Effects of Two Distinct Types of Noise on the Directional Alignment of a Self-Propelled Particles System

Dorilson Silva Cambui

Secretaria de Estado de Educação de Mato Grosso, 78049-909, Cuiabá, Mato Grosso, Brazil
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An important tool for describing the collective motion of autonomous particles is based on the concept of self-propelled particles, where each particle interacts locally with its neighbors, subject to the interference of some noise. There are two ways to introduce noise into the system: either during (extrinsic noise) or after (intrinsic noise) the interactions have occurred. In this work, we present a study of the effects of these two types of noise on the directional order of self-propelled particles. Our numerical investigations show that the directional alignment degree is strongly influenced by the way in which one introduces the noise.

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I. INTRODUCTION

Collective behavior, displayed by many types of individuals and arising from diverse social, biological, and physical systems, is a fascinating characteristic [1, 2]. In nature, many organisms display ordered collective motion, e.g., flocks of birds, schools of fish, bacterial populations, ants, and pedestrian motions [3–7]. Another subject of intense investigation is the influence of noise on cell movement or motility, where stochastic resonance, a well-known phenomenon in such systems, shows the possibility of transforming noise into order [8]. A recent study of the fluctuation kinetics of cells showed which fluctuations along the forward-moving direction can be interpreted as the noise strength being modified by external stimuli [9], and cells undergo a damped motion with multiplicative noise [10].

An intrinsic feature of collective behavior in living systems is the ability that certain biological groups have to align themselves and move in the same direction. To describe these phenomena, there has emerged an increasing interest in finding simple models that can describe the collective motion of groups of biological agents [11, 12]. Self-propelled particle (SSP) models have become an important tool in various studies involving collective motion. A variety of studies have been devoted to such systems, in order to model and explore their properties and to contribute to the understanding of the collective motion dynamics [3–16]. In order to study some aspects of the emergence of collective motion, Vicsek et al. [13] proposed a minimal model to mimic the dynamics of biological swarms. In this model, each individual moves at the same speed \( v \) and in the average direction of...
motion of the particles in its neighborhood of radius $R$.

It is interesting to note that, in nature, no perfect movement can be observed. In view of this fact, self-propelled particle models have incorporated random perturbations (noise) into the individual’s motion direction. The noise represents possible “measurement” errors, committed by a particle, when evaluating the motion direction of its neighbors. When noise is introduced, self-propelled particle systems display a phase transition from a disordered state (random directions) to a state of ordered motion.

There are two ways to introduce noise into the system: either during or after the interactions between the particles, and these two interaction rules can produce different types of phase transitions [17]. These two types of noise we call intrinsic and extrinsic noise; they are also referred to by some authors as angular and vectorial noise, respectively. (For more details, see the paper by Aldana et al., Ref. [18] below). We can define intrinsic noise as that arising from the errors committed when particles try to follow the average direction of motion, already perfectly calculated, of neighboring particles. Extrinsic, or vectorial, noise is the randomness that arises from the evaluation of each particle-particle interaction, i.e., the noise is added directly to the interactions between the particle $i$ and each one of its neighbors.

There are systems of self-propelled particles with multiple interaction radii, in which attraction terms and/or repulsion terms between the particles are considered [14, 19]. Based on the position and alignment of an individual’s nearest neighbors, the behavior of the individual emerges from the simple interactions of repulsion, alignment, and attraction.

However, the aim of this study is to investigate the effects of noise (intrinsic or extrinsic) on the original formulation of Vicsek [13]. We are particularly interested in determining the effect that each type of noise can have on the intensity of the directional orientation of the particles. To this end, we examined, as a function of noise, some global statistical quantities that characterize the collective behavior of biological entities, like polarization, which gives the deviation of each agent’s orientation from the group’s average direction.

The remainder of this paper is structured as follows: The detailed formulation of a self-propelled particle model with intrinsic noise, originally used in [13], and a model with extrinsic noise introduced in [15] are presented in the next section. In Section III, we present our results, and discussions are given in Section IV.

