2} + \frac{\partial \chi(\theta, y)}{\partial \theta} \right] \bigg|_{\theta = \theta_{\text{FO}}},
\] 
(19)

For evolution-dominated hydrodynamics, substituting Eq. (8) into the above equation, we acquire
\[
\frac{dS}{dy} = s_{\text{FO}} (g - 1) C e^{\frac{\theta - 1}{T_{\text{FO}}}} \times \left[ I_0 \left( g - \frac{1}{2} \sqrt{\theta_{\text{FO}}^2 - y^2 / g} \right) + \frac{\theta_{\text{FO}}}{\sqrt{\theta_{\text{FO}}^2 - y^2 / g}} I_1 \left( g - \frac{1}{2} \sqrt{\theta_{\text{FO}}^2 - y^2 / g} \right) \right],
\] 
(20)

where $\theta_{\text{FO}} = \ln(T_0/T_{\text{FO}})$. It is related to the initial temperature of the fluid and is therefore dependent on the incident energy and collision centrality. Its specific value can be fixed by fitting the theoretical predictions with the experimental measurements.

As the entropy is proportional to the number of charged particles, we then obtain the rapidity distributions:
\[
\frac{dN(b, \sqrt{s_{\text{NN}}}, y)}{dy} = C(b, \sqrt{s_{\text{NN}}}) \left[ I_0 \left( g - \frac{1}{2} \sqrt{\theta_{\text{FO}}^2 - y^2 / g} \right) + \frac{\theta_{\text{FO}}}{\sqrt{\theta_{\text{FO}}^2 - y^2 / g}} I_1 \left( g - \frac{1}{2} \sqrt{\theta_{\text{FO}}^2 - y^2 / g} \right) \right],
\] 
(21)

where $C(b, \sqrt{s_{\text{NN}}})$, independent of the rapidity $y$, is an overall normalization constant. $b$ is the impact parameter, and $\sqrt{s_{\text{NN}}}$ is the center-of-mass energy per pair of nucleons.
IV. COMPARISONS WITH EXPERIMENTAL DATA

Given the rapidity distribution of Eq. (21), the pseudorapidity distributions of charged particles measured in experiments can be expressed as [26–28]

\[
\frac{dN(b, \sqrt{s_{NN}}, \eta)}{d\eta} = \sqrt{1 - \frac{m^2}{m_T^2 \cosh^2 y}} \frac{dN(b, \sqrt{s_{NN}}, y)}{dy},
\]

(22)

\[
y = \frac{1}{2} \ln \left[ \frac{\sqrt{p_T^2 \cosh^2 \eta + m^2 + p_T \sinh \eta}}{\sqrt{p_T^2 \cosh^2 \eta + m^2 - p_T \sinh \eta}} \right],
\]

(23)

where \(p_T\) is the transverse momentum, and \(m_T = \sqrt{m^2 + p_T^2}\) is the transverse mass.

Substituting Eq. (21) into Eq. (20), we can get the pseudorapidity distributions of the charged particles. Figures 1 and 2 show such distributions for different centrality Au-Au and Cu-Cu collisions at \(\sqrt{s_{NN}} = 19.6\) and 22.4 GeV, respectively. The solid dots are the experimental measurements [29]. The solid curves are the results from the evolution-dominated hydrodynamics of Eq. (21). In calculations, the parameter \(\theta_{FO}\) in Eq. (21) takes the values as listed in Table I. It can be seen that, for a given incident energy and colliding system, \(\theta_{FO}\) increases with centrality cuts. It should, since \(\theta_{FO}\) in Eq. (21) determines the widths of the distributions, which increase with centrality cuts (c.f., Figs. 1 and 2). Since \(\theta_{FO} = \ln(T_0/T_{FO})\), Table I means that the ratio of \(T_0/T_{FO}\) increases with centrality cuts.