II. SELF-PROPELLED PARTICLES

In self-propelled particle models, individuals move with a constant speed, adopting at each time step the average motion direction of the other particles in their local neighborhood. The interactions that occur with their nearest neighbors are subject to some noise. It is important to point out that random perturbations in the direction of motion represent errors committed by a particle when evaluating the real direction of motion of its neighbors. The term ‘neighbor’ will refer only to those particles inside the interactional neighborhood of the particle in question.
When noise is introduced, the model exhibits a phase transition. The nature of the phase transition (whether continuous or discontinuous) in self-propelled particles depends on the type of noise (intrinsic or extrinsic) introduced. Intrinsic noise leads to a continuous phase transition while extrinsic noise makes the transition discontinuous [17, 18]. The phase transition characterized by an order parameter, occurs from a disordered motion state to an ordered state, where the particles move collectively in the same direction.

In the subsections below, there is a brief description of self-propelled particle models with both types of noise, intrinsic and extrinsic.

II-1. SPP Model with Intrinsic Noise

This section gives a brief description of the dynamics of self-propelled particles, employed by Vicsek et al. [13] to describe the collective phenomena of biological organisms. In this model, the motion of self-propelled particles is driven by very simple rules. Organisms are represented by $N$ identical point-like particles characterized, in $t$ time, by position $x_i(t)$ and velocity $v_i(t)$, with $i = 1, 2, 3 \ldots N$, and move in the direction $\theta_i$ in a two-dimensional space.

The modulus of the velocity of all individuals is assumed to be constant, i.e., each agent moves with the same speed $v_0 = 0.03$. Initially the positions and directions are randomly distributed in a square box of sides $L$, with periodic boundary conditions. The interactions between the particles occur in a region of radius $R$, where each individual acquires the average direction of motion of all particles (including $i$ itself) in the neighborhood centered on $i$. The interaction rules, in the Vicsek model, take place at discrete time steps of length $\tau$. Then, at each time step, new positions of the particles are updated according to,

$$x_i(t + \tau) = x_i(t) + v_i(t)\tau. \quad (1)$$

Within the interaction range $R$, disturbed by the presence of noise, the updated direction of motion for the $i$th individual, given by the angle of motion $\theta_i^{t+\tau}$, is determined by,

$$\theta_i^{t+\tau} = \text{arg} \left[ \sum_{(i,j)} e^{i\theta_j^t} + \eta \xi_i \right]. \quad (2)$$

The sum is performed over the $i$th particle and all its neighboring $j$ particles. $\xi$ is a white noise uniformly distributed ($\xi \in [-\pi, \pi]$) and $\eta$ is the strength of the noise. In the first term of Eq. (2), the interaction between the particles occurs; the particles will try to align themselves and move in a common direction. The second term represents errors in “measurement” committed by the particle $i$ when adjusting its direction to the average motion direction of its neighbors. Note that the average motion direction is computed first and then the noise is added.

In order to characterize the collective behavior of the particles, the order parameter adopted to measure the phase transition is the absolute value of the normalized mean
velocity, given by,

\[ \psi = \frac{1}{Nv_0} \left| \sum_{i=1}^{N} v_i(t) \right| . \]  

(3)

### II-2. SPP Model with Extrinsic Noise

The description can be summarized as follows: Proposed by Grégoire et al. [15], errors in “measurement” can be made when a particle, which does not see its neighbors very well, estimates the directions in which they are moving (noisy environment). In other words, the noise arises from each interaction between the \( i \)th particle and each of its neighbors \( j \). In this respect, one has to consider the following updated rule:

\[ \theta_i^{t+\tau} = \arg \left[ \sum_{\langle i,j \rangle} e^{i\theta_j^t} + \eta n_i e^{i\epsilon_i^t} \right], \]  

(4)

where \( n_i \) is the current number of neighbors of particle \( i \). Unlike the rule given by Eq. (2), here the noise is added directly in each particle-particle interaction.

The difference between the interaction rules given by Equations (2) and (4) consists in the way in which the noise is introduced: outside or inside the angle function. Here, the noise is included in a different way, by adding a vector (in a random direction \( \eta e^{i\epsilon_i^t} \)) to the sum, and the index \( \langle i,j \rangle \), indicates that summation is performed over the \( i \)th particle and all its neighboring \( j \) particles. These two rules lead to the different types of phase transitions [17].

### III. SIMULATION PARAMETERS AND RESULTS

Simulations are made (for the original Vicsek model) using intrinsic and extrinsic noise disturbances, given respectively by the update rules (2) and (4). Parameters like the modulus of the velocity \( v_0 \), interaction radius \( R \), and constant time interval \( \tau \), are in the same range as the ones used in [13], with \( v_0 = 0.03 \), \( R = 1 \) and time steps \( \tau = 1 \). We start the simulations with a random distribution of the positions and directions of \( N \) particles moving in a lattice of linear size \( L = 10 \), with periodic boundary conditions. In the following section, we show the definitions and results for the order parameter \( \psi \), the Binder cumulant \( C \), the nearest neighbors distance \( s_i \), and the polarization \( p \). Our measurements have been computed over \( 1 \times 10^6 \) time steps.

#### III-1. Binder cumulant

It is well known in self-propelled particle systems that intrinsic or extrinsic noise produces different phase transitions. Used to characterize the nature of the phase transition, the Binder cumulant displays a minimum characteristic for first-order phase transitions, while its continuous behavior indicates a second-order phase transition [20]. Fig. 1 shows the plot of the fourth order cumulant of the order parameter, obtained for combinations of the
distinct types of noise described in the previous section. This is given by

$$C = 1 - \left[ \frac{\langle \phi^4 \rangle_t}{3 \langle \phi^2 \rangle_t^2} \right].$$

FIG. 1: Binder cumulant with $L = 10$ and $N = 1000$. Note that by the behavior of $C$ in SPP models with intrinsic noise the phase transition seems to be continuous, whereas in SPP models with extrinsic noise it is discontinuous.

The continuous behavior of $C$ suggests, as pointed out in [13], that the system undergoes a continuous phase transition. On the other hand, in the case of extrinsic noise, $C$ exhibits a minimum point; in this case the phase transition is clearly discontinuous.

III-2. Polarization

We are interested in verifying the intensity of the directional orientation of the particles as a function of both types of noise, intrinsic and extrinsic. To this end, we defined the polarization $p$ as the arithmetic mean deviation of the angle of a particle in relation to the total group of particles present in a system [21],

$$p = \left[ \frac{v_i}{|v_i|}, \sum_{i=1}^{N} \frac{v_i}{|v_i|} \right],$$

where the first term of Eq. (5) represents the direction of the velocity of each particle $i$. In the second term, we have the direction of the velocities of the total group of particles. $p$ quantifies the intensity of the parallel orientation, and may assume values between $0^\circ$ and $90^\circ$. $p = 0^\circ$ indicates that the group of particles has ordered motion, and with $p = 90^\circ$ we have a configuration in which the particles have their directions of motion in a completely unaligned state. This measure is computed by using the dot-product between the vector pair. If we let $\mathbf{A} = v_i$ and $\mathbf{B} = \sum_{i=1}^{N} v_i$, since these vectors and their lengths are known at all times, we can solve for $\cos \phi = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}| |\mathbf{B}|}$, and find the angle $\phi$ between the vectors (which we call the polarization $p$) by taking the inverse cosine.
Figure 2: Temporal evolution of the polarization for $N = 200$ and $L = 10$.

Figure 2 presents the temporal evolution of the polarization. For $\tau = 0$ it implies $p = 90^\circ$, i.e., the particles are in disordered motion. As time passes, in a range time too short, the polarization starts to oscillate around the mean value, indicating a stabilization of the motion with a degree of deviation greater when the noise added is of an intrinsic type.

The influence of density on the directional alignment of the particles is shown in Fig. 3a, which exhibits the polarization as a function of density $\rho = N/L^2$, with the noise kept constant. One can see that the system gains order at high densities since the polarization gradually diminishes. For a low value of noise ($\eta = 0.01$), the curves of the polarization, for both types of noise, practically coincide. However, by increasing the noise length ($\eta = 0.3$) the polarization stabilization occurs for different levels depending on the type of noise chosen. The order is more effective when the noise introduced is extrinsic.

This same behavior is seen in Fig. 3b, which shows us the polarization as a function of the orientation radius size $R$. We changed $R$ until it equaled the size $L$ of the lattice. For a small $R$ we have few particles interacting, such that the deviation is higher. For $R = L$ all the particles interact strongly with each other, resulting in a low polarization. When the noise introduced is extrinsic, the polarization stabilization occurs for smaller levels, (see Fig. 3), indicating the formation of a more compact group, with alignment evidently more effective. Smaller values of $p$ indicate an ordered motion of particles and, from a biological point of view, means a greater cohesion and stability of animal aggregates. In a realistic system, the effect observed of extrinsic noise on directional alignment, with polarization stabilization occurring at smaller levels, indicates that such aggregates move as a coherent entity with strongly aligned collective motion.