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<tbody>
<tr>
<td>(\theta_{FO}) (Au-Au)</td>
<td>1.15</td>
<td>1.22</td>
<td>1.34</td>
<td>1.45</td>
<td>1.51</td>
<td>1.59</td>
<td>1.65</td>
<td>1.68</td>
<td>1.72</td>
<td>1.75</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>(\theta_{FO}) (Cu-Cu)</td>
<td>1.31</td>
<td>1.37</td>
<td>1.42</td>
<td>1.49</td>
<td>1.55</td>
<td>1.63</td>
<td>1.69</td>
<td>1.75</td>
<td>1.81</td>
<td>1.88</td>
<td>1.96</td>
<td>2.02</td>
</tr>
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</table>

Figs. 1 and 2 shows that the evolution-dominated hydrodynamics alone can give a good description for the pseudorapidity distributions of the charged particles measured in nucleus-nucleus collisions at low energies at BNL-RHIC. This is unlike the situations in collisions at the maximum energy of \(\sqrt{s_{NN}} = 200\) GeV at BNL-RHIC or at \(\sqrt{s_{NN}} = 2.76\) TeV at CERN-LHC, where the leading particles [30] are essential in explaining the experimental measurements [5]. The reasons for this difference may lie in the degree of transparency of participants at different incident energies. It is known from Ref. [31] that, in the top 5% most central Au-Au collisions at \(\sqrt{s_{NN}} = 200\) GeV, the rapidity loss of the participants is up to \(\langle \delta y \rangle \approx 2.45\), then the leading particles should locate at

\[
y_0 = y_{\text{beam}} - \langle \delta y \rangle = 5.36 - 2.45 = 2.91.
\]
FIG. 1: The pseudorapidity distributions of the charged particles produced in different centrality Au-Au collisions at $\sqrt{s} = 19.6$ GeV. The solid dots are the experimental measurements [29]. The solid curves are the results obtained from the evolution-dominated hydrodynamics of Eq. (21).

This position is so far away from the mid-rapidity that the effect of leading particles should be taken into account separately, since at rapidity near $y_0$ this effect is evidently due to the low yield of charged particles resulting from the freeze-out of fluid. In this case, the mid-rapidity region is nearly net baryon-free, or the participants are almost transparent. On the other hand, for collisions at low energies, the rapidity loss is about $\langle \delta y \rangle \approx 0.58 y_{\text{beam}}$. Thus the leading particles should locate at

$$y_0 = y_{\text{beam}} - \langle \delta y \rangle = 0.42 y_{\text{beam}} = \left\{ \begin{array}{l} 1.28, \ (\sqrt{s_{\text{NN}}} = 19.6 \text{ GeV}) , \\ 1.33, \ (\sqrt{s_{\text{NN}}} = 22.4 \text{ GeV}) . \end{array} \right.$$
FIG. 2: The pseudorapidity distributions of the charged particles produced in different centrality Cu-Cu collisions at $\sqrt{s} = 22.4$ GeV. The solid dots are the experimental measurements [29]. The solid curves are the results obtained from the evolution-dominated hydrodynamics of Eq. (21).

These positions are so close to the mid-rapidity that the effect of leading particles does not need to be considered separately, since in the vicinities of above $y_0$ this effect is hidden by the large yield of charged particles resulting from the freeze-out of fluid. In this case, the mid-rapidity region is high-baryon dense, or the participants are almost full stopping.
V. CONCLUSIONS

The matter with high temperature and density created in nucleus-nucleus collisions is assumed to expand according to evolution-dominated hydrodynamics. Compared with the great pressure gradient along the longitudinal direction, the effect of initial motion of the matter is negligible. Its expansion is mainly governed by the following evolution. This thus guarantees the rationality of the evolution-dominated hydrodynamic model. With the scheme of the Khalatnikov potential, this model can be solved analytically. The exact solution is then applied to build the rapidity distributions of the charged particles in terms of the 0th and 1st order modified Bessel function of the first kind with only two parameters $g = 1/c_s^2$ and $\theta_{FO} = \ln(T_0/T_{FO})$. $g$ takes the value from experiments. $\theta_{FO}$ is determined by comparing the theoretical results with the experimental data.

Due to the poor transparency of participants in nucleus-nucleus collisions at low energies, there is no need in this case to take the effect of leading particles into account separately. Comparing with the pseudorapidity distributions of the charged particles measured by the PHOBOS Collaboration at BNL-RHIC in Au-Au and Cu-Cu collisions at respective energies of $\sqrt{s_{NN}} = 19.6$ and 22.4 GeV, we can see that the results from evolution-dominated hydrodynamics are in good accord with the experimental data.

Acknowledgments

This work is partly supported by the Transformation Project of Science and Technology of Shanghai Baoshan District with Grant No. CXY-2012-25, the Shanghai Leading Academic Discipline Project with Grant No. XTKX 2012, the National Training Project with Grant No. 14XPM03 and the Hujiang Foundation of China with Grant No. B14004.

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