A principle of organization in biological aggregates is the existence of a region at nearest neighbor distances [22] (NND), where an individual uses the distance as a relevant measure of influence and moves towards its nearest neighbors. Fish, for example, interact differently according to the range of distances in which a neighbor is positioned [12], and swim closer when the density is larger. We observed that for larger densities, the nearest-
FIG. 3: Polarization in function of: a) density $\rho$, with $L = 10$ and ($20 \leq N \leq 3000$); b) interaction radius $R$, with $L = 10$, $N = 1000$ and ($0.01 \leq R \leq L$). Note that the polarization, for small noise ($\eta = 0.01$), is equivalent for the two types of noise considered.

neighbor distance distribution between the fish is narrower [23], indicating a more coherent motion. In this sense, we intend now to verify the relationship between these three different properties: polarization, nearest-neighbor distance, and density of particles as a function of the two types of noise analyzed. It is important to consider that, given that the system with extrinsic noise presents a lower level of polarization (see Fig. 3), it is expected that it presents smaller distances between neighboring particles.

The mean nearest-neighbor distance is defined by

$$\langle s \rangle = \frac{1}{N} \sum_{i=1}^{N} (s_i),$$

where the distance $s_i = \min(|x_j(t) - x_i(t)|)$ for any particle $i$, is found by computing the minimum distance between $i$ to all its neighbors $j$.

When individuals are close enough, they interact by changing their direction of motion, exhibiting an effective alignment at short distances. We can observed in Fig. 4 that,
FIG. 4: Polarization as a function of the mean nearest neighbor distance for various densities. Each point on the graph corresponds to a density that ranged from $\rho = 0.2$ to $\rho = 30$. The parameters are: $L = 10$ and $\eta = 0.3$.

independently of the way in which the noise is introduced, lower polarization (better alignment) of the agents is observed for smaller distances (strong cohesion), by increasing the density and for the same noise strength. However, for the system in which the noise was added vectorially, we observed a lower level of polarization, where the directional orientation and cohesion of the particles increases significantly. Still in Fig. 4, it is shown that the value of $p$ increases as the mean minimum distance (NND) increases, and it decreases as the density and NND decreases, leading to collective states in which the agents move in the same direction.

Although the noise was introduced into the system differently [see Equations (2) and (4)], we realized the comparisons in order to verify their effects on the directional alignment degree between self-propelled particles. The systems contained the same noise length, since we are interested in investigating how the system behaves (via polarization $p$) in this particular case, see Figs. 3 and 4. An especially meaningful effect is the one that refers to the phase transition, in which two different types of noise can produce different order-disorder phase transitions. To illustrate this, Fig. 1 shows typical phase transition curves dependent on the same amplitude $\eta$ for both types of noise, intrinsic and extrinsic.

IV. CONCLUSIONS

The principal aim of this study was to investigate the effects that two distinct types of noise, extrinsic and intrinsic, have on orientational ordering in a two-dimensional self-propelled particles system, in the original formulation of the Vicsek model.

We measured the polarization $p$, which quantifies the intensity of the directional orientation. Higher $p$ values indicate greater disorder, whereas smaller values indicate ordered motion. In our simulations, regardless of the type of noise incorporated, the value
of $p$ increases as the mean minimum distance (NND) increases; the value of $p$ decreases as the density and NND decreases, leading to collective states in which the agents move in the same direction. Our results indicate that both models, with extrinsic and intrinsic noise, agreed reasonably well in the amount of polarization for small enough noise ($\eta = 0.01$; see Fig. 3), while large noise produces different levels of polarization. In this case, the formation of a strongly aligned and cohesive group is more and more pronounced when the noise added is extrinsic.

In fact, it was observed that at the higher level of noise, ($\eta = 0.3$), directional alignment (low $p$) applies to the case of extrinsic noise, whereas it is decreased (high $p$) for intrinsic noise. The angular noise is added to the average motional direction of the neighboring particles after it has been perfectly measured by the particle, which may then decide to move in a different direction. This leads us to conclude that angular noise does not depend on the degree of local order given by the alignment interactions, and for this reason can be thought of as having a strong influence in the directional orientation of the system. In contrast, in the case of extrinsic noise, the noise is added directly into the particle-particle interactions, where the particle, after calculating the motional direction, follows it instead of moving in a different direction. One could argue, in this case, that the effects of the noise (on the orientational order) in this interactional mechanism between particles, tends to be reduced, i.e., its influence decreases, with increasing local order.

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References


